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Example 1. *A shoe maker estimates that the profit for selling shoes as a function of price (accounting for the market equilibrium) is* $-5 + p - \frac{p^2}{40}$ *per shoe.*

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The derivative of exponential functions

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But notice that a^x has no dependence on h so we can pull it out of the limit and get $a^x \lim_{h\to 0}\frac{a^h-1}{h}$. This last limit is going to be some constant, independent of x. It turns out to be equal to $\ln(a)$!

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Knowing just the derivative of e^x allows us to compute

the derivatives of functions made from e^x and polynomials using the chain, product and quotient rules.

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The fact that the derivative of e^{rx} is r times itself makes it useful in modeling populations, investments, temperatures...

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And again, once we combine with other rules, we can now go wild taking derivatives of complicated functions.

Example 5. *Find the derivatives of the functions*

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