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Example 4. *Find the derivatives of* $(3x^2-4)^2$, $f(x)=\sqrt{x^3-2x+577}$, *and* $(-2x+\sqrt{5})^{100}$.

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Example 8. *Find the derivative of the function* $f(x) = x\ln(x)$.