The exam will cover the following topics:

(1) Sections 3.1, 3.4, 3.5: Solving second-order linear, homogeneous equations with constant coefficients.

(2) Section 3.2: Existence and uniqueness results for second-order linear equations; operator notation; the Wronskian.

(3) Section 3.5: Reduction of order.

(4) Section 3.6: The method of undertermined coefficients.

In addition, you should continue to be able to solve linear and separable differential equations, and to evaluate integrals of the types listed on the first review sheet.

Practice questions:

1. (a) \( L[y] = y'' - 3y' + 2y \). Evaluate \( L[e^t], L[e^{2t}], L[e^{-t}] \).
   
   (b) \( L[y] = y'' - ty' + 5y \). Evaluate \( L[t^3] \) and \( L[t^2 + \cos(t)] \).

2. Suppose \( y_1 \) is a solution of \( L[y] = t^2 \), where \( L[y] = t^2y'' + ty' + y \). Evaluate \( L[y_1 + t^2 - 2t + 1] \).

3. Find the largest open interval for which the initial value problem
   \[
   y'' + \frac{1}{t}y' + \frac{1}{t-2}y = \frac{1}{t-3}, \quad y(1) = 3, \quad y'(1) = 2
   \]
   is guaranteed to have a unique solution.

4. (a) Verify that \( y_1(t) = t \) and \( y_2(t) = t^{-1} \) are solutions to the differential equation \( t^2y'' + ty' - y = 0 \).
   
   (b) Compute the Wronskian \( W(y_1, y_2) \).
   
   (c) Find the solution of the initial value problem \( t^2y'' + ty' - y = 0, \ y(1) = 2, \ y'(1) = 4 \).

5. Find the general solution of the differential equation \( y'' - y' = 4t \).

For problems 6–8 find the general solution of the homogeneous system in part (a) and then find the form of a particular solution to the nonhomogeneous systems in parts (b) and (c). Do not solve for the undetermined coefficients.

6. (a) \( y'' - 5y' + 6y = 0 \)
   
   (b) \( y'' - 5y' + 6y = t^2 \)
   
   (c) \( y'' - 5y' + 6y = 2e^{2t} + 5 \cos 3t \)

7. (a) \( y'' - 6y' + 9y = 0 \)
   
   (b) \( y'' - 6y' + 9y = te^{3t} \)
   
   (c) \( y'' - 6y' + 9y = e^t + \cos(3t) \)

8. (a) \( y'' - 2y' + 10y = 0 \)
(b) $y'' - 2y' + 10y = e^t + \cos(3t)$
(c) $y'' - 2y' + 10y = e^t \cos(3t)$

9. You are told that $y_1(t) = e^t$ is a solution to the differential equation

$$(t - 1)y'' - ty' + y = 0.$$ 

Find the general solution to this differential equation.

Answers:

(1) (a) $L[e^t] = 0$, $L[e^{2t}] = 0$, $L[e^{-t}] = 6e^{-t}$;
(b) $2t^3 + 6t$ and $4\cos(t) + t\sin(t) + 3t^2 + 2$.

(2) $6t^2 - 4t + 1$

(3) $0 < t < 2$

(4) (b) $-2t^{-1}$; (c) $y = 3t - t^{-1}$.

(5) $y = C_1 + C_2e^t - 2t^2 - 4t$

(6) (a) $y = C_1e^{2t} + C_2e^{3t}$;
(b) $y = At^2 + Bt + C$;
(c) $y = Ate^{2t} + B\cos(3t) + C\sin(3t)$.

(7) (a) $y = C_1e^{3t} + C_2te^{3t}$;
(b) $y = t^2(At + B)e^{3t}$;
(c) $y = Ae^{3t} + B\cos(3t) + C\sin(3t)$.

(8) (a) $y = [C_1\cos(3t) + C_2\sin(3t)]e^t$;
(b) $y = Ae^t + B\cos(3t) + C\sin(3t)$;
(c) $y = t(A\cos(3t) + B\sin(3t))e^t$.

(9) $y(t) = At + Be^t$. 

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