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Notice that in our first two examples, one function, namely $2x$ had two anti-derivatives, namely x^2 and $x^2 + 7$. In fact, any function will have many anti-derivatives, which makes taking anti-derivatives different in character from taking derivatives or doing algebraic manipulations.

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For example, we saw that $x^2 + 7$ is an anti-derivative for $2x$. And so is x^2, which is equal to $(x^2 + 7) + -7$.

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To make matters more or less confusing (depending on your point of view), the collection of anti-derivatives of a function has another name.

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