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Example 5. *The collection*

$$\begin{aligned} -2x + y &= 1 \\ x + y &= 4 \end{aligned}$$

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Solutions to a system of linear equations are places where the lines of points which satisfy those equations intersect, as can be illustrated with the system in the previous example. This observation leads to the first of three methods we develop for solving these systems.

Methods for solving linear systems

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Substitution: We use one equation solve for one variable as a function of the other, and then substitute this function in for that variable in the other equation(s).

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Example 6. *Solve the following system of linear equations using the three methods:*

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Example 8. *Almonds cost \$3 per pound and Cashews \$8. These two types of nuts make Nuttyfun.*^{TM}

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