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- Differentiate the equation, treating y as a function of x even though it is not expressed as one. Remember to use the chain rule to differentiate y, since we are treating it as a function of x!

- The resulting expression has $\frac{dy}{dx}$ in it, since we have used the chain rule on y.

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Example 2. *Two manufacturers of widgets are in direct competition. Because of the many variables in pricing and* *publicity, the number of widgets they sell does not add up to a constant, but* $6x^2 + xy + 5y^2 = 120,000$*, where* x *is the number of widgets sold per day by the first company and* y *by the second.*

publicity, the number of widgets they sell does not add up to a constant, but $6x^2 + xy + 5y^2 = 120,000$*, where* x *is the number of widgets sold per day by the first company and* y *by the second. If both companies are currently selling* 100 *widgets, what would be the effect on the first company if the second is increase sales by ten widgets per day.*

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Example 4. *When the price of gadgets is p dollars each, the manufacturer is willing to supply x hundred units where $x^2 - 2\frac{x}{5+p} - p^2 = 20$.*

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Theorem 5. *The derivative of* $\ln(x)$ *with respect to* x *is* $\frac{1}{x}$.

Proof: Start with the fact that $e^{\ln x} = x$, and differentiate both sides. Remember that we don't really know yet what the derivative of $\ln x$ is. To take the derivative of the left side, we must use the chain rule; the

derivative of x is straightforward:

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Back when we were determining where a function increased or decreased, we saw that the first step was finding places where $f'(x) = 0$.

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Definition 6. *A number x in the domain of a function $f(x)$ is a stationary number if $f'(x) = 0$ and is a singular number if the derivative at x does not exist.*

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Example 7. *Find all the stationary points for the function* $f(x) = x^3 - \frac{9}{2}x^2 + 2x - 5$*. Find all the singular points of the function* $g(x) = |5 + 4x - x^2|$*.*

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