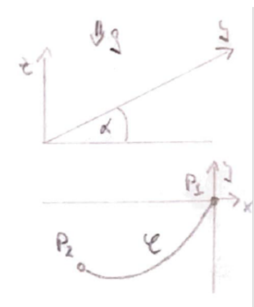
PHYS 622 Classical Electrodynamics W 2021

Problem Assignment # 1 01/08/2021
due 01/15/2021

0.2.1 **The brachistrochrone problem**

A point mass glides without friction on an inclined plane (inclination angle α) from point P_1 to point P_2 on a path $\mathfrak{C}$ according to Galilean mechanics.



a) Use energy conservation to find the velocity as a function of y, using the coordinate system in the sketch.

b) Write the passage time from P_1 to P_2 in the form

$$T = \int_{x_1}^{x_2} dx\, L(y, y')$$

with y considered a function of x and $y' = dy/dx$, and determine the Lagrangian L. Use the fact that Jacobi's integral is constant to find an ODE for y.

c) Substitute $y' = \operatorname{ctg} t$ to write the brachistochrone in a parametric form, $y = y(t)$, $x = x(t)$.

d) Express the passage time as a function of the terminal y-velocity, $y_2' = (dy/dx)_{P_2}$.

e) Find the passage time for the shortest path from P_1 to P_2 (as opposed to the brachistochrone) as a function of the same y_2'.

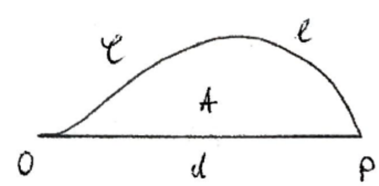
f) Discuss the ratio of the two passage times as a function of y_2'.

hint: The parameter value t_2 for the brachistochrone at the end point P_2 is a known function of y_2'. It therefore suffices to discuss the passage time as a function of t_2.

(18 points)

0.2.2 **Dido's problem**

An area A in the x-y-plane is enclosed by straight line between two points O and P that are a distanced d apart, and a path $\mathfrak{C}$ with end points O and P and length $\ell > d$. Find the path $\mathfrak{C}$ that maximizes A.



(6 points)

0.2.3 **Geodesics on the 2-sphere**

Show that the geodesics on the 2-sphere are great circles.

hint: There are various ways of doing this. One is to set up the problem of geodesics in $\mathbb{R}_3$ with the constraint that the desired paths $\vec{x}(t)$ must lie on the sphere. Now use the Euler-Lagrange equations for the constrained problem to show that $\vec{\ell} = \vec{x} \times \vec{p} = \text{const}$, where $\vec{p} = \partial L/\partial \dot{\vec{x}}$, with L the appropriate Lagrangian.

(5 points)