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Example 2. *Find all the stationary points for the function $f(x) = x^3 - \frac{9}{2}x^2 + 2x - 5$. Find all the singular points of the function $g(x) = |5 + 4x - x^2|$. Graph these functions and say what you see at critical points.*

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- *P is not a relative extremum if $f'(x)$ has the same sign on both sides of c.*

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