The exam will cover the following topics:

(1) Section 2.4: Existence and uniqueness theorems for first-order differential equations.

(2) Section 2.3: Application problems. You should be able to set up and solve the following types: Mixing problems, problems where \( y' \) is proportional to \( y \) (radioactive substances and investment problems), and Newton’s law of cooling.

(3) Section 2.6: Exact equations.

(4) Section 2.7: The Euler tangent line method for approximating solutions.

In addition, you should continue to be able to solve linear and separable differential equations, and to evaluate integrals of the types listed on the first review sheet.

Practice questions:

1. Determine whether each of the following differential equations is separable, homogeneous, linear, or exact (determine all types that apply):
   
   (a) \[ 2x + y + (x + 3y)\frac{dy}{dx} = 0 \]
   
   (b) \[ x + 3y + (2x + y)\frac{dy}{dx} = 0 \]
   
   (c) \[ x + 3y + 1 + (2x + y + 1)\frac{dy}{dx} = 0 \]
   
   (d) \[ 2xy + 1 + (x^2 + 1)\frac{dy}{dx} = 0 \]
   
   (e) \[ x^2 + 1 + (y^2 + 1)\frac{dy}{dx} = 0 \]

2. Find an implicit solution of the initial value problem

   \[ 2xy + 1 + (x^2 + 2y)\frac{dy}{dx} = 0, \quad y(1) = -1. \]

3. Determine an approximate value at \( x = 0.5 \) of the solution of the initial value problem \( y' = 3x + y, \) \( y(0) = 1 \) using the Euler tangent line method with \( h = 0.25. \)

4. Consider the initial value problem \( y' = xy + y^2, \) \( y(3) = -1. \)
   
   (a) Is the solution increasing or decreasing near \( x = 3? \)
   
   (b) Is the solution concave up or concave down near \( x = 3? \)
   
   (c) Are the Euler tangent line approximations of the solution near \( x = 3 \) greater than or less than the actual solution?

5. A thermometer reads \( 36^\circ \) when it is moved into a \( 70^\circ \) room. Five minutes later the thermometer reads \( 50^\circ. \) Find the thermometer reading \( t \) minutes after being in the room. What will it read 10 minutes after being moved into the room?
6. A 500 gallon tank contains 200 gallons of solution with salt concentration 2oz/gal. Pure water flows into the tank at a rate of 10gal/min, while the mixture flows out of the tank at 5gal/min. Find the salt concentration in the tank at the time the tank becomes completely filled.

7. Suppose \( y' \) is proportional to \( y \), \( y(0) = 4 \), and \( y(2) = 2 \). Find \( y \) in terms of \( t \). For what value of \( t \) does \( y(t) = 3 \)?

8. For each of the initial value problems below, determine the largest interval for which a unique solution is guaranteed:
   (a) \( y' - \frac{2}{t} y = \frac{1}{t}, \ y(1) = 0 \)
   (b) \( y' + (\tan t)y = \sec t, \ y(0) = 0 \)
   (c) \( y' + \frac{t}{t^2 - 9} y = \frac{1}{t - 2}, \ y(0) = 1 \)
   (d) \( (t + 4)y' - ty = \frac{1}{t}, \ y(-2) = 1 \).

9. For each of the initial value problems determine all initial points \((t_0, y_0)\) for which a unique solution is guaranteed in some interval \( t_0 - h < t < t_0 + h \):
   (a) \( y' = t^2 + y^2, \ y(t_0) = y_0 \)
   (b) \( y' = \sqrt{t^2 + y^2}, \ y(t_0) = y_0 \)
   (c) \( y' = \frac{t}{y}, \ y(t_0) = y_0 \)
   (d) \( y' = t^{1/3}y^{2/3}, \ y(t_0) = y_0 \).

Answers:

1. (a) homogeneous, exact; (b) homogeneous; (c) none of these types; (d) linear, exact; (e) separable, exact.
2. \( x^2y + x + y^2 = 1 \)
3. 1.75
4. (a) Decreasing; (b) Concave down; (c) Euler approximations are greater than the actual solution.
5. \( T = 70 - 34e^{(\ln(10/17))t/5}; \ T(10) = 58.2^\circ \).
6. 0.32oz/gal
7. \( y = 4e^{(\ln(0.5)t/2)}; \ t = 0.83. \)
8. (a) \( t > 0 \); (b) \( -\frac{\pi}{2} < t < \frac{\pi}{2} \); (c) \(-3 < t < 2; -4 < t < 0. \)
9. (a) All \((t_0, y_0)\); (b) All \((t_0, y_0)\) except \((0, 0)\); (c) All \((t_0, y_0)\) with \( y_0 \neq 0 \); (d) All \((t_0, y_0)\) with \( y_0 \neq 0 \).