SUPPORTING DOCUMENT

This supporting document presents the proofs of Lemma 1 and Proposition 4. It also provides the extended versions of the proofs of Proposition 1 and Theorem 1. We also provide a discussion about the existence of the Market Equilibrium in the special market structure.

Appendix S.1: Proof of Lemma 1

For notational convenience, we let $\sigma_{A_n}$ be the utilization of agents in sub-pool $A_n$, and $P_{nm}$ be the probability that a customer choosing sub-pool $A_n$ being served by sub-pool $m$ for all $n \in \{1, \ldots, N_A\}$.

1. We first show that $\rho_A \geq \rho_0^\ell$ when $U_{A_1}^c = r_{A_1}$. Note that this claim is trivially true for $\ell = 1$. Therefore, we just focus on the case where $\ell > 1$. Note that $r_{A_1} > r_{A_\ell}$, and thus we should have that $\rho_A D_{A_1}^c = \frac{r_{A_1}}{r_{A_\ell}} y_{A_1} > y_{A_1}$. (If “=” were “>”, then we would have that $U_{A_1}^c < r_{A_1}$ while $D_{A_1}^c > 0$, which contradicts with the fact that $U_{A_1}^c = r_{A_1}$.) As we have $\rho_A D_{A_1} > y_{A_1}$, sub-pool-1 should serve only its own customers, i.e. $P_{n1} = 0$ for all $n \in \{2, \ldots, N_A\}$ because of its balance equation (i.e. rate-in equals rate-out). If $\ell = 2$, we are done.

Otherwise, we should also have that $\rho_A D_{A_2}^c = \frac{r_{A_2}}{r_{A_\ell}} y_{A_2} > y_{A_2}$ since $r_{A_2} > r_{A_\ell}$ (This is true as we show for sub-pool-1 as a result of the fact that $P_{21} = 0$.) As we have $\rho_A D_{A_2}^c > y_{A_2}$, sub-pool-2 should serve only its own customers, i.e. $P_{n2} = 0$. Finally, we can continue like this until we show that $\rho_A D_{A_n}^c = \frac{r_{A_n}}{r_{A_\ell}} y_{A_n}$ for any $n < \ell$, and $P_{nm} = 0$ for any $n > \ell$ and $m < \ell$. Then, we have that

$$\rho_A \sum_{n=1}^{N_A} D_{A_n}^c \geq \rho \sum_{n=1}^{\ell-1} D_{A_n}^c = \sum_{n=1}^{\ell-1} \frac{r_{A_n} y_{A_n}}{r_{A_\ell}} = \rho_0^\ell$$

Now we show that $\rho_A \sum_{n=1}^{N_A} D_{A_n}^c \leq y_{A_\ell}$, which contradicts with the fact that $U_{A_\ell}^c > r_{A_\ell}$ because customers choosing sub-pool $A_\ell$ cannot be served by any sub-pool offering a higher net-reward. Furthermore, we should have that $P_{n\ell} = 1$ for any $n > \ell$ with $D_{A_n}^c > 0$ because if $P_{n\ell} < 1$, we would have that $U_{A_n}^c < r_{A_\ell}$. Finally, there must be enough service capacity to serve all customers requesting service from sub-pool $A_\ell$ (otherwise some customers would be rejected and lead to $U_{A_n}^c < r_{A_\ell}$). Thus, we have that

$$\rho_A \sum_{n=\ell}^{N_A} D_{A_n}^c = \rho D_{A_\ell}^c + \rho_A \sum_{n=\ell+1}^{N_A} D_{A_n}^c P_{n\ell} = \sigma_{A_\ell} y_{A_\ell} \leq y_{A_\ell},$$

where the first inequality holds since $P_{n\ell} = 1$ for any $n > \ell$ with $D_{A_n}^c > 0$.

2. As $U_{A_1}^c < r_{A_1}$, we should have that $\rho_A D_{A_1}^c > \frac{r_{A_1}}{r_{A_\ell}} y_{A_1}$ because otherwise we would have that $U_{A_1}^c \geq r_{A_1}$, which contradicts with the fact that $U_{A_1}^c < r_{A_1}$. Furthermore, this implies that
\( \rho_A D_{A_1}^{ce} > y_{A_1} \), and thus sub-pool-1 serve only its own customers, i.e. \( P_{n1} = 0 \) for all \( n \in \{2, \ldots, N_A\} \). As we discuss above, then we can show that \( \rho_A D_{A_n}^{ce} = \frac{\tau_{An}}{r_{A_n}} y_{A_n} \) for any \( n < \ell \), and \( P_{nm} = 0 \) for any \( n > \ell \) and \( m < \ell \).

