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- *P is not a relative extremum if $f'(x)$ has the same sign on both sides of c.*

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Example 4. *Classify the critical points of the function* $\ln(|x^2 - 2| + 1)$ *over the interval from* $-e$ *to* e*.*

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Theorem 5. [First Optimization Theorem] *A continuous function defined everywhere on some interval I obtains its absolute maxima and minima either at the end(s) of I or at some critical number(s) in I.*

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Example 6. *Maximize profit when the price at which* q *units can be sold is* $p(q) = 25 - q$ *and it costs $10 to produce each unit.*

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- Compared values at all of these points - the largest is the max and the smallest is the min.

Example 7. *Find the maximum and minimum values of the function* $\frac{x^2+3}{x+1}$ *as* x *can take values from* 0 *to* 5*.*

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Example 8. *Suppose advertising costs \$1000 per unit (say for magazine adds), and product development costs \$20000 per unit. Suppose that the profits generated from* x *units of advertising and* y *units of product development are* xy^2 *thousands of dollars. If a company has \$10000 to spend on advertising and product development together,*

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We will formalize our techniques next time.