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sp16 (w7)
L6.1

a) Fourier Background

b) Fourier Lab

a) Intro to Fourier

here's how we might start talking about Fourier Transforms (in a math class):

FT.
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

I.F.T.
$$f(t) = \int_{-\infty}^{\infty} F(\omega) e^{+i\omega t} d\omega$$

done, right? questions? assumptions?

Comments: need to know $f(t)$ for all times, t , or conversely know $F(\omega)$ for all ω

Moving to the real world, we note that:

i) all ^{real} Adats is discrete

⇒ on board, then URL: lectures.m-AA

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- a common data type is a "time series" which implies data is sampled at regular (and user settable) time intervals. Eg. any data coming from an A/D converter.

So $y(t) = [y_0(t), y_1(t), y_2(t), \dots]$

a sample freq. is associated w/ this T.S.

$$\nu_s = \frac{1}{\Delta t}$$

(eg. in B.M. lab $\nu_s = \frac{1}{10} \text{ Hz}$ (?)

Important Fourier Transform Point 1 (IFTPI)
"for T.S. data sampled at rate ν_s , we can't see' freqs $> 0.5 \nu_s$

$$\nu_N = \frac{1}{2} \nu_s = \text{"Nyquist"}$$

IFTPI 2

T.S. always sampled for finite duration defined as N points

$$T_s = T_{\text{sample}} = N \Delta t = \frac{N}{\nu_s}$$

we can't see' freqs below $\nu_0 = \frac{1}{T_s} = \frac{\nu_s}{N}$

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Thus by setting $v_s + N$, we set the range of freqs that we can "see"

$$v_e = \frac{v_s}{N} < v < \frac{1}{2} v_s = v_N$$

from a practical point of view, we might use

$$3v_e = \frac{3v_s}{N} \leq v \leq \frac{1}{2} v_s = \frac{1}{2} v_s$$

are we limited w/ DFT's, no?

DFT 3 'aliasing' and 'foldover' mean that signals of freq above v_N foldover & alias, i.e. masquerade as lower-freq. signals.

Mc lectures on BB + CC

interpret for students

back to Fourier Theory

thinking first about F.T. in integral form

$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

some observed "spectra" "basis" functions

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PTP 4

this all works b/c this particular set of $\cos, \sin, e^{j\omega t}$ are "orthonormal"

defined at $\sum_{\text{cycle}} \cos(\omega_a t + \phi_a) = \cos(\omega_b t + \phi_b)$

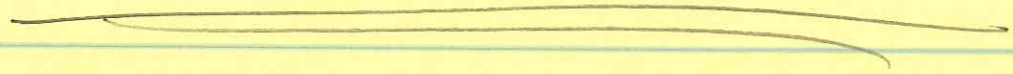
time window $\begin{cases} = 0 & \text{if } \omega_a \neq \omega_b \\ = \text{some number} & \text{if } \omega_a = \omega_b \end{cases}$

ML lectures on DP

show how this works, sum of point-by-point cross-multiplication

ML lectures on EE

non orthogonality



Demo PTP 6.6 (LQ, etc)

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in 1870 Joseph Fourier pointed
that:

$$y(t) = A_0 + A_1 \cos(\omega_s t + \phi_1) + A_2 \cos(2\omega_s t + \phi_2) + \dots + A_n \cos(n\omega_s t + \phi_n)$$

in words, any time series can be described as
the sum of cosines (or sines) for particular
frequencies. The amount of each one
(A_j) and the related phase (ϕ_j) are
related to the "Fourier coefficients"

Q1: do we know $y(t)$ for all time?

$$Q2: \text{what is } \omega_s? = \frac{2\pi N_s}{N} = \frac{2\pi}{N\Delta t} = \frac{2\pi}{T_s}$$

↑ ↑
time window

remainder component to time series is

"composed" of cosines or sines for
particular, integer-multiple freqs ($j\omega_s$)

Fabrizio Fourier App

so lets rewrite:

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$$\psi(t) = A_0 + A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(2\omega_1 t + \phi_2) + \dots + \frac{A_N}{2} \cos\left(\frac{N}{2}\omega_1 t + \phi_{\frac{N}{2}}\right)$$

only $A_{\frac{N}{2}}$ (+ A_0) coeffs + ϕ_s
to account for
 $v < v_N = \frac{v_s}{2}$

ϕ 's are phases relative to ... ?
(start of time window)

because $e^{int} = \cos(nt) + i \sin(nt)$
we can rewrite in complex notation, and
at a sum

$$\psi(t) = C_0 + \sum_{k=1}^{N/2} \left[C_k e^{ik\omega_1 t} + C_k^* e^{-ik\omega_1 t} \right]$$

C_0, C_k s called Fourier coefficients
they are complex (except C_0), real + imag
parts such that:

$$\text{amp}(C_k) = \sqrt{(\text{real}(C_k))^2 + (\text{imag}(C_k))^2}$$

$$\text{phase}(C_k) = \tan^{-1} \frac{\text{imag}(C_k)}{\text{real}(C_k)}$$

and $C_k^* = \text{real}(C_k) - i \text{imag}(C_k)$ complex
conj.