Problem Assignment # 6   02/21/2024
due 02/28/2024

1.3.1. Energy density

Show that the argument from ch. 1 §3.6 remark (3) for $u(\boldsymbol{x}, t)$ being the energy density of the electromagnetic field still holds if the field is coupled to N relativistic particles rather than one nonrelativistic one.

(3 points)

1.3.2. Addition of velocities

Consider a particle that has a velocity $\boldsymbol{v}$ in some inertial frame. Find the velocity of the particle in another inertial frame that moves with a velocity $\boldsymbol{V}$ with respect to the first one. Use the result to show that the velocity in the second frame is less than c, provided it was less than c in the first one.

(3 points)

1.3.3. Galileo transformations of Maxwell's equations

a) Show explicitly which of Maxwell's equations are or are not invariant under Galileo transformations. *hint:* Consider the transformations of all vectors (the 4-gradient, the fields, and the 4-current) to zeroth order in $1/c$, but keep the terms of $O(1/c)$ in Maxwell's equations. In other words, note that if you do a Lorentz transformation consistently to a given order in $1/c$, then of course all of Maxwell's equations are invariant.

b) Suppose you had never heard of Lorentz transformations, but were familiar with Galilean mechanics. What are the two logical conclusions you could draw from the result of part a)? (Obviously, one of them by now is of historical interest only.)

(4 points)

1.3.4. Lorentz transformations of fields

Consider static and homogeneous fields $\boldsymbol{E}$ and $\boldsymbol{B}$ that are not parallel to one another in some inertial frame.

a) Show that there exists an inertial frame in which $\boldsymbol{E}$ and $\boldsymbol{B}$ are parallel, and that the two frames are related by a Lorentz boost whose velocity is given by the solution of the equation

$$\frac{\boldsymbol{V}}{c}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right) = (1 + \boldsymbol{V}^2/c^2)\,\boldsymbol{E} \times \boldsymbol{B}$$

b) Show explicitly that this equation has one and only one physical solution that obeys $|\boldsymbol{V}|/c < 1$, that there always is a physical solution, and that the result in the limit of almost parallel fields in the original reference frame is sensible.

c) Are there other inertial frames in which $\boldsymbol{E}$ and $\boldsymbol{B}$ are parallel? If so, how many?

(7 points)