

Problem Assignment # 3

10/11/2023
due 10/18/20231.2.3 The group S_3

- a) Compile the group table for the symmetric group S_3 . Is S_3 abelian?
b) Find all subgroups of S_3 . Which of these are abelian?

(6 points)

1.2.4 Subgroups

Let (G, \vee) be a group and let $H \subset G$ with $H \neq \emptyset$. Show that H is a subgroup of G if and only if $a, b \in H$ implies $a \vee b^{-1} \in H$.

(5 points)

1.3.1 Fields

- a) Show that the set of rational numbers \mathbb{Q} forms a commutative field under the ordinary addition and multiplication of numbers.
b) Consider a set F with two elements, $F = \{\theta, e\}$. On F , define an operation “plus” (+), about which we assume nothing but the defining properties

$$\theta + \theta = \theta \quad , \quad \theta + e = e + \theta = e \quad , \quad e + e = \theta$$

Further, define a second operation “times” (\cdot), about which we assume nothing but the defining properties

$$\theta \cdot \theta = e \cdot \theta = \theta \cdot e = \theta \quad , \quad e \cdot e = e$$

Show that with these definitions (and **no** additional assumptions), F is a field.

(7 points)

1.4.1. Function space

Consider the set C of continuous functions $f : [0, 1] \rightarrow \mathbb{R}$. Show that by suitably defining an addition on C , and a multiplication with real numbers, one can make C an additive vector space over \mathbb{R} .

(2 points)