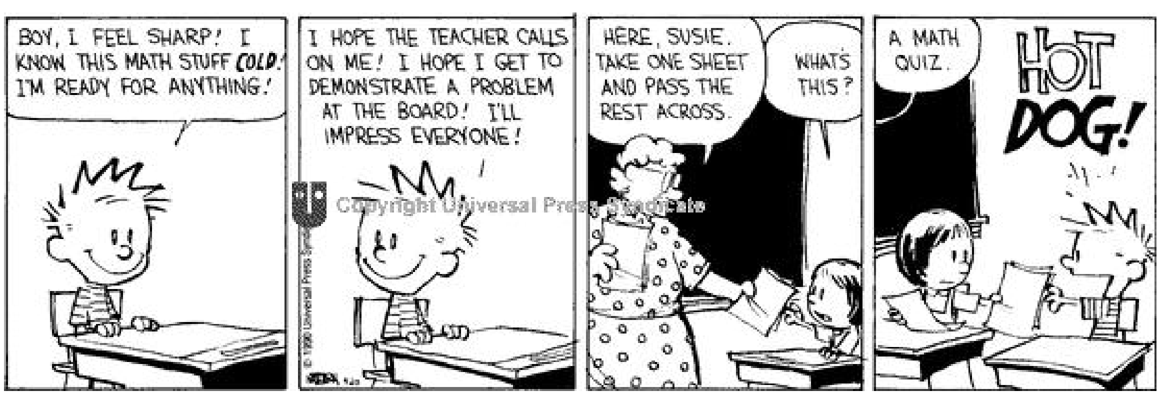
Name_______________

Midterm (11/02/09)

Directions: Make sure to show all of your work to receive full credit. You may not use your neighbor's papers, or anything else that would be construed as cheating. Please turn off all cell phones, mp3 players, or pretty much anything else that can turn off. You are allowed a calculator.



Problem 1. [**True/False**] (3 points each) - Indicate if the statements are true or false.

a) __________

$$p \veebar q \Leftrightarrow \neg(p \leftrightarrow q)$$

b) __________

$$[(\neg p \vee \neg q) \wedge (r \rightarrow q) \wedge (\neg p)] \rightarrow (\neg r)$$

c) __________ For a given open statement $p(x, y)$, the statement

$$\exists x \forall y \, [p(x, y)]$$

is always equivalent to the statement

$$\forall y \exists x \, [p(x, y)]$$

Suppose that I know the following facts:

- All mathematicians are cool.
- Ted is a mathematician.
- Frank is not a mathematician.
- Charles is cool.
- George is not cool.

Problem 2. (3 points) If $p(x)$ is the statement "x is a mathematician", and $q(x)$ is the statement "x is cool". Write the statement "All mathematicians are cool" in symbolic form.

Problem 3. (3 points each) For the following statements, indicate whether or not we can draw this conclusion from the given information.

a) Charles is a mathematician.

b) Ted is cool.

c) George is not a mathematician.

Problem 4. A new government is formed in a small country of 27 citizens. There are 7 public officials: President, Prime Minister, Czar, Prince, and 3 Senators.

a. (5 points) How many ways can the offices be assigned from the 27 citizens?

b. (5 points) If Bob and Sue are each guaranteed that they will be one of the offices besides Senator, how many ways can the offices be assigned?

Problem 5. (7 points) Simplify

$$(-p \wedge q) \vee (p \vee (p \vee q))$$

Problem 6. Suppose that Mr. Smith's 4th grade class is taking a class picture. He has 12 students.

a) (4 points) How many ways can they be arranged for a class picture if they are all standing in a line.

b) (4 points) How many ways can they be arranged for a class picture if they are standing in two lines with 5 children in the front row, and 7 children in the back row.

c) (4 points) How many ways can they be arranged for a class picture if they are standing in two lines (with at least one student in each line).

Problem 7. (5 points) In how many ways can Lisa toss 50 (identical) dice so that at least three of each type of face will be showing?

Problem 8. (8 points) Recall that an integer n is odd if $n = 2k + 1$ for some integer k. Prove that in the universe of integers if $a + b$ is an odd number, then $a - b$ is an odd number.

Problem 9. (5 points) Suppose a license plate contains 3 letters followed by 3 numbers. If there is no repetition among the letters, no repetition among the numbers, and the 3-digit number must be divisible by 5, how many license plates can be formed?

Problem 10. (Bonus - 3 points) How many nonnegative integer solutions are there to the pair of equations

$$x_1 + x_2 + x_3 + \cdots + x_7 = 37$$

$$x_1 + x_2 + x_3 = 6$$

(i.e. the x_1, x_2, and x_3 satisfy both equations).

Problem 11. (Bonus - 3 points) Write the sum

$$(-3)^{100} + \frac{100}{1} \cdot 2 \cdot (-3)^{99} + \frac{100 \cdot 99}{2 \cdot 1} \cdot 2^2 \cdot (-3)^{98} + \frac{100 \cdot 99 \cdot 98}{3 \cdot 2 \cdot 1} \cdot 2^3 \cdot (-3)^{97} + \cdots$$

$$\cdots + \frac{100 \cdot 99 \cdot 98 \cdot \cdots \cdot \ldots \cdot 4 \cdot 3 \cdot 2}{99 \cdot 98 \cdot 98 \cdot 97 \cdot \ldots \cdot 3 \cdot 2 \cdot 1} \cdot 2^{99} \cdot (-3) + 2^{100}$$

as a single integer.

$\neg\neg p \Leftrightarrow p$	Law of Double Negation
$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	DeMorgan's Laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	
$p \vee q \Leftrightarrow q \vee p$	Commutative Laws
$p \wedge q \Leftrightarrow q \wedge p$	
$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	Associative Laws
$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	Distributive Laws
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	
$p \vee p \Leftrightarrow p$	Idempotent Laws
$p \wedge p \Leftrightarrow p$	
$p \vee F_0 \Leftrightarrow p$	Identity Laws
$p \wedge T_0 \Leftrightarrow p$	
$p \vee \neg p \Leftrightarrow T_0$	Inverse Laws
$p \wedge \neg p \Leftrightarrow F_0$	
$p \vee T_0 \Leftrightarrow T_0$	Domination Laws
$p \wedge F_0 \Leftrightarrow F_0$	
$p \vee (p \wedge q) \Leftrightarrow p$	Absorption Laws
$p \wedge (p \vee q) \Leftrightarrow p$	

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

Modus Ponens

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Law of the Syllogism

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Modus Tollens

$$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$$

Rule of Conjuction

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$$

Rule of Disjunctive Syllogism

$$\begin{array}{l} \neg p \to F_0 \\ \hline \therefore p \end{array}$$

Rule of Contradiction

$$\begin{array}{l} p \land q \\ \hline \therefore p \end{array}$$

Rule of Conjunctive Simplification

$$\begin{array}{l} p \\ \hline \therefore p \lor q \end{array}$$

Rule of Disjunctive Amplification

$$\begin{array}{l} p \land q \\ p \to (q \to r) \\ \hline \therefore r \end{array}$$

Rule of Conditional Proof

$$\begin{array}{l} p \to r \\ q \to r \\ \hline \therefore (p \lor q) \to r \end{array}$$

Rule for Proof by Cases

$$\begin{array}{l} p \to q \\ r \to s \\ p \lor r \\ \hline \therefore q \lor s \end{array}$$

Rule of the Constructive Dilemma

$$\begin{array}{l} p \to q \\ r \to s \\ \neg q \lor \neg s \\ \hline \therefore p \lor \neg r \end{array}$$

Rule of the Destructive Dilemma