Finally, we should have that \( \rho_A D_{A_\ell}^{ce} > y_{A_\ell} \) because otherwise we would have that \( U_{A_\ell}^{ce} = r_{A_\ell} \).

Combining all these observations, we have that

\[
\rho_A \sum_{n=\ell}^{N_A} D_{A_n}^{ce} \geq \rho_A \sum_{n=1}^{\ell-1} D_{A_n}^{ce} + \rho D_{A_\ell}^{ce} > \frac{\sum_{n=1}^{\ell-1} r_{An} y_{A_n}}{r_{A_\ell}} + y_{A_\ell} = \rho_\ell^{0} + y_{A_\ell}.
\]

3. In part 3, we should have that \( \rho_A D_{A_1}^{ce} < \frac{r_{A_1}}{r_{A_1}} y_{A_1} \) because otherwise we would have that \( U_{A_1} < r_{A_1} \) which contradicts the fact that \( U_{A_1}^{ce} > r_{A_1} \). Furthermore, since \( U_{A_1}^{ce} < r_{A_{1-1}} \), we should have that \( \rho_A D_{A_1}^{ce} > \frac{r_{A_1}}{r_{A_{1-1}}} y_{A_1} \geq y_{A_1} \). Hence, as we show in parts 1 and 2 sub-pool-1 serves only its own customers. We can apply the same arguments to any \( n < \ell \), and conclude that \( \rho_A D_{A_n}^{ce} < \frac{r_{An}}{r_{A_n}} y_{A_n} \) for any \( n < \ell \), and \( P_{nm} = 0 \) for any \( n > \ell \) and \( m < \ell \).

Finally, since \( P_{nm} = 0 \) for any \( n > \ell \) and \( m < \ell \), customers choosing sub-pool \( n \) for \( n \geq \ell \) cannot obtain an expected utility greater than \( r_{A_n} \). Therefore, we should have that \( D_{A_n}^{ce} = 0 \) for any \( n \geq \ell \) because we suppose \( U_{A_n}^{ce} > r_{A_n} \) in part 3. Combining all these observations, we have that

\[
\rho_A \sum_{n=\ell}^{N_A} D_{A_n}^{ce} < \rho_A \sum_{n=1}^{\ell-1} D_{A_n}^{ce} < \frac{\sum_{n=1}^{\ell-1} r_{An} y_{A_n}}{r_{A_\ell}} = \rho_\ell^{0}.
\]

4. We already have the result for \( r_{A_\ell} < U_{A_\ell}^{ce} < r_{A_{\ell-1}} \). Now consider the case where \( U_{A_\ell}^{ce} = r_{A_{\ell-1}} \). Then by part 1, we know that \( \rho_A \sum_{n=1}^{N_A} D_{A_n}^{ce} \leq \rho_\ell^{0} + y_{A_{\ell-1}} < \rho_\ell^{0} \). Next, for the case where \( r_{A_{\ell-1}} < U_{A_\ell}^{ce} < r_{A_{\ell-2}} \), we have the result again by part 3 because \( r_{A_{\ell-1}} < U_{A_\ell}^{ce} < r_{A_{\ell-2}} \) implies that \( \rho_A \sum_{n=\ell}^{N_A} D_{A_n}^{ce} < \rho_\ell^{0} + \rho_{\ell-1}^{0} \) and \( \rho_\ell^{0} < \rho_{\ell-1}^{0} \).

**Appendix S.2: Extended Proof of Proposition 1**

Below, we state and prove an extended version of Proposition 1. Our claim in Proposition 1 is a direct implication of Proposition 3. When \( \rho_A > \rho_\ell^{0} + y_{A_\ell} \), the result holds since \( \rho_A D_{A_\ell}^{ce} > y_{A_\ell} \) by parts 2 and 3 (We only need part 3 to show that sub-pool \( N_A \) will be over-utilized when \( \rho_A > \rho_{N_A}^{0} + y_{A_{N_A}} \)).

When, \( \rho_A \in [\rho_\ell^{0}, \rho_\ell^{0} + y_{A_\ell}] \), this time the result holds by part 1. Finally, when \( \rho_A < \rho_\ell^{0} \), the result holds again by part 1.

