We continue our study of Riemann sums, today emphasizing the connection to areas.

We continue our study of Riemann sums, today emphasizing the connection to areas.

Definition 1. *If f is a function over some interval $[a, b]$ then the Riemann sum of f with n terms is defined as*

$$RS_n f|_a^b = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_n)],$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

Example 2. *Compute the Riemann sum for the function* $f(x) = x^2 - 5$ *over the interval from* 0 *to* 4 *with* $n = 8$*.*

Riemann sums are easy to do with the help of a computer.

Example 2. *Compute the Riemann sum for the function* $f(x) = x^2 - 5$ *over the interval from* 0 *to* 4 *with* $n = 8$.

Riemann sums are easy to do with the help of a computer.

Example 3. *Use Excel to compute the Riemann sum with everything as in the previous example but with* $n = 20$.

Areas and Riemann sums

Areas and Riemann sums

We have seen that Riemann sums arise natural when one is trying to compute a total displacement from velocity data, total cost from marginal cost data, etc.

Areas and Riemann sums

We have seen that Riemann sums arise natural when one is trying to compute a total displacement from velocity data, total cost from marginal cost data, etc. They first arose arose historically in computing areas under curves,

Areas and Riemann sums

We have seen that Riemann sums arise natural when one is trying to compute a total displacement from velocity data, total cost from marginal cost data, etc. They first arose arose historically in computing areas under curves, a first step to computing areas of arbitrary shapes.

Areas and Riemann sums

We have seen that Riemann sums arise natural when one is trying to compute a total displacement from velocity data, total cost from marginal cost data, etc. They first arose arose historically in computing areas under curves, a first step to computing areas of arbitrary shapes.

Example 4. *Use rectangles to approximate the area of the trapezoid which is under the graph of* $y = 100 - 32x$*, above the* x*-axis and between the lines* $x = 3$ *and* $x = 4$*.*

Areas and Riemann sums

We have seen that Riemann sums arise natural when one is trying to compute a total displacement from velocity data, total cost from marginal cost data, etc. They first arose arose historically in computing areas under curves, a first step to computing areas of arbitrary shapes.

Example 4. *Use rectangles to approximate the area of the trapezoid which is under the graph of* $y = 100 - 32x$*, above the* x*-axis and between the lines* $x = 3$ *and* $x = 4$*. Compare this question with the first example from the* *previous lecture.*

previous lecture. Use geometry to get an exact answer.

previous lecture. Use geometry to get an exact answer.

As we see in this example, the areas of individual rectangles used to approximate area are exactly the terms which occur in a Riemann sum.

previous lecture. Use geometry to get an exact answer.

As we see in this example, the areas of individual rectangles used to approximate area are exactly the terms which occur in a Riemann sum. This relationship is so natural, we won't muddy the waters by trying to formalize it. We'll just give one more example.

previous lecture. Use geometry to get an exact answer.

As we see in this example, the areas of individual rectangles used to approximate area are exactly the terms which occur in a Riemann sum. This relationship is so natural, we won't muddy the waters by trying to formalize it. We'll just give one more example.

Example 5. *Write down and (use computer to) evaluate a Riemann sum with twenty terms which approximates the area under the curve* $y = \frac{x}{x^2+2}$ *between* $x = 1$ *and* $x = 3$.

The Fundamental Theorem

The Fundamental Theorem

We return to the the Fundamental Theorem of Calculus.

The Fundamental Theorem

We return to the the Fundamental Theorem of Calculus.

The way mathematicians usually organize these ideas is by defining $\int_a^b f(x)dx$ to be the limit as $N \longrightarrow \infty$ of $RS_N|_a^b f(x)$.

Theorem 6. [Fundamental Theorem] *$\int_a^b f(x)dx = F(b) - F(a)$ where F is any anti-derivative for f.*

The Fundamental Theorem

We return to the the Fundamental Theorem of Calculus.

The way mathematicians usually organize these ideas is by defining $\int_a^b f(x)dx$ to be the limit as $N \to \infty$ of $RS_N|_a^b f(x)$.

Theorem 6. [Fundamental Theorem] $\int_a^b f(x)dx = F(b) - F(a)$ *where* F *is any anti-derivative for* f*.*

This theorem now makes some intuitive sense, in the case where $f(x)$ is measuring the marginal change in some quantity $F(x)$.

some quantity $F(x)$. Adding up all of those changes yields the total change $F(b) - F(a)$.

some quantity $F(x)$. Adding up all of those changes yields the total change $F(b) - F(a)$. Indeed, we can almost (ignoring issues about limits) see a mathematically complete proof by writing in the definition of the derivative of $F(x_i)$ everywhere $f(x_i)$ appears in the Riemann sums defining $\int_a^b f(x)dx$.

some quantity $F(x)$. Adding up all of those changes yields the total change $F(b) - F(a)$. Indeed, we can almost (ignoring issues about limits) see a mathematically complete proof by writing in the definition of the derivative of $F(x_i)$ everywhere $f(x_i)$ appears in the Riemann sums defining $\int_a^b f(x)dx$.

Example 7. *Use the Fundamental Theorem to find exact answers for each of the questions we have considered so far in this lecture.*