Midterm Review

Intro: This list is not necessarily comprehensive. To study for this midterm, I would know everything on this sheet, and I would practice as many problems as I could. Check out some of the problems from the chapters 1 and 2 chapter review. Also, look over old homeworks, quizzes, and the worksheet. The solutions to the quizzes and worksheet are up, along with partial solutions to the homework problems.

1.1 - The Rules of Sum and Product

◇ The Rule of Sum says that if one task can be performed m ways, another can be performed n ways, and neither they can't be performed simultaneously, then performing either task can be accomplished in $m + n$ ways.

For example, how many ways can you choose a soup OR a salad from a menu?

◇ The Rule of Product says that if a procedure can be brokend down into first and second stages, and if there are m possible outcomes for the first stage, and if for each of these outcomes there are n possibilities for the second stage, then the procedure can be carried out in mn ways.

For example, how many ways can you choose a soup and a salad?

1.2 - Permutations

Formulas:

$$0! = 1$$
$$n! = n(n-1)(n-2)\cdots(3)(2)(1) \quad n \geq 1$$

$$P(n,r) = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$$

◇ If there are n distinct objects and r is an integer, with $1 \leq r \leq n$, then by the rule of product, the number of permutations of size r for the n objects is $P(n,r)$.

For example, if you want to give out prizes for 1st, 2nd, and 3rd place, how many ways can this be done?

◇ If there are n objects with n_1 indistinguishable objects of a first type, n_2 indistinguishable objects of a second type, ... and n_r indistinguishable objects of an r^{th} type, where $n_1 + n_2 + \cdots + n_r = n$, then there are

$$\frac{n!}{n_1! n_2! \cdots n_r!}$$

linear arrangements fo the given n objects.

For example, if you want to know how many arrangements there are of the word Tennessee.

1.3 - Combinations

Formulas:

$$\binom{n}{r} = C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}, \ 0 \leq r \leq n$$

$$(x+y)^n = \binom{n}{0}x^0y^n+\binom{n}{1}x^1y^{n-1}+\binom{n}{2}x^2y^{n-2}+\cdots+\binom{n}{n-1}x^{n-1}y^1+\binom{n}{n}x^ny^0 = \sum_{k=0}^{n}\binom{n}{k}x^ky^{n-k}$$

◇ If we start with n distinct objects, each combination of r of these objects, with no reference to order, corresponds to $r!$ permutations of size r from the n objects. Thus the number of combinations of size r from a collection of size n is

$$\binom{n}{r} = C(n,r)$$

For example, how many ways can we choose 5 numbers for a lottery ticket? The order of the number doesn't matter.

◇ Make sure you understand how to use summation notation. That is, what does

$$\sum_{i=1}^{n} a_i$$

mean?

◇ Make sure you know the binomial theorem which is stated above.

1.4 - Combinations with Repetition

◇ There is really only one principle in this section. When we wish to select, with repetition, r of n distinct objects, we find that there are

$$\frac{(n+r-1)!}{r!(n-1)!} = \binom{n+r-1}{r} \text{ combinations}$$

Consequently, the number of combinations of n objects taken r at a time, with repetition, is $C(n+r-1,r)$.

◇ It seems that all questions from this section boil down to one problem. How many nonnegative integer solutions are there to the equation

$$x_1 + x_2 + \cdots + x_n = r$$

2.1 - Basic Connectives and Truth Tables

◇ Understand the symbols and truth values for

$$\neg,\ \wedge,\ \vee,\ \veebar,\ \rightarrow,\ \leftrightarrow$$

p	q	$-p$	$p \wedge q$	$p \vee q$	$p \veebar q$	$p \rightarrow q$	$p \leftrightarrow q$
0	0	1	0	0	0	1	1
0	1	1	0	1	1	1	0
1	0	0	0	1	1	0	0
1	1	0	1	1	0	1	1

◇ Be able to translate symbolic statements into English statements, and vice versa.