**Proposition 3.** Given any \( r_A, y_A \), let \( D_{A_n}^{ce} \) be the fraction of customers choosing sub-pool \( A_n \) in a customer equilibrium.

1. If \( \rho_A \in [\rho_\ell^{0}, \rho_\ell^{0} + y_{A_\ell}] \) for some \( \ell \in \{1, \ldots, N_A\} \), then we have that \( D_{A_n}^{ce} = \frac{r_{An} y_{A_n}}{r_{A_n} + y_{A_n}} \) for any \( n < \ell \), and \( \sum_{n=\ell}^{N_A} D_{A_n}^{ce} = \frac{\rho_{\ell-1}^{0} - \rho_\ell^{0}}{\rho_A} \). Furthermore, all customers choosing sub-pool \( n \) are served by sub-pool \( n \) for any \( n < \ell \), while the remaining customers are served by sub-pool \( \ell \).
2. If \( \rho_A \in (\rho_0^\ell + y_A, \rho_{\ell+1}) \) for some \( \ell \in \{1, \ldots, N_A - 1\} \), then we have that \( D_{A_n}^{ce} = \frac{r_{A_n} y_{A_n}}{\sum_{m=1}^{n} (r_{A_m}) y_{A_m}} \) for any \( n \leq \ell \), and \( D_{A_n}^{ce} = 0 \) for any \( n > \ell \). Furthermore, all customers choosing sub-pool \( n \) are served by sub-pool \( n \) for any \( n \leq \ell \).

3. If \( \rho_A \in (\rho_0^{N_A}, \infty) \), then we have that \( D_{A_n}^{ce} = \frac{r_{A_n} y_{A_n}}{\max\{\sum_{m=1}^{N_A} (r_{A_m}) y_{A_m}, r_A\}} \) for any \( n \leq N_A \).

Furthermore, all customers choosing sub-pool \( n \) are served by sub-pool \( n \) for any \( n \leq N_A \).

Here, we let \( \rho_0^n \) and \( \rho_1^n \) be the expected utility of customers in the equilibrium from the sub-pools attracting positive demand.

Proof: We let \( U_{A_n}^{ce} \) be the expected utility of customers in the equilibrium from the sub-pools.

1. We first show that \( U_{A_n}^{ce} = r_A \) in part 1. To do this, we first suppose \( U_{A_n}^{ce} < r_A \). Then, by Lemma 1.2, we have that \( \rho_A \sum_{n=1}^{N_A} D_{A_n}^{ce} > \rho_0^\ell + y_A \), which implies that \( \sum_{n=1}^{N_A} D_{A_n}^{ce} > 1 \) since \( \rho_A \leq \rho_0^\ell + y_A \). However, this is clearly a contradiction since \( D_{A_n}^{ce} \)'s are the fractions, and \( \sum_{n=1}^{N_A} D_{A_n}^{ce} < 1 \).

Now suppose \( U_{A_n}^{ce} > r_A \). This time, by Lemma 1.4, we have that \( \rho_A \sum_{n=1}^{N_A} D_{A_n}^{ce} < \rho_0^\ell \), which implies that \( \sum_{n=1}^{N_A} D_{A_n}^{ce} < 1 \) since \( \rho_A \geq \rho_0^\ell \). However, this is again a contradiction since we should have that \( \sum_{n=1}^{N_A} D_{A_n}^{ce} = 1 \) whenever \( U_{A_n}^{ce} > 0 \).

Hence, we should have that \( U_{A_n}^{ce} = r_A \). As a direct implication of that we have \( \rho_A D_{A_1}^{ce} = \frac{(r_A) y_{A_1}}{r_A} \).

Furthermore, sub-pool-1 serves only its customers since \( \rho_A D_{A_1}^{ce} > y_{A_1} \). Therefore, customers picking sub-pool-2 can only get their service from sub-pool-2, and because of that we should \( \rho_A D_{A_2}^{ce} = \frac{(r_A) y_{A_1}}{r_A} \). Moreover, sub-pool-2 serves only its own customers since \( \rho_A D_{A_2}^{ce} > y_{A_2} \). We actually apply this argument to any sub-pool \( n \) for \( n < \ell \), and conclude that \( \rho_A D_{A_n}^{ce} = \frac{(r_A) y_{A_n}}{r_A} \) and sub-pool \( n \) serves only its own customers for any \( n < \ell \).

