

Problem Assignment # 15  
(we'll skip #16)

05/06/2021  
due 05/20/2021

#### 4.4.1. Radiation by an accelerated point particle

Show that the result of ch.4 §4.4 example (2) for the energy radiated by an accelerated point particle,

$$\frac{dU}{d\omega} = \frac{2}{3} \frac{e^2}{\pi c^3} |\dot{\mathbf{v}}(\omega)|^2$$

is consistent with the Larmor formula for the radiated power, ch.4 §3.3:

$$\mathcal{P}(t) = \frac{2e^2}{3c^3} (\dot{\mathbf{v}}(t))^2$$

(2 points)

#### 4.4.2. Classical model of an atom

Consider the classical model of a radiating atom from ch.4 §4.5; i.e., a damped harmonic oscillator with charge  $e$  and equation of motion

$$\ddot{y} = -\omega_0^2 y - \gamma \dot{y}$$

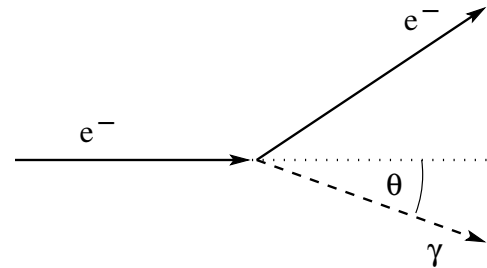
Do NOT assume the damping to be small and solve the equation of motion exactly.

- Assuming that the oscillator represents an electron, and that the spectrum is peaked somewhere in the range of visible light, use the approximate result from ch.4 §4.5 to show that one does not need to consider the case of the oscillator being overdamped.
- Calculate again the radiation spectrum  $dU/d\omega$ , and the total energy  $U$  radiated, exactly. Compare with the approximate result obtained in class. Show that the result self-consistently validates the assumption made in part a) above.

(8 points)

#### 4.5.1. Čerenkov radiation

- Consider a particle with rest mass  $m_0$  that travels through a medium with index of refraction  $n$ . The particle emits a photon that moves at an angle  $\theta$  with respect to the particle's initial trajectory. Use relativistic kinematics to express  $\theta$  in terms of the particle's initial energy and momentum, the wave number of the photon, and  $n$ .
- Show that to leading order for  $c \rightarrow \infty$  one recovers the result of the nonrelativistic approximation used in ch. 4 §5.2.
- The core of a <https://www.youtube.com/watch?v=mgNwtepP-6M> water-cooled reactor emits a 1.42 MeV electron that travels through the water ( $n = 1.33$ ). Calculate  $\theta$  for blue light ( $\lambda = 4,000 \text{ \AA}$ ). How good or bad is the nonrelativistic approximation in this case?



(6 points)