

**Problem 1.** Suppose a new government of a country of 25 people has three primary political offices: President, Vice President, and Sheriff.

a. How many ways can the country assign the offices to 3 of its citizens?

b. If Bob and Sue each get one of the 3 offices, how many ways will it be possible to assign the offices? (Hint: Figure out how many ways you can assign offices to Bob and Sue first)

c. Suppose a new political office of Deputy is introduced. There are two Deputies in this country. How many ways can the country assign 5 people to these political offices?

**Problem 2.** How many ways can the letters in the word "RIEMANNIAN" be arranged?

In the example above regarding the new government, the Deputies were chosen but not arranged in a particular order (i.e. there is no first Deputy and second Deputy, there are just 2 Deputies). If we have  $n$  distinct objects and we want to pick  $r$  of them in a particular order, there are

$$P(n, r) = \frac{n!}{(n-r)!}$$

of them. If order ceases to matter, then given a particular arrangement, there are  $r!$  equivalent arrangements. Thus, if we have  $n$  distinct objects and we want to choose  $r$  of them without regards to order, there are

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{(n-r)!r!} = \binom{n}{r}$$

of them.

**Example 1.** If one wants to draw 5 cards (a hand) from a 52 card deck, there are

$$C(52, 5) = \binom{52}{5} = \frac{52!}{47!5!} = 2598960$$

possible different hands that you could end up with.

**Problem 3.** Suppose that Andy wins a boat trip for 5 people (including himself). Andy has 34 friends. How many ways can he select 4 friends to go with him on the boat ride?

**Problem 4.** A statistician is conducting a survey of college students at a party where there are 35 underclassmen and 57 upperclassmen.

a. She wants to select 9 people for her survey. How many ways can she select 9 random people from the party?

b. She decides that of these 9 people, she wants exactly 5 of them to be upperclassmen. How many ways can she select 9 people from the party now?