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- The Fundamental Theorem cannot always be applied. Some functions such as e^{x^2} do not have anti-derivatives which are easy to describe.

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- In most real-world applications, definite integrals are computed using sums (with the help of computers).

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Example 1. *Suppose a ball thrown up in the air has velocity* $v(t) = -32t + 100$ *feet per second. What is its velocity at* 3, 3.1 *and* 3.2 *seconds? Using its velocity at* 3.1 *seconds, estimate how far it travels between* 3.1 *and*

3.2 *seconds.*

3.2 *seconds. Using approximations at tenths, hundredths and thousandths of a second, estimate how far it travels between three and four seconds.*

Example 2. *Suppose the marginal cost for producing CD's at a factory is* $\frac{10}{\sqrt{x}}$ *cents for the* x*th CD.*

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Definition 3. *If f is a function over some interval $[a, b]$ then the Riemann sum of f with n terms is defined as*

$$RS_n f\big|_a^b = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_n)],$$

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where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$. If f represents the change in some quantity, the Riemann sum approximates the total amount of that quantity.

Example 4. *Identify the parameters* f, a, b, n, Δx *and* x_3 *in each of the previous examples.*

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Example 6. *Use Excel to compute the Riemann sum with everything as in the previous example but with* $n = 20$.

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Example 8. *Write down and evaluate a Riemann sum with twenty terms which approximates the area under the curve* $y = \frac{x}{x^2+2}$ *between* $x = 1$ *and* $x = 3$*.*

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