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Thus, practically speaking, if you were walking along some surface which you could then think of as the graph of a two-variable function, some surface which you could then think of as the graph of a two-variable function, then the partial derivatives tell you about how steep your climb or descent will be if you walk parallel to the x or y axes (due north-south or east-west).

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Example 8. *Find the rate of change of the function* $x^2 + y^2$ *at the point* $(1, 1)$ *in the direction of the unit vectors* $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

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