1. (10 points)
   (a) Give the general solution of the equation \( y^{(4)} + 4y^{(3)} + 13y^{(2)} = 0 \).
   
   Answer: \( y_h(t) = \)

   (b) For \( y^{(4)} + 4y^{(3)} + 13y^{(2)} = 5e^{3t} - 6t \sin(3t) \), give the form of the particular solution (you do not have to solve for the undetermined coefficients).

   Answer: \( y_p(t) = \)

   (c) Give the form of the particular solution of \( y^{(4)} + 4y^{(3)} + 13y^{(2)} = 2t^2 - e^{-2t} \sin(3t) \) (you do not have to solve for the undetermined coefficients).

   Answer: \( y_p(t) = \)
2. (10 points) The differential equation $ty'' - (t+1)y' + y = 0$ has solutions $y_1(t) = 1 + t$ and $y_2(t) = e^t$. Use variation of parameters to find a particular solution to the differential equation

$$ty'' - (t+1)y' + y = t^2 e^{2t}, \quad t > 0.$$ 

You may need the integral $\int x e^x \, dx = (x - 1)e^x$.

Answer:
3. (11 points) A spring-mass system leads to the initial value problem

\[ y'' + 36y = 2 \cos(5t), \quad y(0) = 3, \quad y'(0) = 0. \]

The homogeneous solution is \( y_h(t) = C_1 \cos(6t) + C_2 \sin(6t). \)

(a) Find the function \( y(t) \) describing the motion.

Answer: \( y(t) = \)

(b) If the equation is instead \( y'' + ky = 2 \cos(5t) \), for what values of \( k \) will the solution \( y(t) \) become unbounded as \( t \to \infty \)? Explain your answer.

Answer: \( \)
4. (9 points) An object is bobbing up and down on a spring. It’s displacement from equilibrium (measured in feet) is given by $x(t) = \cos(t/6) + \sqrt{3}\sin(t/6)$.

(a) Determine $R$, $\omega$, and $\phi$ for which $x(t) = R\cos(\omega t - \phi)$. Here $R$ should be positive.

Answer:

\[R = \quad \omega = \quad \phi =\]

(b) What is the maximum distance from equilibrium obtained by this object?

Answer:

(c) When $t = 0$ the particle is at $x = 1$. After this moment, what is the first time that the particle arrives back at $x = 1$? (Be careful).

Answer: $t = \quad$

5. (5 points) Let $L[y] = 3y'' + t^2y' - 5ty$. If $L[y_1] = 7\cos t$, find $L[3y_1 + t^2]$. 


6. (5 points) Let \( x(t) = \begin{bmatrix} \cos(2t) \\ \sin(2t) \end{bmatrix} \). Is this a solution to the differential equation 
\[ x'(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot x(t) \] 
Explain why or why not.

7. (5 points) Compute the matrix expression
\[
\begin{bmatrix}
1 & 2 & 3 \\
0 & -2 & x \\
1 & y & 5
\end{bmatrix}
\cdot
\begin{bmatrix}
1 \\
2 \\
z
\end{bmatrix}
+ 
\begin{bmatrix}
2 \\
1 \\
3
\end{bmatrix}
\]
Your answer should contain the variables \( x, y, \) and \( z \).

8. (5 points) If \( A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \), compute the determinant of \( A - rI_2 \) where \( I_2 \) is the \( 2 \times 2 \) identity matrix.