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Functions can be about measurements of a quantity, such as a cost function which measures for each time the amount it costs to produce a given good.

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Even if you are more interested in functions which measure real-world quantities, part of the power of calculus, and of mathematics in general, is connecting those functions with algebraic functions. For example, both $F = ma$ (Newton) and $E = mc^2$ (Einstein) changed the world, relating fundamental physical quantities through simple functions.

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Example 4. *If $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3x + 2$, what*

is the domain of the function $\frac{f(x)}{g(x)}$*?*

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Example 5. *To measure speed, we take distance travelled and divide it by time. What if we tried to measure the speed of a jet by using a stopwatch over 100 yards?*

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Definition 8. *The slope of the line $y = mx + b$ is equal to m. It measures the change in y if x is increased by one. If one is not given the slope explicitly, it can be computed by $m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are*

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The different descriptions we will use are as follows, listed from simplest to most complicated. We translate each to the standard form, and give an example.

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