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Example 1. *If $f(x) = x$ and $g(x) = x$ then $\frac{d}{dx}f(x) = 1 = \frac{d}{dx}g(x)$. So the product of the derivatives is $1 \cdot 1 = 1$.*

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$$\frac{d}{dx}\left(f(x)g(x)\right) = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)].$$

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- *Apply the theorem with* $f(x) = x^2$, $g(x) = (x - 5)$, *and check the result by multiplying the two and expanding and then taking the derivative.*

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- *Find the tangent line to the curve* $y = (2\sqrt{x} - 1)(x + 4)$ *at* $x = 1$.

Where did the product rule come from? It helps to think geometrically, with $f(x)$ and $g(x)$ measuring lengths of a rectangle, and their product $f(x) \cdot g(x)$ measuring area.

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The at first surprising nature of the product rule has confused many bright people in many settings, including Leibniz who was one of the inventors of calculus.

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Note that there is a big difference between the derivative of the percentage and what you get by "turning the derivative into a percentage" by taking $\frac{f'(x)}{f(x)}$.

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