Finally, since \( \rho_A - \rho_A \sum_{n=1}^{\ell-1} D_{A_n}^{ce} = \rho - \rho_0^\ell \leq y_A \), we should have that \( \sum_{n=1}^{\ell-1} D_{A_n}^{ce} = 1 \) (Otherwise the third condition of the customer equilibrium would be violated.). Thus, we have that \( \sum_{n=1}^{\ell-1} D_{A_n}^{ce} = \frac{\rho_A - \rho_0^\ell}{\rho_A} \). Moreover, we should have that \( P_{n\ell} = 1 \) for any \( n > \ell \) with \( D_{A_n}^{ce} > 0 \) as discussed in the proof of Lemma 1.1.

2. In this case, we first show that \( U_{A_n}^{ce} \in (r_{A_{\ell+1}}, r_A) \). To this end, we first suppose \( U_{A_n}^{ce} \leq r_{A_{\ell+1}} \). Then, by Lemma 1.1 and 1.2, we have that \( \rho_A \sum_{n=1}^{\ell+1} D_{A_n}^{ce} \geq \rho_0^{\ell+1} \), which implies that \( \sum_{n=1}^{\ell+1} D_{A_n}^{ce} \geq \frac{\rho_0^{\ell+1}}{\rho_A} > 1 \) since \( \rho_0^{\ell+1} > \rho_A \). However, this is clearly a contradiction since \( D_{A_n}^{ce} \)'s are the fractions, and \( \sum_{n=1}^{\ell+1} D_{A_n}^{ce} \) cannot exceed 1.

Now, suppose \( U_{A_n}^{ce} \geq r_A \). First, note that \( U_{A_n}^{ce} \geq r_A \) since \( \ell < N_A \) and \( r_A > r_{A_{N_A}} \). Then, by Lemma 1.1 and 1.4, we have that \( \rho_A \sum_{n=1}^{N_A} D_{A_n}^{ce} \leq \rho_0^\ell + y_A \), which implies that \( \sum_{n=1}^{N_A} D_{A_n}^{ce} \leq \frac{(r_A) y_{A_1}}{r_A} \).
$\frac{\rho_{\ell} + y_{A_{\ell}}}{\rho_A} < 1$ since $\rho_{\ell} + y_{A_{\ell}} < \rho_A$. However, this is again a contradiction since we should have that $\sum_{n=1}^{N_A} D_{A_n}^{ce} = 1$ whenever $U_{A_n}^{ce} > u$.

Hence, we should have that $U_{A_n}^{ce} \in (r_{A_n+1}, r_{A_n})$. Let $u = U_{A_n}^{ce}$. Then, as in part 1, we can argue that $\rho_A D_{A_n}^{ce} = \frac{(r_{A_n})y_{A_{n+1}}}{u}$ and sub-pool $n$ serves only its own customers for any $n \leq \ell$. Therefore, customers choosing sub-pool $n$ for any $n > \ell$ cannot earn more than $r_{A_n+1}$, and we should have that $D_{A_n}^{ce} = 0$ for any $n > \ell$. Finally, since $u > r_{A_{\ell+1}} \geq u$, we have that $\sum_{n=1}^{\ell} D_{A_n}^{ce} = 1$, and this implies that $\rho_A(u) = \sum_{n=1}^{\ell}(r_{A_n})y_{A_{n+1}}$. Thus, we have that $D_{A_n}^{ce} = \frac{(r_{A_n})y_{A_{n+1}}}{\sum_{n=1}^{\ell}(r_{A_n})y_{A_{n+1}}}$.

3. The proof is very similar to part 2, so that we only give a sketch and highlight the important differences. We first consider the case where $r_{A_n} > u$. In this case, as above we show that $U_{A_n}^{ce} < r_{A_n}$. To this end, we suppose $U_{A_n}^{ce} \geq r_{A_n}$, which implies that $\sum_{n=1}^{N_A} D_{A_n}^{ce} < 1$ since $\rho_A > \rho_{N_A}^{0}$. This constitutes a contradiction because $U_{A_n}^{ce} \geq r_{A_n} - u$. After establishing $U_{A_n}^{ce} < r_{A_n}$, we let $u = U_{A_n}^{ce}$ and derive that $u = \frac{\sum_{n=1}^{N_A} r_{A_n} y_{A_{n+1}}}{\rho_A}$. Note that we cannot guarantee that $u \geq u$ for large $\rho_A$ unlike we could do that in part 2. Therefore, once $\rho_A > \frac{\sum_{n=1}^{N_A} (r_{A_n})y_{A_{n+1}}}{u}$, we should have that $u = u$.

