Finding maxima and minima using partial derivatives

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From our main theorem of last time, we can extract the following procedure to find maxima, minima and saddle points of a two-variable function $f(x, y)$.

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- If $D > 0$ and $f_{xx} < 0$ the critical point is a relative maximum.
- If $D > 0$ and $f_{yy} > 0$ the critical point is a relative

minimum.

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- If none of these hold (for example $D = 0$), further analysis (which we will not develop) is needed to understand the critical point.

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For applications we should add to the beginning of our list our standard procedures of identifying relevant variables for a problem and determining the function to be optimized.

Example 2. *A computer company is introducing two new systems marketed to larger businesses. They estimate that if the systems are priced at* x *and* y *hundred dollars, respectively, then* $40 - 8x + 5y$ *customers will buy the first system and and* $50 + 9x - 7y$ *will buy the second.*

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In the next example we see that optimization methods of calculus can lead to solutions of geometric problems, which historically has been one of the main applications of the calculus. To this day, using calculus is central in mathematical areas such as finding as surfaces with least area and areas in physics, biology and chemistry such as finding configurations of molecules which minimize energy.

Example 3. *Four dormitories on campus are located at points with coordinates* $(-5, 0)$, $(1, 7)$, $(9, 0)$ *and* $(0, -8)$ *on a campus map. Where on that map should a cafeteria be placed to minimize the total of distances (squared) from the dorms to the cafeteria?*

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Example 4. *Suppose it costs \$8 per square foot for material on the bottom of a large rectangular fish tank and \$10 per square foot for the sides. What dimensions should the fish tank be to maximize volume with a total cost of \$1000?*