Problem Assignment # 8 03/06/2024
due 03/13/2024

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2.3.5. **Field due to distant charges**

Consider the electric field generated by a charge density $\rho(\boldsymbol{y})$ that vanishes inside a sphere with radius $r_0$: $\rho(\boldsymbol{y}) = 0$ for $|\boldsymbol{y}| \leq r_0$. Show that

a) If $\rho$ is invariant under parity operations, $\rho(-\boldsymbol{y}) = \rho(\boldsymbol{y})$, then the electric field at the origin vanishes.

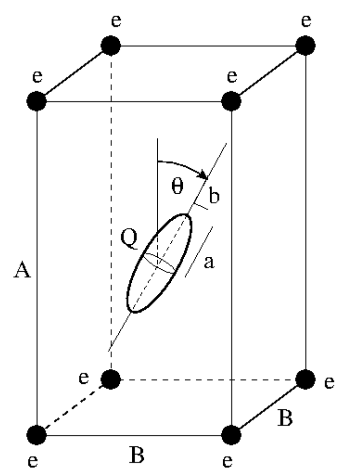
b) If $\rho(\boldsymbol{y})$ is invariant under rotations about the $z$-axis through multiples of an angle $\alpha$ with $|\alpha| < \pi$, then the field-gradient tensor at the origin has the form $\varphi_{ij}(\boldsymbol{x} = 0) = \begin{pmatrix} \varphi & 0 & 0 \\ 0 & \varphi & 0 \\ 0 & 0 & -2\varphi \end{pmatrix}$

c) If $\rho(\boldsymbol{y})$ has cubic symmetry, i.e., if $\rho(\boldsymbol{y})$ is invariant under rotations through $\pi/2$ about any of the three axes $x$, $y$, and $z$, then the field-gradient tensor at the origin vanishes.

(6 points)

2.3.7. **Electrostatic interaction: Quadrupole in an external electric field**

Consider the following classical model for a nuclear quadrupole moment in a crystal lattice: A rectangular parallelepiped (height $A$, length and width $B$) carries a charge $e$ at each of its eight corners. At the center of the parallelepiped is a homogeneously charged spheroid (charge $Q$, semi-axes $a$ and $b$). The symmetry axis of the spheroid forms an angle $\theta$ with the $A$-axis of the parallelepiped. The center of the spheroid is fixed, but the angle $\theta$ can vary. Let $A \gg a$, $B \gg b$.




a) Calculate the electrostatic interaction energy $U$ of this system to quadrupolar order. Show that $U$ can be expressed in terms of $e$, the lattice constants $A$ and $B$, and the quadrupole moment $Q_{33}$ of the spheroid in the coordinate system of the lattice.

b) Calculate the quadrupole moment $Q'_{33}$ of the spheroid in its principal-axes system, and then calculate $Q_{33}$ by transforming into the lattice system. Express $U$ as a function of the angle $\theta$.

*hint:* In general, lining up the principal-axes systems would require three Euler angles. However, due to the symmetries of the problem $Q'_{33}$ and $Q_{33}$ in the present case are related by only one angle, viz., $\theta$.

c) Find the equilibrium positions of the spheroid. Make sure to distinguish the cases of prolate and oblate spheroids ($a > b$ and $a < b$, respectively), as well as between the cases $A > B$ and $A < B$.

(15 points)