

This is an open-book exam. You can use any inanimate resources you want, but please do not consult any live resources.

Treat this exam exactly as you would treat an in-class exam, except that you have four hours instead of two: Once you open it, you can work on it for up to four contiguous hours. Do NOT stop and resume later. You can choose any four-hour period you like, but you must submit your work by 5pm PDT on Thu, March 21, 2024 at the latest. Submit a scan with your name on it to dbelitz@uoregon.edu, NOT to your grader.

1. Magnetic monopoles

In ch. 1 we noticed that the Maxwell equations are weirdly asymmetrical. This can be “fixed” as follows. Suppose nature had decided that one 4-vector potential A^μ and one 4-current J^μ was not enough, and there was another 4-vector field \tilde{A}^μ and another 4-current \tilde{J}^μ . Now define a modified field tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \partial_\kappa \epsilon_{\mu\nu}{}^{\kappa\lambda} \tilde{A}_\lambda$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the 4-dimensional Levi-Civita symbol, and a modified Lagrangian density

$$\mathcal{L} = \frac{-1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{c} A_\mu J^\mu - \frac{1}{c} \tilde{A}_\mu \tilde{J}^\mu$$

- a) Show that the resulting Euler-Lagrange equations have the form

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu \quad , \quad \partial_\mu \tilde{F}^{\mu\nu} = \frac{4\pi}{c} \tilde{J}^\nu$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ is the dual field tensor from Problem 1.1.1 (6 pts). Compare these field equations with the corresponding ones in Maxwell theory (2 pts).

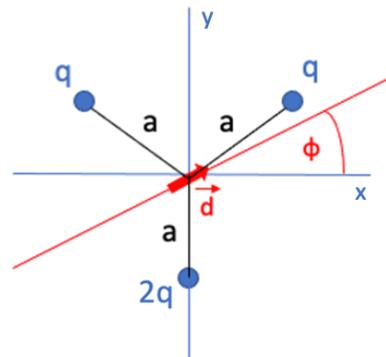
- b) Define electric and magnetic fields \mathbf{E} and \mathbf{B} by parameterizing the field tensor $F^{\mu\nu}$ as in Maxwell theory, express the field equations from part a) in terms of \mathbf{E} and \mathbf{B} , and compare the result with the usual Maxwell equations (2 pts).
- c) Show that the 4-current \tilde{J}^μ obeys a continuity equation and briefly discuss its physical meaning (3 pts).

(13 points)

2. Electric dipole in an external field

Three charges, q , q , and $2q$, are sitting on the corners of an equilateral triangle in the x - y plane, each at a distance a from the origin. At the center of the triangle sits an electric dipole \mathbf{d} . The positions of the charges and the dipole are fixed, but the dipole is free to rotate in the x - y plane.

- a) Calculate the electrostatic interaction energy of this system to dipolar order (6 pts), and determine the equilibrium orientation of the dipole (i.e., the angle ϕ that minimizes the energy) (1 pt).
- b) What happens if all three charges are equal to q ? (2 pts)



(9 points)