Problem Assignment # 4 10/18/2023
due 10/25/2023

1.4.2. **The space of rank-2 tensors**

a) Prove the theorem of ch.1 §4.3: Let V be a vector space V of dimension n over K. Then the space of rank-2 tensors, defined via bilinear forms $f : V \times V \to K$, forms a vector space of dimension n^2.

b) Consider the space of bilinear forms f on V that is equivalent to the space of rank-2 tensors, and construct a basis of that space.

hint: On the space of tensors, define a suitable addition and multiplication with scalars, and construct a basis of the resulting vector space.

(5 points)

1.4.3. **Cross product of 3-vectors**

Let $x, y \in \mathbb{R}_3$ be vectors, and let ϵ_{ijk} be the Levi-Civita symbol. Show that the (covariant) components of the cross product $x \times y$ are given by

$$(x \times y)_i = \epsilon_{ijk} x^j y^k$$

(1 point)

1.4.5. $\mathbb{R}$ **as a metric space**

Consider the reals $\mathbb{R}$ with $\rho : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by $\rho(x, y) = |x - y|$. Show that this definition makes $\mathbb{R}$ a metric space.

(3 points)

1.4.6. **Limits of sequences**

a) Show that a sequence in a metric space has at most one limit.

   hint: Assume there are two limits, and use the triangle inequality to show that they must be the same.

b) Show that every sequency with a limit is a Cauchy sequence.

(3 points)