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This example marks the beginning of the subject of *differential equations*,

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Techniques of integration

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- If we can integrate $\int g(u)du = G(u) + C$ then we can substitute $u(x)$ for u to find the original integral.

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