Math 242:

Math 242: Calculus for Business and the Social Sciences,

Math 242: Calculus for Business and the Social Sciences, II

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Finally, we will develop optimization techniques, Finally, we will develop optimization techniques, as were the main focus at the end of last term, Finally, we will develop optimization techniques, as were the main focus at the end of last term, in the setting of multiple variables.

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Multivariable optimization through calculus is ubiquitous in economics and other social sciences. Graduate programs in economics often give crash courses in this material to their incoming students.

Practicalities

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Any of the familiar examples of functions (polynomials, exponentials) can be *restricted* to be discrete functions. We often use n to name the variable instead of x, so that the squaring function is named $f(n) = n^2$.

Difference functions

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Definition 2. *Given a discrete function* $f(n)$ *define its difference function* $Df(n)$ *by* $Df(n) = f(n) - f(n-1)$.

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Definition 4. *Given a discrete function $f(n)$ define the aggregate function $Af(n)$ to be $f(1) + f(2) + \cdots + f(n)$.*

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Notice that the derivative of $Af(n)$ looks related to $f(n)$ itself.

The fundamental theorems of discrete calculus

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Theorem 6. [First fundamental theorem] *If $f(n)$ is a discrete function, then $D(Af(n)) = f(n)$.*

Proof:

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\begin{aligned}
D(Af(n)) &= Af(n) - Af(n-1) \\
&= \{f(1) + \cdots + f(n)\} - \{f(1) + \cdots + f(n-1)\}
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Along similar lines, we have the following.

Theorem 7. [Second fundamental theorem] *If* $f(n)$ *is a discrete function then* $A(Df(n)) = f(n) - f(0)$.

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