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Definition 1. *The inverse of the exponential function* a^x *is called the logarithm function (with a base of* a*) denoted* $\log_a(x)$*. By this definition,* $\log_a a^x = x$ *and* $a^{\log_a x} = x$*.*

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The properties of the logarithm function follow from those of the exponential function.

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Technology permitting, we may elaborate on this by looking at graphs of these and other functions.

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Question 6. *How could we calculate exactly how fast the ball is going after two seconds?*

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What do you notice about this and the previous

problem?

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Example 10. *Analyze different measured and predicted* *rates of change for world population according to:*

`http://www.unfpa.org/6billion/pages/worldpopgrow`