Math 231 **Name** ________________ **Worksheet 10/02/09**

Problem 1. Suppose a new government of a country of 25 people has three primary political offices: President, Vice President, and Sheriff.

a. How many ways can the country assign the offices to 3 of its citizens?

$$P(25,3) = \frac{25!}{22!}$$

b. If Bob and Sue each get one of the 3 offices, how many ways will it be possible to assign the offices? (Hint: Figure out how many ways you can assign offices to Bob and Sue first)

$$P(3,2) \times P(23,1) = \frac{3!23!}{1!22!} = 6 \times 22 = 132$$

c. Suppose a new political office of Deputy is introduced. There are two Deputies in this country. How many ways can the country assign 5 people to these political offices?

There is no ordering for the Deputies, so for example if Bob and Sue are voted in as Deputies, that is the same as if Sue and Bob are voted in. If they were different, then there would be

$$P(25,3) \times P(22,2)$$

but we would be double counting. Hence, there are

$$\frac{P(25,3) \times P(22,2)}{2!} \text{ ways}$$

Problem 2. How many ways can the letters in the word "RIEMANNIAN" be arranged?

If the Ns, As, and Is were different, it would be

$$P(10,10) = 10! \text{ ways}$$

however, we would be over counting. The question we have to answer is how many of these do we need to get rid of. The answer is that for each arrangement (with Ns indistinguishable from each other, Is and As as well), there are 3!2!2! was to distinguish the Ns, Is, and As. Thus there are

$$\frac{10!}{3!2!2!}$$

ways to arrange the word RIEMANNIAN.

In the example above regarding the new government, the Deputies were chosen but not arranged in a particular order (i.e. there is no first Deputy and second Deputy, there are just 2 Deputies). If we have n distinct objects and we want to pick r of them in a particular order, there are

$$P(n,r) = \frac{n!}{(n-r)!}$$

of them. If order ceases to matter, then given a particular arrangement, there are $r!$ equivalent arrangements. Thus, if we have n distinct objects and we want to choose r of them without regards to order, there are

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!r!} = \binom{n}{r}$$

of them.

Example 1. If one wants to draw 5 cards (a hand) from a 52 card deck, there are

$$C(52,5) = \binom{52}{5} = \frac{52!}{47!5!} = 2598960$$

possible different hands that you could end up with.

Problem 3. Suppose that Andy wins a boat trip for 5 people (including himself). Andy has 34 friends. How many ways can he select 4 friends to go with him on the boat ride?

$$C(34,4) = \binom{34}{4}$$

Problem 4. A statistician is conducting a survey of college students at a party where there are 35 underclassmen and 57 upperclassmen.

a. She wants to select 9 people for her survey. How many ways can she select 9 random people from the party?

$$\binom{92}{9}$$

b. She decides that of these 9 people, she wants exactly 5 of them to be upperclassmen. How many ways can she select 9 people from the party now?

$$\binom{57}{5}\binom{35}{4}$$