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Example 2. *Analyze the system of equations*

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2x - y &= 5 \\
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We may use matrices to represent and efficiently solve systems of equations by putting both the coefficients of the system and the values of the equations in a matrix and mimicking our usual procedure for solving them.

Example 4. *Translate into matrix notation and solve the system*

$$\begin{aligned} x + 2y &= 2 \\ x + 3y &= 5. \end{aligned}$$

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Matrix notation has uses well beyond solving systems.

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$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 6 & -1 \end{bmatrix}.$$

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$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}-\begin{bmatrix}4 & 2 & -1\\3 & -5 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}-3 & 0 & 4\\1 & 10 & 5\frac{1}{2}\end{bmatrix}.$$

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Some manipulations of matrices are not just translation from those on numbers.

Definition 7. *The transpose of an n by m matrix M is the m by n matrix called M^T whose i,jth entry is the j,ith entry of M.*

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Example 8. *The transpose of* $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ *is* $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$.

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Definition 10. *The dot product of a row vector* $\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$ *and a column vector* $\begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$ *is the sum* $a_1 b_1 + a_2 b_2 + \cdots a_n b_n$.

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