Problem Assignment # 18 05/27/2021
due 06/03/2021

4.6.2. **Synchrotron radiation** (continued)

c) Use the integral representation of the Bessel function J_{2m} from Problem 4.6.1 to show that P_m, the power radiated into the m-th harmonic, can be expressed in terms of Bessel functions as

$$P_m = \frac{e^2}{R}\, m\, \omega_0 \left[2\beta^2\, J'_{2m}(2m\beta) - (1-\beta^2) \int_0^{2m\beta} dx\, J_{2m}(x) \right]$$

where $\beta = v/c$ and J'_{2m} is the derivative of J_{2m} with respect to its argument.

note: One can also obtain this by integrating the final result from ch. 4 §6.2 over the angles, but that's harder.

d) Show that for $\beta \approx 1$ the power peaks at $m_{\text{max}} \propto \gamma^3$, where $\gamma = 1/\sqrt{1-\beta^2}$.

hint: Analyze the J' contribution in detail and do what you can on the second term.

e) Estimate the peak frequency of the power spectrum and the corresponding wave length for the Advanced Light Source (1.9 GeV electrons in a circular orbit with radius $R \approx 20$m), and for a typical radio galaxy (5 GeV electrons in a field $B \approx 5\mu$G).

(7 points)

f) **This part is optional and meant for people who would like to gain a deeper understanding of the power distribution.** Work through Jackson ch. 14.6 to derive his result (14.84) for the angular distribution of the synchrotron radiation. Start with the expression for the power spectrum in the parallel-polarization state in § 6.3 and integrate over T to undo the Wigner structure. Then follow Jackson's logic and approximations, taking into account the difference between his coordinate system and ours, and also the different zeros of time. Repeat this for the perpendicular polarization, then add up the two and integrate over the frequency to obtain the angular distribution.

note: Note that Jackson's approximations are valid only for large γ and small angles about the orbital plane, and also involve a large-frequency approximation. Coming up with a *complete* expression for the angular distribution for all angles that captures both the ultrarelativistic and nonrelativistic limits is remarkably difficult.

4.7.1. **Scattering by a dielectric sphere**

a) Argue on general grounds that the dipole moment of a dielectric sphere (radius a, dielectric constant ϵ) subject to an external electric field $\boldsymbol{E}_{\text{ext}}$ is given by

$$\boldsymbol{d} = f(\epsilon)\, a^3\, \boldsymbol{E}_{\text{ext}} \ ,$$

where the function f has the properties $f(\epsilon \to 1) = 0$, $f(\epsilon \to \infty) = \text{const.}$ (We will determine $f(\epsilon)$ explicitly next week, see Problem 49.)

b) Find the scattering cross section for radiation with wavelength $\lambda \gg a$ scattered by the sphere.

(6 points)