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We can understand the terminology when we think of the meaning of the second derivative, as the rate of change of the first derivative. A positive second derivative means that if the first derivative were positive it is becoming more positive, and if it were negative it would become less negative. In either case, the graph of the function "curls up", which is what it means to be concave upward.

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Example 2. *Find where the function* $x^4 - 4x^3 - 18x^2 + 57x - \sqrt{7}$ *is concave upward or downward.*

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Concavity information will be useful in graphing curves.

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Theorem 3. *Let* $P = (c, f(c))$ *be a stationary point of a function* $f(x)$.

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- *P is a relative minimum if $f''(c) > 0$.*

Example 4. *A gag store can buy whoopie cushions at \$1.25 each and estimates that if they are sold for x dollars each, they can sell $10e^{-0.02x}$ each week. Express the profit as a function of x and find the price at which profit is maximized.*

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- Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, if they exist, in order to see where the graph "begins" and "ends" and draw in these ends if they can be determined.

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- Calculate the second derivative and find where it is zero, positive and negative in order to determine where f is concave upward and downward.

- Check where the sign of the derivative changes or use the second derivative test to determine which critical points are local maxima or minima.

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- Draw in “cups” at local minima, “caps” at local maxima, and one of four kinds of curve, as sketched on the board, in the regions where the the signs of the first and second derivative do not change.

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- Fill in the parts of the graph in between the curves you have put in.

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