Practice Midterm

Problem 1. In how many ways can the symbols $\diamondsuit, \clubsuit, \spadesuit, \blacksquare, \bigstar, \bigstar, \bigstar, \bigstar$ be arranged so that no $\bigstar$ is adjacent to another $\bigstar$?

Problem 2. (a) How many arrangements are there of the letters RIEMANNIAN?

(b) In how many of the arrangements in part (a) are the R and M adjacent (i.e next to each other)?

(c) In how many of the arrangements in part (a) are all the vowels adjacent?

Problem 3. A student is to answer 7 out of 10 questions on an examination. In how many ways can he make his selection if

(a) there are no restrictions?

(b) he must answer the first two questions?

(c) he must answer at least four of the first six questions?

Problem 4. Determine the number of integer solutions of

$$x_1 + x_2 + x_3 + x_4 = 53$$

where

(a) $x_i \geq 0$, $1 \leq i \leq 4$.

(b) $x_i \geq -2$, $1 \leq i \leq 4$.

Problem 5. In how many ways can Lisa toss 100 (identical) dice so that at least three of each type of face will be showing?

Problem 6. Determine all of the truth values assignments of p, q, and r that would make the following statement false:

$$q \leftrightarrow (-p \vee -q)$$

(you don't need a truth table, but you could determine this by constructing one ... in fact, you should try it out just so that you make sure that you remember how to do it).

Problem 7. Simplify

$$(-p \vee q) \wedge (p \wedge (p \wedge q))$$

Problem 8. Verify the following logical implication

$$\begin{array}{l} p \wedge q \\ p \rightarrow (r \wedge q) \\ r \rightarrow (s \vee t) \\ -s \\ \hline \therefore t \end{array}$$

Problem 9. Let $p(x)$, $q(x)$, and $r(x)$ denote the following open statements

$p(x)$: $x^2 - 8x + 15 = 0$

$q(x)$: x is odd

$r(x)$: $x > 0$

For the universe of all integers, determine the truth or falsity of each of the following statements.

a) $\forall x[p(x) \rightarrow q(x)]$

b) $\exists x[p(x) \rightarrow q(x)]$

c) $\forall x[(p(x) \vee q(x)) \rightarrow r(x)]$

Problem 10. Let n be an integer. Prove that n is even if and only if $31n + 12$ is even.