1. (10 points) Consider the initial value problem

\[(x - 2)y' + (\tan x)y = \frac{1}{\sin^2 x}, \quad y(3) = 5.\]

Determine the largest interval on which this IVP is guaranteed to have a unique solution.

Answer: ___________________________________________
2. (10 points) A tank has a capacity of 300 liters. It is initially full of salt water where the concentration of salt per water is 0.2 kg/liter. Water with a salt concentration of 0.05 kg/liter flows into the tank at a rate of 2 liters/min, and the well-stirred mixture is allowed to flow out at the same rate.

(a) Write down a differential equation concerning the amount of salt in the tank as a function of time. Be sure to explicitly state what your variables represent.

Answer:

(b) Solve this differential equation to determine an expression for the concentration of salt in the tank as a function of time.

Answer:
3. (10 points) Determine an approximate value at $t = 2$ of the solution of the initial value problem

$$y' = t^2 + y, \quad y(1) = 2.$$ 

Use the Euler-tangent-line method with step size $h = 0.5$, and only give two decimal points in the approximation.

Answer: 


4. (10 points) For each of the following differential equations, identify it as either linear, homogeneous, separable, or exact.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^3 \frac{dy}{dx} = (\tan y - 5) , dx$</td>
<td></td>
</tr>
<tr>
<td>$(x^2 - 2xy + y^3) , dx + (3xy^2 - x^2 + 5y) , dy = 0$</td>
<td></td>
</tr>
<tr>
<td>$\frac{dy}{dx} = \frac{x - \cos y}{6y - x \sin y}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{dy}{dx} = \frac{5x^2 \cos(x/y) - 2xy}{y^2 \sin(y/x)}$</td>
<td></td>
</tr>
<tr>
<td>$x^3 , dy = \left[(\cos^2 x)y - x^4\right] , dx$</td>
<td></td>
</tr>
</tbody>
</table>
5. (10 points) Determine an implicit solution of the initial value problem

\[(2x + y + 2xy^2) + (x + 2x^2y + 3y^2 + 1)y' = 0, \quad y(3) = 2.\]

Answer:
6. (10 points) The water in a small lake is draining in such a way that the rate of drainage is proportional to the amount of water left in the lake. The lake initially has $50 \times 10^6$ gallons of water. Assuming that half of the lake has drained after one week, determine the formula for the volume $V(t)$ of the lake after $t$ weeks.

Answer: $V(t) =$