PHYS 391
Sp 16
L 4.1

1) weighted averages

2) background: Brownian Motion theory

3) about lab writeups

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1) weighted averages

have 2 separate experiments intending to measure the "same thing" $x_A \pm \sigma_A$, $x_B \pm \sigma_B$
however, the $\bar{x}$ + σ's are different for 2

→ goal: how to combine? ←
or, specifically, what do we state as the "best value" and error for these 2 measurements?

we approach as before: ask for the best value X that maximizes the J.P. (maximum likelihood)

$$JP = \frac{1}{\sqrt{2\pi}\sigma_A} e^{\frac{-(x_A - X)^2}{2\sigma_A^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_B} e^{\frac{-(x_B - X)^2}{2\sigma_B^2}}$$

$$= \frac{1}{2\pi} \frac{1}{\sigma_A \sigma_B} e^{-\frac{\chi^2}{2}}$$

with $\chi^2 = \frac{(x_A - X)^2}{\sigma_A^2} + \frac{(x_B - X)^2}{\sigma_B^2}$

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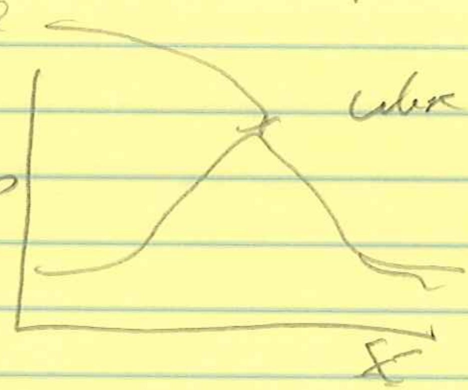
to maximize JP, set appropriate partial derivative $= \phi$

ie find



equiv to minimizing χ^2

solving $\Rightarrow$
$$\bar{X} = \frac{\frac{x_A}{\sigma_A^2} + \frac{x_B}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}}$$

define $w_i = \frac{1}{\sigma_i^2}$ $\Rightarrow$

$$\bar{X} = \frac{w_A x_A + w_B x_B}{w_A + w_B}$$ weighted avg.

in general $\bar{X} = \frac{\sum w_i x_i}{\sum w_i}$

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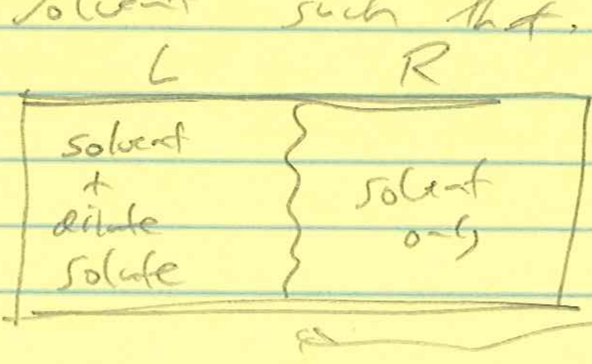
2 BM Theory

consider a system of a solute (large molecules) and solvent such that.



all at same initial pressure

moveable membrane can pass only solvent molts

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what happens next?

* solute contributes to pressure on L side of barrier
  some solute mols bounce off it

* thus more solvent moves L ← R than vice versa

* net solvent flow is L ←

* pressure on L ↑, barrier moves → R

Einstein (1905)
* solute mols can be viewed as moving in vacuum
* Brownian particles (Brown's pollen) are analogous to solute particles
* $\omega_{sim} = \overline{KE} = \frac{3}{2} k_b T$

(effort shifts to Paul Langevin, looks @ $\overline{KE}$)

following motion of Brownian particle / solute mol.

