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Our rules are far from being enough to take the derivatives of many familiar functions.

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Limits at finite points are conceptually trickier. In order to get the logic straight sometimes one has to pretend that the function does not exist at that point and use the values at nearby points to, if possible, come up with a single possible value which fits.

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