You should be able to recognize the order of a differential equation, and identify the special types of linear, separable, homogeneous, and autonomous differential equations. You should understand how to obtain direction fields, and understand how they relate to solutions. You should be able to solve

(i) First-order linear equations
(ii) First-order separable equations
(iii) First-order homogeneous equations

and associated initial value problems. You should understand how to find the range on which a given solution is defined, given appropriate information (like a graph).

You should be able to sketch solutions to autonomous differential equations, identify the equilibrium solutions, and identify each equilibrium solution as stable, unstable, or semi-stable.

You should be able to evaluate the following types of integrals:

1. \( \int (\text{polynomial}) \, dx \)
2. \( \int e^{\alpha x} \, dx \)
3. \( \int \frac{1}{t} \, dt \)
4. \( \int (ax + b)^\alpha \, dx \)
5. \( \int \cos(ax) \, dx \) and \( \int \sin(ax) \, dx \)
6. \( \int \frac{ax + b}{(x - r_1)(x - r_2)} \, dx \) (use partial fractions).

You should also be able to evaluate simple integrals associated to these by substitution, like \( \int \frac{5t}{1 + t^2} \, dt \).

Practice questions:

1. Consider the differential equation \( \frac{dy}{dt} = y(y - 2) \).
   
   (a) Sketch the graph of the solution of the differential equation for each of the initial values \( y(0) = -\frac{2}{3} \), \( y(0) = 0 \), \( y(0) = \frac{2}{3} \), \( y(0) = \frac{4}{3} \), \( y(0) = 2 \), \( y(0) = \frac{8}{3} \).

   (b) What are the equilibrium solutions?

   (c) Which equilibrium solutions are stable?

   (d) For which values of \( y(0) \) is the graph of \( y(t) \) increasing for \( t > 0 \)?

   (e) For which values of \( y(0) \) is the graph of \( y(t) \) concave up for \( t > 0 \)?

   [Answers for (b)-(e): (b) \( y = 0 \) and \( y = 2 \); (c) \( y = 0 \); (d) \( y < 0 \) and \( y > 2 \); (e) \( 0 < y < 1 \) and \( y > 2 \).]
1. Determine the order of each of the differential equations; also state whether the equation is linear or nonlinear.
   (a) \(yy' + t = 1\)
   (b) \(ty' + y = 1\)
   (c) \((y')^2 + ty = 1\)
   (d) \(y'' + \sqrt{ty} = 1\)

2. (a) Which of the functions \(y_1(t) = t\) and \(y_2(t) = -1\) are solutions of the initial value problem \(yy' = t, \ y(0) = 0?\)
   (b) Which of the functions \(y_1(t) = t\) and \(y_2(t) = -t\) are solutions of the initial value problem \(yy' = t, \ y(1) = 1?\)

3. (a) Show that \(y = t^3\) is a solution of the initial value problem \(y' = 3y^{2/3}, \ y(0) = 0.\)
   (b) Find a different solution of the initial value problem.

4. For what value(s) of \(r\) is \(y = e^{rt}\) a solution of the differential equation \(y'' - 5y' + 6y = 0?\)

5. Find the general solution of the differential equation \(y' + \frac{1}{t+1}y = 1.\)
   Assume \(t+1 > 0.\)

6. Find the solution of the initial value problem \(ty' = y + 1, \ y(1) = 2.\)

7. For what value(s) of \(a\) is the solution of the initial value problem \(y' - y + 2e^{-t} = 0, \ y(0) = a\) bounded on the interval \(t \geq 0?\)

8. Use the given direction field of \(y' = (y-1)(y-3)\) to determine the behavior of \(y\) as \(t\) increases for each initial value \(y(0) = a.\)

9. Use the given direction fields to sketch the solution of the corresponding initial value problem for the indicated initial value \((t_0, y_0).\) Extend your sketch in both directions as far as seems possible and explain why the domain of the solution may be restricted.
   (a) \((t_0, y_0) = (1, 1)\)
   (b) \((t_0, y_0) = (0, -1)\)

10. Sketch the direction field of \((a) \ y' = -\frac{y}{t} \) and \((b) \ y' = -\frac{t}{y}.\)

11. Find an implicit solution of the initial value problem \(y' = \frac{1-2t}{1+3y^2}, \ y(1) = 2.\)

12. Find an explicit solution of the initial value problem \(t^2y' = y^2, \ y(1) = 1/2.\) Indicate the interval in which the solution is valid.

13. Find the slope of the solution of the differential equation \(y' = 2y^3 + 4t\) at the point \((2, -1)\).
1. (a) first order, nonlinear, (b) first order, linear, (c) first order, nonlinear, (d) third order, linear
2. (a) $y_1$ and $y_2$, (b) $y_2$
3. (a) $y' = 3t^2 = 3(t^3)^{2/3} = 3y^{2/3}$, (b) $y = 0$
4. $r = 2, 3$
5. $y = \frac{t^2}{2(t+1)} + \frac{t}{t+1} + \frac{C}{t+1}$ or $y = \frac{t+1}{2} + \frac{C}{t+1}$
6. $y = -1 + 3t$
7. $a = 1$
8. $y \to \infty$ as $t$ increases if $a > 3$; $y \to 3$ as $t$ increases if $a = 3$; $y \to 1$ as $t$ increases if $a < 3$.
9. (a) The graph becomes vertical near $(t, y) = (\pm 2, 0)$.
(b) The graph approaches a vertical asymptote near $t = 2$.
10. (a)
(b)
11. $y + y^3 = t - t^2 + 10$
12. $y = \frac{t}{t+1}$, $t > -1$
13. slope = 6