Quiz #6 11/20/09

Name: ____________________

Directions: Make sure to read each problem carefully. To receive full credit, you must show all of your work.

Problem 1. (3 points) Find the quotient (q) and remainder (r), when $a = -47$ is divided by $b = 7$.

$$-47 = 7(-7) + 2$$

That is, the quotient is -7, and the remainder is 2.

Problem 2. (5 points) Prove (using induction) that for all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

Proof. Base Case:

$$\sum_{i=1}^{1} i(i+1) = 2 = \frac{1(1+1)(1+2)}{3}$$

Inductive Step:
Assume that this is true for $n = k$, i.e.

$$\sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$$

Then,

$$\sum_{i=1}^{k+1} i(i+1) = \left(\sum_{i=1}^{k} i(i+1)\right) + (k+1)(k+2)$$
$$= \frac{k(k+1)(k+2)}{3} + \frac{3(k+1)(k+2)}{3} = \frac{(k+1)(k+2)(k+3)}{3}$$

which is exactly what we wanted to show. □

Problem 3. (3 points) If the sequence $\{a_n\}$ is explicitly defined by

$$a_n = 2^n - 1$$

then define a_n recursively (i.e. in terms of a_{n-1} and not n).

Note that $2a_{n-1} = 2^n - 2$. Thus if we add one more, we get a_n. Therefore

$$a_n = 2 \times a_{n-1} + 1$$

Problem 4. (5 points) Prove that if n is odd, then $8|(n^2 - 1)$.

Proof. If n is odd, then there exists a number k such that $n = 2k + 1$.

$$n^2 - 1 = (2k + 1)^2 - 1 = 4k^2 + 4k = 4(k^2 + k) = 4(k + 1)k$$

which is clearly divisible by 4. Since $k \in \mathbb{Z}$, it is either even or odd.

If k is even, then there exists some integer m such that $k = 2m$, and

$$n^2 - 1 = 4(2m)(k + 1) = 8m(k + 1)$$

which is divisible by 8. If k is odd, then $k + 1$ is even, and the proof is analagous. □