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Sp16 (w7)
L6.1

a) Fourier Background

b.) Fourier Lab

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a) Intro to Fourier

heres how we might start talking about Fourier Transforms (in a math class):

F.T. $$\Psi(\omega) = \int_{-\infty}^{\infty} \psi(t)\, e^{-i\omega t}\, dt$$

I.F.T. $$\psi(t) = \int_{-\infty}^{\infty} \Psi(\omega)\, e^{+i\omega t}\, d\omega$$

done, right? questions? assumptions?

comments: need to know $\psi(t)$ for all times, t, or conversely know $\Psi(\omega)$ for all ω

moving to the real world, we note that:

i) all real data is <u>discrete</u>

⟹ on board, then ML: Lecture8.m — AA

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- a common data type is a "time series" which implies data is sampled at regular (and user settable) time intervals. E.g. any data coming from an A/D converter.

So $\psi(\Delta t) = [\psi_0(\emptyset), \psi_1(\Delta t), \psi_2(2\Delta t), \text{etc}]$

a sample freq. is associated w/ this T.S.

$$\nu_s = \frac{1}{\Delta t}$$

(e.g. in B.M. lab $\nu_s = \frac{1}{1\emptyset}$ hz (?))

Important Fourier Transform Point 1 (IFTP1)
"for TS data sampled at rate ν_s, we can't 'see' freqs > $\emptyset.5\ \nu_s$

$$\nu_N = \frac{1}{2}\nu_s = \text{"Nyquist"}$$

IFTP 2
TS always sampled for finite duration defined as N points

$$T_s = T_{sample} = N\Delta t = \frac{N}{\nu_s}$$

we can't "see" freqs below $\nu_l = \frac{1}{T_s} = \frac{\nu_s}{N}$

Thus by setting ν_s + N, we set the range of freqs that we can "see"

$$\nu_\ell = \frac{\nu_s}{N} < \nu < \frac{1}{2}\nu_s = \nu_N$$

from a practical point of view, we might use

$$3\nu_\ell = \frac{3\nu_s}{N} \leq \nu \leq \frac{1}{2}\nu_N = \frac{1}{4}\nu_s$$

are we finished w/ IFTP's, no?

IFTP3 'aliasing' and 'foldover' mean that signals of freqs above ν_N foldover + alias, ie masquerade as lower-freq. signals

Mc lecture8.m $BB^{(t)}$ + $CC^{(\omega)}$

interpret for students

back to Fourier Theory

thinking first about F.T in integral form

$$\psi(t) = \int_{-\infty}^{\infty} \Psi(\omega) e^{+i\omega t} d\omega$$

some abstract "spectrum" "basis" functions

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IFTP 4

this all works b/c this particular set of cos, sin, $e^{\sim}$ are "orthonormal"

defined as $\sum_{\text{cycle}} \cos(w_a t + \phi_a) \cdot \cos(w_b t + \phi_b)$

time window

$= 0$ if $w_a \neq w_b$

$=$ some number if $w_a = w_b$

ML lecture8.m DD
show how this works,
sum of point-by-point cross-multiplication

ML lecture8.m EE
non orthogonality

Demo FFT L6 (CQ, etc)

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in 1822 Joseph Fourier posited that:

$$y(t) = A_0 + A_1 \cos(w_f t + \phi_1) + A_2 \cos(2w_f t + \phi_2) + \dots A_n \cos(nw_f t + \phi_n)$$

in words, any time series can be described as the sum of cosines (or sines) for particular frequencies. The amount of each one (A_j) and the related phase (ϕ_j) are related to the "Fourier coefficients"

Q1: do we know $y(t)$ for all time?

Q2: what is w_f ? $= \frac{2\pi V_s}{N} = \frac{2\pi}{N\Delta t} = \frac{2\pi}{T_s}$

time window

reminder comment: time series is "composed" of cosines or sines for particular, integer-multiple freqs ($j w_f$)

Fabulous Fourier App

So let's rewrite :

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$y(t) = A_0 + A_1 \cos(\omega_f t + \phi_1) + A_2 \cos(2\omega_f t + \phi_2)$
$+ \ldots A_{\frac{N}{2}} \cos(\frac{N}{2}\omega_f t + \phi_{\frac{N}{2}})$

only $A_{\frac{N}{2}}$ ($+ A_0$) coeffs + ϕs
to account for
$\nu < \nu_N = \frac{\nu_s}{2}$

ϕ's are phases relative to .....?
(start of time window)

because $e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$
we can rewrite in complex notation, and
as a sum

$$y(\Delta t) = C_0 + \sum_{k=1}^{N/2} \left[C_k e^{ik\omega_f t} + C_k^* e^{-ik\omega_f t} \right]$$

C_0, C_k's called Fourier coefficients
they are complex (except C_0), real + imag
parts such that:

$$\text{amp}(C_k) = \sqrt{(\text{real}(C_k))^2 + (\text{imag}(C_k))^2}$$

$$\text{phase}(C_k) = \tan^{-1} \frac{\text{imag}(C_k)}{\text{real}(C_k)}$$

and $C_k^* = \text{real}(C_k) - i\,\text{imag}(C_k)$ complex conjg.