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- $\lim_{x\to\infty} \frac{3x}{x^2+4}$.

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- $\lim_{x\to\infty} \frac{5x^2+1}{x-3x^2}$.

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We can now justify this theorem carefully. The key is to understand a general formula for $(x+h)^n$, which starts out $x^n + n \cdot x^{n-1} + \cdots$.

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The main idea mathematically is that since the derivative is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ the derivative is approximately equal to the difference quotient when $h=1$, if $f(x)$ is much larger than one (for example, if $f(x)$ is national debt).

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This leads us to the following definitions.

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Example 4. *If it costs approximately* $1000 - \frac{500}{x}$ *dollars to produce* x *CD's, what is the marginal cost of producing the* 20*th CD?*

Example 5. *Suppose the value of the marginal profit function for producing the* 1000*th jar of peanut butter was* -0.25.

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- *The actual cost and profit for producing the tenth duckie.*

Moral of the story - the marginal cost, revenue and profit functions are only *approximations* to the amounts needed/earned for producing one more item. But they are good approximations and (because of the rules we learn for calculus) are easier to work with than the true values.