◇ Be able to construct a truth table for a given statement.

◇ Be able to define a tautology and a contradiction.

2.2 - Logical Equivalence: The Laws of Logic

◇ Understand what it means for 2 logical statements to be equivalent. (i.e. have the same truth values)

$--p \Leftrightarrow p$	Law of Double Negation
$-(p \vee q) \Leftrightarrow -p \wedge -q$	DeMorgan's Laws
$-(p \wedge q) \Leftrightarrow -p \vee -q$	
$p \vee q \Leftrightarrow q \vee p$	Commutative Laws
$p \wedge q \Leftrightarrow q \wedge p$	
$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	Associative Laws
$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	Distributive Laws
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	
$p \vee p \Leftrightarrow p$	Idempotent Laws
$p \wedge p \Leftrightarrow p$	
$p \vee F_0 \Leftrightarrow p$	Identity Laws
$p \wedge T_0 \Leftrightarrow p$	
$p \vee -p \Leftrightarrow T_0$	Inverse Laws
$p \wedge -p \Leftrightarrow F_0$	
$p \vee T_0 \Leftrightarrow T_0$	Domination Laws
$p \wedge F_0 \Leftrightarrow F_0$	
$p \vee (p \wedge q) \Leftrightarrow p$	Absorption Laws
$p \wedge (p \vee q) \Leftrightarrow p$	

◇ Understand the dual of a statement.

◇ Understand the contrapositive, converse, and inverse of an implication.

◇ Be able to simplify a logical statement.

2.3 - Logical Implication: The Rules of Inference

◇ This section was concerned with arguments of the form

$$(p_1 \wedge p_2 \wedge p_3 \wedge \cdots \wedge p_n) \rightarrow q$$

To show that an argument of this form is valid, one can show that this is a tautology, but really we only need to check to see that the conclusion is true if all of the premises are true.

◇ Rules of Inference

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$ Modus Ponens

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$ Law of the Syllogism

$$\begin{array}{l} p \rightarrow q \\ -q \\ \hline \therefore -p \end{array}$$ Modus Tollens

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Rule of Conjuction

$$\begin{array}{l} p \vee q \\ -p \\ \hline \therefore q \end{array}$$

Rule of Disjunctive Syllogism

$$\begin{array}{l} -p \to F_0 \\ \hline \therefore p \end{array}$$

Rule of Contradiction

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$$

Rule of Conjunctive Simplification

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

Rule of Disjunctive Amplification

$$\begin{array}{l} p \wedge q \\ p \to (q \to r) \\ \hline \therefore r \end{array}$$

Rule of Conditional Proof

$$\begin{array}{l} p \to r \\ q \to r \\ \hline \therefore (p \vee q) \to r \end{array}$$

Rule for Proof by Cases

$$\begin{array}{l} p \to q \\ r \to s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

Rule of the Constructive Dilemma

$$\begin{array}{l} p \to q \\ r \to s \\ -q \vee -s \\ \hline \therefore p \vee -r \end{array}$$

Rule of the Destructive Dilemma

2.4 - The Use of Quantifiers

⋄ Know what is meant by open statement, universe, existential quantifier, and universal quantifier.

⋄ Be able to establish the truth value of a statement involving a universal quantifier, and be able to establish the truth value of a statement obtained from an open statement by replacing the free variable with an element of the universe.

⋄ Know how to define contrapositive, converse, and inverse for a universally quantified implication.

⋄ Know the interpretation of the negation of a quantified statement. i.e. $-[\forall x p(x)] \Leftrightarrow \exists x - p(x)$

⋄ Know the commutativity relations among universal quantifiers. i.e. Is $\forall x \exists y$ the same as $\exists y \forall x$?

2.5 Quantifiers, Definitions, and the Proofs of Theorems

◇ Know that in mathematical definitions, we sometimes use the implication when it is clear that the biconditional is what was intended.

◇ Know the rules of universal specification and generalization and how to apply them in verifying an argument.