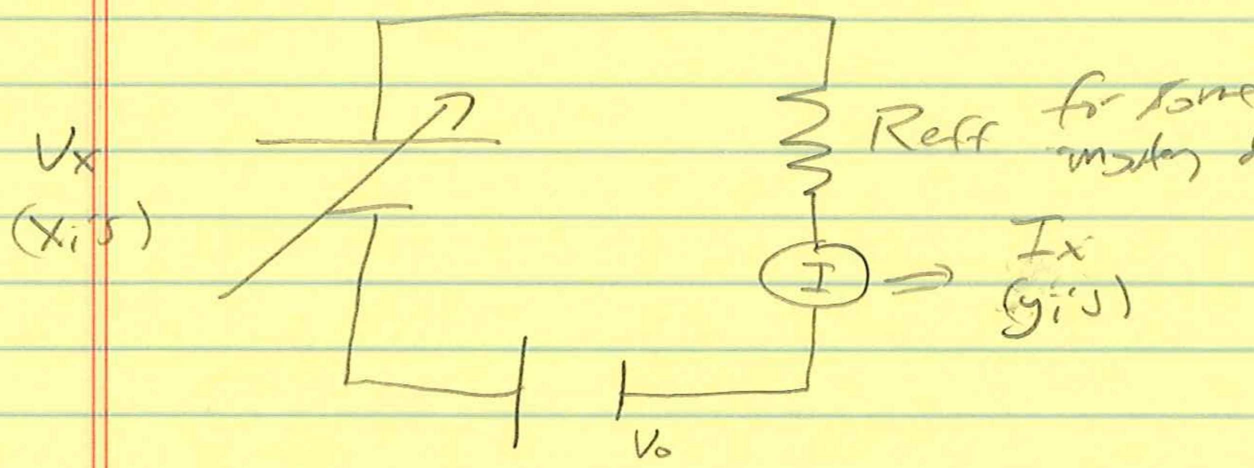
391
Sp16
L5.1

1) LS fitting

2) a bit of Astrophysics (w/ care-fr)

1) have N responses from experiment with y_i responses from x_i (manipulated) inputs. (Changing x)

for example.



our goal is to determine Reff

assume Ohm's Law applies $V = IR_{eff} = V_x + V_0$

solve for $I = \frac{V_0}{R_{eff}} + \frac{V_x}{R_{eff}}$

of form $y_i = A + Bx_i$ $\left(A = \frac{V_0}{R_{eff}} \quad B = \frac{1}{R_{eff}}\right)$

writing $y_i = A + Bx_i$ could imply a pair A, B for each pair (x_i, y_i)

as in $y_i = A_i + B_i x_i$

our goal, however, is to determine one pair A, B that provide a "best fit" for all (x_i, y_i) pairs.

to start, consider a set of "optimal Y_i's" corresponding to x_i's w/ $Y_i \neq y_i$ b/c of statistics

we also assume that each observed y_i is normally distributed about its respective Y_i

so: $$P_{Y_i}(y_i) = \frac{1}{\sigma_y} e^{-\frac{[y_i - Y_i]^2}{2\sigma_y^2}}$$

and, for one pair (x_i, y_i)
$$= \frac{1}{\sigma_y} e^{-\frac{[y_i - (A + Bx_i)]^2}{2\sigma_y^2}}$$

[to recap, we have assumed there is a set of optimal Y_i's, generated from $Y_i = A + Bx_i$, and that our observed y_i's are normally distributed about these Y_i's]

so "think & share" question: "what is probability of observing our N y_i's for our x_i's?"

the Joint Probability is

$$JP \propto \frac{1}{(\sigma_y)^N} e^{-\frac{\chi^2}{2}}$$

with $\chi^2 = \sum_{i=1}^{N} \frac{[y_i - A - Bx_i]^2}{\sigma_y^2}$

chi-squared, sum of residuals

as per usual, want to maximize $JP \Leftrightarrow$ min. χ^2

so $\frac{\partial}{\partial A}(JP) = 0$
$\frac{\partial}{\partial B}(JB) = 0$ $\Rightarrow$ best A + B

3rd
[illegible]
LSQ

As per usual, we want to
maximize the JP (find the true $\bar{Y}_i$'s).
This means minimizing χ^2 (negative exp)

for example, set

$$\frac{\partial(JP)}{\partial A} = 0 \quad \text{and solve}$$

also $$\frac{\partial(JP)}{\partial B} = 0$$

$\Rightarrow$ this gives:

$$A\,N + B\,\Sigma x_i = \Sigma y_i$$
$$A\,\Sigma x_i + B\,\Sigma x_i^2 = \Sigma x_i y_i$$

2 eqns, 2 unknowns, but wait!
This is a job for linear algebra

Write this as $\tilde{m} \cdot \vec{F} = \vec{K}$

matrix, vectors

$$\tilde{m} \rightarrow \begin{pmatrix} N & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} A \\ B \end{pmatrix}}_{\vec{F}} = \underbrace{\begin{pmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{pmatrix}}_{\vec{K}}$$

solve by "dividing" both sides by $\tilde{m}$

2) continued

$$\vec{F} = \mathrm{inv}(\vec{m}) \circ \vec{K}$$

(Matlab L3Fits.m polyo1.m)

what if variances vary eg $\sigma_{y_1}^2 \neq \sigma_{y_2}^2 \neq$ etc.

then weight individual y_i pieces of χ^2 by their own variance

$$\chi^2 \equiv \sum_{i=1}^{N} \frac{(y_i - A - Bx_i)^2}{\sigma_i^2}$$

that's different

as with weighted avgs, define $w_i \equiv \frac{1}{\sigma_i^2}$

$$\Rightarrow \vec{m} = \begin{pmatrix} \sum w_i & \sum w_i x_i \\ \sum w_i x_i & \sum w_i x_i y_i \end{pmatrix}$$

$$\vec{K} = \begin{pmatrix} \sum w_i y_i \\ \sum w_i x_i y_i \end{pmatrix}$$

easy to fix in ML (python)

(polyo1_wgt.m)

note ~~help~~ comments & help

3xx
W6
L5.7

2) cont'd

note them $\Longrightarrow$ finally, what about higher order or other "fit" functions?

define $Y_i = A + Bx_i + Cx_i^2$ quad. fit fctn

$$JP \propto e^{-\frac{\chi^2}{2}}$$

$$\chi^2 \equiv \sum_{i=1}^{N} w_i \cdot \left[y_i - (A + Bx_i + Cx_i^2) \right]^2$$

($w_i = \frac{1}{\sigma_i^2}$ as before)

then minimize $\chi^2 \Rightarrow A, B, C$

set $\left.\begin{array}{l} \frac{\partial \chi^2}{\partial A} = 0, \text{ solve} \\ \frac{\partial \chi^2}{\partial B} = 0, \text{ solve} \\ \frac{\partial \chi^2}{\partial C} = 0, \text{ solve} \end{array}\right\} \Longrightarrow$ 3 eqns for 3 unknowns (A, B, C)

again, solve using linear-eqns

show polyo2_wgt