Practice with the definite integral and the Fundamental Theorem

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Example 1. *Use both the Fundmental Theorem and a Riemann sum with 50 terms to evaluate* $\int_1^3 \frac{x}{x+1}$.

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Areas between curves

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Example 4. *Find the area of the region between the functions* $f(x) = x$ *and* $g(x) = x^2$ *and the vertical lines at* $x = -3$ *and* $x = -1$.

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Example 5. *Find the total area of the football-shaped region between the graphs of* $f(x) = x^2 - 2$ *and* $g(x) = 2 - x^2$.

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