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The derivative of percentage should not be confused with the notion of *percentage rate of change*. If $f(x)$ is measuring a quantity, the percentage rate of change of that quantity is $\frac{f'(x)}{f(x)} \times 100\%$. In practical terms, the derivative of percentage can be volatile, while the percent rate change is more of a “big picture” number.

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- Composing two functions is so natural that you've done it without thinking about it.

- It makes no sense to take the composite of two numbers so unlike addition of functions we cannot rely on previous intuition.

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- In some important ways, composition behaves quite differently from addition and multiplication of functions.

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Sometimes composition of functions is denoted $f \circ g$, which makes it look even more akin to multiplication.

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