Midterm Extra Credit

Directions: Make sure to show all of your work to receive full credit. Do these problems on a separate sheet of paper. Make sure your work is clean and easy to follow. There are 68 points on this sheet just as there were on the midterm. Your midterm score will be replaced with the minimum of (the average of this exam and your original exam) and (15 percentage points above your original exam).

Problem 1. (6 points) Verify the following argument:

$$[(\neg p \vee \neg q) \wedge (r \to q) \wedge (\neg p)] \to r$$

Problem 2. (6 points) Find a counterexample:
For a given open statement $p(x, y)$, the statement

$$\exists x \forall y \, [p(x, y)]$$

is always equivalent to the statement

$$\forall y \exists x \, [p(x, y)]$$

Problem 3. A new government is formed in a small country of 27 citizens. There are 7 public officials: President, Prime Minister, Czar, Prince, and 3 Senators.

a. (6 points) How many ways can the offices be assigned from the 27 citizens?

b. (6 points) If Bob and Sue are each guaranteed that they will be one of the offices besides Senator, how many ways can the offices be assigned?

Problem 4. (10 points) Simplify

$$\neg[\neg[(p \wedge q) \vee r] \wedge \neg q]$$

Problem 5. (6 points) Suppose a license plate contains 3 letters followed by 3 numbers. If

1) there is no repetition among the letters,
2) no repetition among the numbers,
3) the 3-digit number must be divisible by 2
4) and the 3rd letter must be a vowel (that is A,E,I,O, or U)

how many license plates can be formed?

Problem 6. (8 points) Using truth tables, show that

$$p \veebar q \Leftrightarrow [(p \wedge \neg q) \vee (\neg p \wedge q)]$$

and

$$p \veebar q \Leftrightarrow \neg(p \leftrightarrow q)$$

Problem 7. (8 points) Suppose that John has 15 cupcakes and 17 doughnuts to distribute among 4 children. If each child gets at least one cupcake or one doughnut, how many ways can John distribute the cupcakes and doughnuts among the 4 children?

Problem 8. Define an Gaussian integer to be a number of the form $a + bi$, where $a, b \in \mathbb{Z}$, and $i = \sqrt{-1}$. Define the conjugate of a Gaussian integer $a + bi$ (denoted $\overline{a + bi}$) to be

$$\overline{a + bi} = a - bi$$

a) (6 points) Prove that if $x = a + bi$ is a Gaussian integer with $a + b$ odd, then $x\overline{x}$ is an odd integer (i.e. a number of the form $c + 0i$).

b) (6 points) Prove that if $x = a + bi$ is a Gaussian integer with a and b odd, then $\frac{x\overline{x}}{2}$ is an integer.

(For both parts you may use the fact that if k is an integer, then so is k^2).