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Informally, a tangent line “kisses” the curve. The derivative at x which measures instantaneous change at x, is the slope of the tangent line at x.

We already know one point on the tangent line, namely the point on the curve which it is kissing.

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Example 3. *Calculate the equation of the line tangent to* $f(x) = \frac{1}{x}$ *when* $x = 1$*.*

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Theorem 4. *A function is increasing at* x *if its derivative there is positive. A function is decreasing at* x *if the derivative at* x *is negative.*

Example 5. *Find where the function* $f(x) = 2x - x^2$ *is increasing or decreasing.*

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Example 6. *The derivative game: given some graphs of derivative functions, sketch possible graphs for the original functions.*

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Example 9. *Suppose price index, measuring the aggregate price for a large cross-section of household goods measured in thousands of dollars, has values* $2 - \frac{1}{1+x}$ *over two years. What is the rate of price increase over* *two years?*

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