PHYS 623 Classical Electrodynamics S 2021

Problem Assignment # 10 04/01/2021
due 04/08/2021

2.3.1. **Quadrupole moments (continued)**

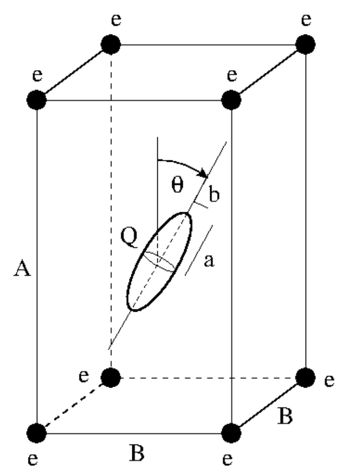
This is a continuation of Problem #2.3.1.

e) Consider the homogeneously charged ellipsoid from part c) and calculate the quadrupole moments Q_{2m} as defined in ch.2 §3.5.

(3 points)

2.3.7. **Electrostatic interaction II: Quadrupole in an external electric field**

Consider the following classical model for a nuclear quadrupole moment in a crystal lattice: A rectangular parallelepiped (height A, length and width B) carries a charge e at each of its eight corners. At the center of the parallelepiped is a homogeneously charged spheroid (charge Q, semi-axes a and b). The symmetry axis of the spheroid forms an angle θ with the A-axis of the parallelepiped. The center of the spheroid is fixed, but the angle θ can vary. Let $A \gg a$, $B \gg b$.



a) Calculate the electrostatic interaction energy U of this system to quadrupolar order. Show that U can be expressed in terms of e, the lattice constants A and B, and the quadrupole moment Q_{33} of the spheroid in the coordinate system of the lattice.

b) Calculate the quadrupole moment Q'_{33} of the spheroid in its principal-axes system, and then calculate Q_{33} by transforming into the lattice system. Express U as a function of the angle θ.

hint: In general, lining up the principal-axes systems would require three Euler angles. However, due to the symmetries of the problem Q'_{33} and Q_{33} in the present case are related by only one angle, viz., θ.

c) Find the equilibrium positions of the spheroid. Make sure to distinguish the cases of prolate and oblate spheroids ($a > b$ and $a < b$, respectively), as well as between the cases $A > B$ and $A < B$.

(15 points)

2.3.8. **Electric charges in an external field**

Consider a static electric charge distribution $\rho(\boldsymbol{x})$ subject to a static potential $\varphi(\boldsymbol{x})$. Consider the force $\boldsymbol{F}_{\text{el}}$ on the charge distribution and show that $\boldsymbol{F}_{\text{el}} = -\boldsymbol{\nabla} U$, with U the electrostatic energy calculated in ch.3 §3.6. In particular, convince yourself that the dipole term in the multipole expansion of U gives the correct potential energy for an electric dipole moment $\boldsymbol{d}$ in an electric field $\boldsymbol{E}$.

(3 points)