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Sp16
L3.1

1) Normal distributions – PDFs & the Gaussian

2) SDOM – standard deviation of the mean

~~3) weighted averages~~

3 ~~4~~) Lab 2 Brownian Motion

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1) typically ~~with~~ plan for N measurements of same thing for an experiment. (then we might change a variable and go again, that's later

$x_1, x_2, x_3 \ldots x_N$

→ Our goal, then, is to develop a "best estimate" & "best width estimate" from these N ←

We start our statistical analysis process by assuming our distribution of measures x_i has a well-defined mean & standard deviation $\bar{x}$ & σ.

Thus the central limit thm says that M sets of these N measures would be "normal", ⇒

~~Look at stationline & 2 results~~

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1st assumption: dist. is normal

then, it's probability density fctn would be Gaussian

$$\mathrm{Prob}\,[x_i, x_i + dx] = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(x_i - \bar{X})^2}{2\sigma^2}}\, dx$$

So now we can estimate the (joint) probability of observing our original N x's.

$$\mathrm{Prob}_{\bar{X},\sigma}(x_1, x_2 \ldots x_N) = \underset{\bar{X},\sigma}{\mathrm{Prob}}(x_1) \cdot \mathrm{Prob}(x_2) \cdots \mathrm{Prob}(x_N)$$

2nd point: joint probabilities are multiplicative

Each of these is a Gaussian

note $e^x \cdot e^y = e^{x+y}$

3rd point: JPs mult. → Σ of exponents

So joint probability ⇒

$$\mathrm{Prob}_{\bar{X},\sigma}(x_1, x_2 \ldots x_N) \propto \frac{1}{\sigma^N}\, e^{\frac{-\Sigma (x_i - \bar{X})^2}{2\sigma^2}}$$

Q assumptions? − all σ's are same, and $\bar{X}$'s

fine, but we don't really know $\bar{X}$ + σ

4th: maximize JP

to pursue this, we'll use the "principle of maximum likelihood"

e.g. find $\bar{X}$ + σ that maximize JP

Q?: how to maximize SP?

4 th step $\equiv$ minimize exp.

— minimize the (negative) exponent —

Q: how to do this?

$\frac{\partial}{\partial \bar{X}}$ of $\sum_{i=1}^{N} \frac{(x_i - \bar{X})^2}{2\sigma^2}$, set to $\emptyset$

(a min or max)

$\Rightarrow \quad \sum (x_i - \bar{X}) = \emptyset \Rightarrow$

$$\bar{X} = \frac{\sum x_i}{N} = \bar{x}$$

$\Rightarrow$ So the simple average is the best estimate assuming normal distribution, well-defined mean + std. dev (σ)

to find σ_b, $\frac{\partial}{\partial \sigma}$ [SP exp.] $\overset{set}{=} \emptyset$ and solve

$\boxed{\text{Hw 5.26}}$

$\Rightarrow \quad \sigma_b = \sqrt{\frac{1}{N} \sum (x_i - \bar{x})^2} =$ our defn σ_x

sometimes $\frac{1}{N-1}$

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2) SDOM

have established, given certain (reasonable) assumptions, $\bar{X} = \bar{x}$ and $\sigma_b = \sigma$

Still working w/ N x_i's, what is "reliability" of $\bar{X} = \bar{x}$?

additional assumption here that orig dist represented by x_i's is normal

to consider, look at dist of $\bar{X}_k = \frac{\sum_N x_{i,k}}{N}$ $k = 1, M$ (sets)
dist. of $\bar{x}$'s over M "sets"

we note that if $x_1, x_2 \ldots x_N$ is normally distributed then $\bar{x}$ is normally distributed (simple fctn of x_i's)

2nd, each x_i has the same true value X

$$\text{so} \quad \bar{X} = \frac{X + X + X \ldots}{N} = \text{"true value"}$$

$\boxed{\text{Now look at stat-online 2 results}}$

So measuring M sets doesn't change $\bar{X} = \bar{x}$ (of all)

Q: What is width estimate of $\bar{x}$? (add errors in x in quadrature)

$$\sigma_{\bar{x}} = \sqrt{\left(\frac{\partial \bar{x}}{\partial x_1}\sigma_{x_1}\right)^2 + \left(\frac{\partial \bar{x}}{\partial x_2}\sigma_{x_2}\right)^2 + \cdots \left(\frac{\partial \bar{x}}{\partial x_N}\sigma_{x_N}\right)^2}$$

but $\sigma_{x_1} = \sigma_{x_2} = \sigma_{x_N} = \sigma_x$

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and: $\frac{\partial \bar{x}}{\partial x_1} = \frac{\partial \bar{x}}{\partial x_2} = \ldots \frac{\partial \bar{x}}{\partial x_N} = \frac{1}{N}$

$$\Rightarrow \quad \sigma_{\bar{x}} = \sqrt{\left(\frac{1}{N}\sigma_x\right)^2 + \ldots \left(\frac{1}{N}\sigma_x\right)^2}$$

$$= \frac{\sigma_x}{\sqrt{N}} \quad \text{SDOM}$$

(What we use in reporting error)

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3) Brownian Motion

- Say we put ~5 micron spheres in deionized water at room temp

- what would we see? (nada)
  " " " " with 40x mag.? (still nada)

- what would we see if made movies with 40x digital microscope (still nada) (depends on frame rate)

- lets make frame rate 0.05 fps ← Q (ask for what they mean)

- (show movie)

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we note: (ask for this)

- spheres appear to move
- "motion" is called Brownian Motion
- there may be other factors affecting their motion (query)

Why do this?

$$\overline{\Delta x^2} = 2 D_{\Delta x} \Delta t$$

(also $\overline{\Delta y^2} = 2 D_{\Delta y} \Delta t$
$\overline{\Delta r^2} = 4 D_{\Delta r} \Delta t$)

- first, why Δx (Δy, Δr)?

- next what does $\overline{\Delta x^2}$ mean?

- note that $\sigma^2_{\Delta x} = \overline{\Delta x^2} - (\overline{\Delta x})^2$

what do expect $\overline{\Delta x}$ to be for B.M?

[mention it should be ϕ, but this might be 'experiment zero' $\Rightarrow$
$(\overline{\Delta x})^2 << \overline{\Delta x^2}$
thus $\overline{\Delta x^2} \approx \sigma^2_{\Delta x}$

- what is Δt? Could we vary it? how is that useful?

#1 M question, what is D?
(the 'diffusion constant' first
hypothesized by Einstein

with $Df = k_B T$ (T=?, units?)

and $f = 6\pi\eta R$ (drag factor, calculable)

⇒ If we can estimate D & its error (δD) ⇐
then we can " k_B & its error

In this lab we will:

1) make movies of spheres undergoing BM

2) analyze these, first using Logger Pro

3) get LP data ⇒ python (or MC)

4) ? calc Δ's, $\overline{\Delta x}^2$ vs $\overline{\Delta x^2}$
P_{random} (data quality)

5) ? do statistics

6) ? est D & δD

7) calc k_B & δk_B

8) 'Time Evolution'

ask students to complete list