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Informally, an unbounded region has points which go to infinity in some direction.

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Remarkably, the same basic principal applies in many variables!!

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