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Theorem 1. *If money is paid at a rate of $R(t)$ dollars per unit of time, the total amount paid between time a and b is $\int_a^b R(t)dt$.*

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But as any pensioner or lottery winner will tell you, getting a constant amount of money means that your purchasing power will go down over time. How can we determine the current or future value of installment payments?

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Theorem 4. *The present value of a revenue stream* $R(t)$ *between times* a *and* b *is* $\int_a^b R(t)e^{r(a-t)}dt$.

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Let's justify this again by analyzing the present and future value of the money deposited over a short period of time.

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Theorem 6.

$$\int t^n e^{rt} dt =$$

$$\frac{1}{r} t^n e^{rt} - \frac{n}{r^2} t^{n-1} e^{rt} + \frac{n(n-1)}{r^3} t^{n-2} e^{rt} -$$

$$\cdots \pm \frac{n(n-1)(n-2)\cdots 1}{r^{n+1}} e^{rt} + C.$$

We are mainly concerned in the cases of $n = 0$ or 1, in which case we can verify this by taking the derivative.

We can use this for more complicated revenue stream problems.

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Example 8. *The revenues of Startup.com is modeled by*

$10e^{1.2t} - 20$ *millions of dollars. Adjusting for inflation of 4%, estimate the present value of its revenues over its first five years.*