Quiz #4 10/30/09

Name: _____

Directions: Make sure to read each problem carefully. To receive full credit, you must show all of your work.

Problem 1. (2 points each part) Let $p(x, y)$ be the open statement $|x| \leq |y|$ in the universe of real numbers. Which of these statements are true, and which are false?

a) $\forall y \exists x[p(x, y)]$

b) $\forall x \exists y[p(x, y)]$

c) $\exists y \forall x[p(x, y)]$

Problem 2. (6 points) Recall that an odd number is defined to be a number of the form $2k + 1$ where k is an integer. Using this definition, prove that if the product of two integers is odd, then both of the integers themselves must be odd.

Problem 3. (3 points) For the universe of all integers, let

$$\begin{aligned} p(x): &\quad x \text{ is divisible by } 4 \\ q(x): &\quad x \text{ is even} \\ r(x): &\quad x \text{ is a perfect square} \end{aligned}$$

Write the statement

If x is even and x is a perfect square, then x is divisible by 4

in symbolic form.

Problem 4. (Bonus - 3 points) Prove that the above statement is true.