PHYS 622 Classical Electrodynamics W 2021

Problem Assignment # 7 02/19/2021
due 02/26/2021

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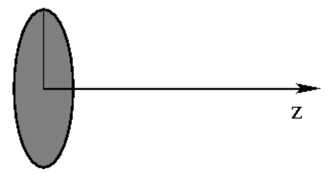
2.2.1. **Planar charge distributions**


a) Consider a homogeneously charged infinitesimally thin ring with radius $R$ and total charge $Q$ that is oriented perpendicular to the $z$-axis. Calculate the electric field on the $z$-axis.




b) The same for a homogeneously charged disk with charge density $\sigma$ and radius $R$. Consider the limits $z \to \infty$, $z \to 0$, and $R \to \infty$, and ascertain that they makes sense.

(4 points)

2.2.2. **Spherically symmetric charge distributions**

Consider a spherically symmetric static charge distribution (in spherical coordinates): $\rho(\boldsymbol{x}) = \rho(r)$.

a) Express the electric field in terms of a one-dimensional integral over $\rho(r)$, and the electrostatic potential by a one-dimensional integral over the field.

*hint:* Make an *ansatz* for a purely radial field, $\boldsymbol{E}(\boldsymbol{x}) = E(r)\,\hat{e}_r$, and integrate Gauss's law over a spherical volume.

Explicitly calculate and plot the field $\boldsymbol{E}(\boldsymbol{x})$ and the potential $\varphi(\boldsymbol{x})$ for

b) a homogeneously charged sphere

$$\rho(\boldsymbol{x}) = \begin{cases} \rho_0 & \text{if } r \leq r_0 \\ 0 & \text{if } r > r_0 \ . \end{cases}$$

c) a homogeneously charged spherical shell

$$\rho(\boldsymbol{x}) = \sigma_0\,\delta(r - r_0) \ .$$

(8 points)

2.2.3. **Electrostatics in $d$ dimensions (to be continued later)**

Consider the third Maxwell equation in $d$ dimensions:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}(\boldsymbol{x}) = S_d\,\rho(\boldsymbol{x})$$

with the electric field $\boldsymbol{E}$ a $d$-vector, and $S_d$ the area of the $(d-1)$-sphere: $S_{2n} = 2\pi^n/(n-1)!$ and $S_{2n+1} = 2^{2n+1}n!\pi^n/(2n)!$ for even and odd dimensions, respectively. Define a scalar potential $\varphi(\boldsymbol{x})$ in analogy to the $3-d$ case, such that

$$\boldsymbol{E}(\boldsymbol{x}) = -\boldsymbol{\nabla}\varphi(\boldsymbol{x})$$

and consider Poisson's equation

$$\boldsymbol{\nabla}^2\varphi(\boldsymbol{x}) = -S_d\,\rho(\boldsymbol{x})$$

*note:* Here we consider a generalization of electrostatics to $d$-dimensional space, NOT a $d$-dimensional charge distribution embedded in 3-dimensional space.

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a) Show that the Green function $G_d(\boldsymbol{x})$ function for Poisson's equation, i.e., the solution of

$$\boldsymbol{\nabla}^2 G_d(\boldsymbol{x}) = -S_d\, \delta(\boldsymbol{x})$$

is given by

$$G_d(\boldsymbol{x}) = \frac{1}{d-2} \frac{1}{|\boldsymbol{x}|^{d-2}}$$

for all $d \neq 2$, and by

$$G_2(\boldsymbol{x}) = \ln(1/|\boldsymbol{x}|)$$

for $d = 2$.

*hint:* For $d = 1$, differentiate directly, using $d \operatorname{sgn} x/dx = 2\,\delta(x)$. For $d \geq 2$, show that $G_d(\boldsymbol{x})$ is a harmonic function for all $\boldsymbol{x} \neq 0$, then integrate $\boldsymbol{\nabla}^2 G_d$ over a hypersphere around the origin and use Gauss's law.

(4 points)