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Example 1. *Take the derivatives of* $(\frac{e^x \ln x}{x^2+1})^9$ *and* $\ln(\frac{e^x}{\sqrt{x}})^9$*.*

Implicit Differentiation

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Example 2. *Find the equation of the tangent line to the circle* $x^2 + y^2 = 25$ *at the point* $(3, 4)$.

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• *Find* $\frac{dy}{dx}$ *when* $e^{xy+4x-3} = 5x^2$*. (Do this twice, the second time by taking the natural logarithm of both sides).*

Example 4. *Two manufacturers of widgets are in direct competition. Because of the many variables in pricing and publicity, the number of widgets they sell does not add up to a constant, but* $6x^2 + xy + 5y^2 = 120,000$*, where* x *is the number of widgets sold per day by the first company and* y *by the second.*

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