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One of the main uses of averages is as part of a summary of a set of data. Moving averages summarize data over periods of time.

Definition 2. *The moving average of a function* $f(t)$ *with lag-time* n*-units is the function* $\overline{f}(x)$ *whose value at time* x *is the average value of* $f(t)$ *between times* $x - n$ *and* x.

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Example 3. *Find the four-quarter moving average of the following quarterly sales data.*

1	3	2	4	1.5	3.2	2.8	4.1
1.4	3.6	2.5	5	2	4	3	4.8

Why would you want to do this?

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