goal: write out $\overline{m \frac{d^2x}{dt^2}}$

Simplify to eqn of motion for 1 particle and in 1-D

(radius, viscosity)

$$m \frac{d^2x}{dt^2} = -6\pi a \eta \frac{dx}{dt} + X$$

drag force — net push from solvent mols

mult by x

$$x m \frac{d^2x}{dt^2} = -6\pi a \eta x \frac{dx}{dt} + xX$$

clever rewrite — "corrects" for

math trick A →

$$m \frac{d}{dt}\left(x \frac{dx}{dt}\right) - m \left(\frac{dx}{dt}\right)^2 = -3\pi a \eta \frac{d}{dt}(x^2) + xX$$

trick B → average over a "long time" { relative to? collision rate of solvent on solute mols

$$\overline{m \frac{d}{dt}\left(x \frac{dx}{dt}\right)} - \overline{m \left(\frac{dx}{dt}\right)^2} = \overline{-3\pi a \eta \frac{d}{dt}(x^2)} + \overline{xX} \rightarrow 0$$

assume $(\overline{X})^2 \rightarrow 0$

$$\frac{1}{2} m \frac{d^2}{dt^2} \overline{x^2} + 3\pi a \eta \frac{d}{dt} \overline{x^2} = k_B T$$

* Einstein says $\overline{\frac{1}{2} m \left(\frac{dx}{dt}\right)^2} = KE = \frac{3}{2} k_B T$ for 3-D

$= \frac{1}{2} k_B T$ for 1-D

to solve, set $y \equiv \frac{d}{dt}\overline{x^2}$

$$\Rightarrow \quad \frac{dy}{dt} + \frac{6\pi a\eta}{m} y = \frac{2k_BT}{m}$$

soln $\quad y(t) = \frac{k_BT}{3\pi a\eta} + Ce^{-\frac{6\pi a\eta}{m}t}$

goes to ϕ quickly
ignore

$$\Rightarrow y(t) = \frac{d}{dt}\overline{x^2} = \frac{k_BT}{3\pi a\eta}$$

or $\quad \overline{x^2} = \frac{k_BT}{3\pi a\eta} t$

$$\equiv \quad \overline{\Delta x^2} = \frac{k_BT}{3\pi a\eta} \Delta t$$

define $D \equiv \frac{k_BT}{6\pi a\eta} \Rightarrow$

$$\overline{\Delta x^2} = \sigma_{\Delta x}^2 = 2D\Delta t$$

(our expression is $\overline{Df = k_BT}$

with $f \equiv 6\pi a\eta$)

3ss
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L4.4
L4.6

1) confirmed

$\Rightarrow$
$$y(t) = \frac{d}{dt}\overline{x^2} = \frac{k_B T}{3\pi a \eta}$$

$$\Rightarrow \quad \overline{x(t)^2} = \frac{k_B T}{3\pi a \eta} t$$

$$\equiv \quad \overline{\Delta x^2} = \frac{k_B T}{3\pi a \eta} \Delta t$$

with $D \equiv \frac{k_B T}{6\pi a \eta} \Rightarrow$ not true for Δr

$$\overline{\Delta x^2} = \sigma_{\Delta x}^2 = 2D\Delta t$$

in lab we use $Df = k_B T \Rightarrow$

$$f \equiv 6\pi a \eta$$

2) notes about lab

$d = 3.2$ microns — but Bisham may have used larger spheres 5.4μ for some marked slides

a reasonable value for D?
($\sim 6.8\times10^{-14}$)

units for everything? stated everywhere

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wtf spe
L4.87

2) Lab writeups

5 sections

show last → 1) abstract (write last?)
e.g. JR

2) intro & background
goals for lab
theory + assumptions { also experiment diagram

can cycle here through "tasks" in lab writeup

3) results → sample data plots
→ tables
→ quality indicators e.g. $\overline{(\Delta x)^2} \ll \overline{(\Delta x^2)}$
also $\overline{\Delta x^2} \approx \overline{\Delta y^2}$

4) analysis → summative plots
→ values (e.g. $D_{\Delta x} = ??$)
→ errors

5) conclusions → main findings
→ problems
→ "next time" improvements

general notes:
a) put sample histograms in report
b) numbers have units (always!)
c) time evolution (do it?)