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Example 1. *The derivative game: given some graphs of* *derivative functions, sketch possible graphs for the original functions.*

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Definition 2. *The derivative of $f(x)$ with respect to x*

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Next, we state a rule which relates for example the derivatives of x^3 and $-53x^3$.

Theorem 7. *If* $g(x) = cf(x)$ *then* $\frac{d}{dx}g(x) = c\frac{d}{dx}$*.*

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Theorem 9. $\frac{d}{dx}\{f(x)+g(x)\} = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$

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Example 10. *What is the derivative of* $3x^2 + \frac{1}{x}$*?*

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