PHYS 610 Mathematical Methods for Scientists F 2023

Problem Assignment # 1 09/27/2023
due 10/04/2022

1.1.1 **Russell's Paradox (B. Russell, 1901)**

a) Consider the set M defined as the set of all sets that do not contain themselves as an element: $M = \{x; x \notin x\}$. Discuss why this is a problematic definition.

b) A less abstract version of Russell's paradox is known as the barber's paradox: Consider a town where all men either shave themselves, or let the barber shave them and don't shave themselves. Now consider the statement

The barber is a man in town who shaves all men who do not shave themselves, and only those.

Discuss why this definition of the barber is problematic (assuming there actually is a barber in town).

hint: Ask "Does the barber shave himself?"

c) Suppose the definition of the barber is modified to read

The barber shaves all men in town who do not shave themselves, and only those.

Discuss what this modification does to the paradox.

(3 points)

1.1.2 **Distributive property of the union and intersection relations**

Show graphically that the relations $\cup$ and $\cap$ defined in ch.1, §1.1, def. 3 obey the following distributive properties: For any three sets A, B, C,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(2 points)

1.1.3 **Mappings**

Are the following $f : X \to Y$ true mappings? If so, are they surjective, or injective, or both?

a) $X = Y = \mathbb{Z}$, $f(m) = m^2 + 1$.

b) $X = Y = \mathbb{N}$, $f(n) = n + 1$.

c) $X = \mathbb{Z}$, $Y = \mathbb{R}$, $f(x) = \log x$.

d) $X = Y = \mathbb{R}$, $f(x) = e^x$.

(2 points)

1.1.4 **Parabolic Mapping**

Consider $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = a\,n^2 + b\,n + c$, with $a, b, c \in \mathbb{Z}$.

a) For which triplets (a, b, c) is f surjective?

b) For which (a, b, c) is f injective?

(4 points)