1. Use GeoGebra (or any other interactive geometry software) to do the following:

Construct a quadrilateral $ABCD$ and let $M$, $N$, $P$, and $Q$ be the midpoints of the sides. Construct the quadrilateral $MNPQ$. Find the areas of both quadrilaterals, as well as the four small triangles at the corners of $ABCD$. Find an equation relating the areas of the four triangles, and find an equation relating the areas of the quadrilaterals. Print out a picture with the calculations that demonstrate your equations.

2. Prove your conjectures from #1.


4. In this problem you will prove the leftover case of Theorem 12, from our class discussion. In the following diagram assume that $M$ and $N$ are the midpoints of the segments $AB$ and $AC$, and let the perpendicular bisectors of these segments meet at point $X$. Let $P$ be the midpoint of $BC$, draw the line $XP$, and prove that $XP$ is perpendicular to $BC$. [Hint: Follow a method very similar to what we did in class for the other case; start by drawing the lines from $X$ to the vertices of the triangle.]
5. Given: $ABCD$ is a parallelogram, $X$ is a point on line $AB$, and $Y$ is a point on line $BC$. Prove that $\triangle DCX$ has the same area as $\triangle ADY$.

6. Given $AB = AC = BC$. Let $P$ be a point inside the triangle. Let $a$, $b$, and $c$ be the (perpendicular) distances from $P$ to $AB$, $AC$, and $BC$, respectively. Let $h$ be the distance from $A$ to $BC$. To prove: $a + b + c = h$. [Hint: Think about areas.]

7. For any triangle $ABC$, prove that the angle bisectors are concurrent. [Hint: Structure the proof like we did for Theorem 12, by taking two angle bisectors and letting $X$ be the point where they intersect. At some point it will be a good idea to draw in some extra lines.]
8. (a) Given that $A$, $B$, $C$ are on the circle below and that $AC$ is a diameter of the circle, prove that $\angle ABC$ is $90^\circ$.

(b) Now suppose given a right triangle $ABC$, where $AC$ is the hypotenuse. Give two proofs that the points $A$, $B$, and $C$ lie on a circle whose center is at the midpoint of $AC$, following the suggestions below.

**Proof 1:** Let $M$ be the midpoint of $AC$. Show that $BM = AM$. (Hint: Draw lines through $M$ parallel to $BC$ and $AB$, as well as the line $\overrightarrow{AM}$. Use congruent triangles.)

**Proof 2:** Let $M$ be the midpoint of $AC$ and draw the circle with center $M$ and radius $MC$. Assume that $B$ is not on this circle. Then either $B$ lies inside the circle or outside the circle. Do the two cases separately, and in each case use (a) to deduce a contradiction.

9. Given: $A$, $B$, and $C$ lie on the circle with center $O$. Prove that $\angle AOB = 2 \cdot \angle ACB$.

[Note: Compare this to problems 3 and 8 of HW#2. This is another case where one really needs two different pictures, and proofs, to get the full result.]