Now, consider the case $r_{A_n} = u$. In this case, we first establish that $U_{A_n}^{ce} = u$. Then, using this we have that $\rho_A D_{A_n}^{ce} = \frac{(r_{A_n})y_{A_{n+1}}}{u}$ for any $n \leq N_A - 1$ since $r_{A_n} > u$ for any $n \leq N_A - 1$. Finally, we have that $\rho_A D_{A_n}^{ce} = y_{A_n}$ by Condition 3 in the Customer Equilibrium and $\rho_A > \rho_{N_A}^{0} + y_{A_n}$. Also note that when $r_{A_n} = u$, we always have that $\rho_A > \rho_{N_A}^{0} + y_{A_n} = \frac{\sum_{n=1}^{N_A-1} (r_{A_n})y_{A_{n+1}}}{u} + y_{A_n} = \frac{\sum_{n=1}^{N_A-1} (r_{A_n})y_{A_{n+1}}}{u}$. 

Appendix S.3: Proof of Proposition 4

WLOG we assume $r_{i_n} > r_{i_{n+1}}$ for all $n \in \{1, \ldots, N_i - 1\}$. For notational convenience. Since $(r_{i}, y_{i})$ is an Market Equilibrium, there exists a sequence $(r^{k}_{i}, y^{k}_{i})$ such that $(r^{k}_{i}, y^{k}_{i})$ is a $(\epsilon^{k}, \delta^{k})$-FME where $\lim_{k \to \infty} \epsilon^{k} = 0$, $\lim_{k \to \infty} \delta^{k} = 0$, $r_{i} = \lim_{k \to \infty} r^{k}_{i}$, and $y_{i} = \lim_{k \to \infty} y^{k}_{i}$.

1. In this proof, we focus only on class-A and dedicated agents. The proof for the flexible agents and class-B are the same, and thus omitted. Let $N_{AD} = \{n : \exists$ dedicated agents in sub-pool $n, n \in \{1, \ldots, N_A\}\}$ and $\bar{n} = \min_{n \in N_{AD}} n$. Note that $|N_{AD}| \geq 2$.

We prove our claim by contradiction. Therefore, we suppose $V_{A_n,D_0}^{mc} > 0$ for some $\hat{n} \in N_{AD}$ and find a contradiction for $\rho_A \geq \rho_{2+1}^{0}$ and $\rho_A < \rho_{2+1}^{0}$, where $\rho_{\ell}^{0}$ is defined as in Proposition 1 for all $\ell \in \{1, \ldots, N_A\}$.

When $\rho_A \geq \rho_{2+1}^{0}$, consider a deviation from $(r^{k}_{A}, y^{k}_{A})$ where $\hat{\gamma} < \delta^{k}$ fraction of dedicated agents from sub-pool $A_{\hat{n}}$ increase their prices by $\gamma = (r_{A_{\hat{n}}} - r_{A_{\hat{n}+1}})/2 > 0$. Then, by Proposition 1, the revenue of deviating agents is $R_{A} - r_{A_{\hat{n}}} + \zeta$ for large $k$ because we have that $\rho_A \geq \rho_{2+1}^{0} > \frac{\sum_{n=1}^{N_A-1} (r_{A_n})y_{A_{n+1}}}{\hat{n}} + \hat{\gamma}$. This is a contradiction because deviating agents increase their
revenues by more than $\epsilon^k$ for large $k$ as \( \lim_{k \to \infty} \epsilon^k = 0 \) because their revenues before deviation can be at most \( R_A - r_{A_n}^k \).

When \( \rho_A < \rho_0 + 1 \), we should have that \( V_{me}^{me} = 0 \) for all \( n \in \{n + 1, \ldots, N_A\} \), and thus we should have that \( V_{me}^{me} > 0 \), which implies that \( R_A - r_{A_n} > 0 \) and \( \rho_A \geq \rho_0 + \gamma^k > 0 \) for some \( \gamma^k > 0 \) by Proposition 1. Consider a deviation from \( (r_{A_k}, y_{A_k}) \) where \( \gamma^k < \delta^k \) fraction of dedicated agents from sub-pool \( A_n \) change a strictly positive price \( p' = (R_A - r_{A_n})/2 \) for some \( n \in N_{AD} \) with \( n \neq n \).

Then, by Proposition 1, the revenue of deviating agents is \( p' \) for large \( k \) because we have that \( \rho_A \geq \rho_0 + \gamma^k > \lim_{k \to \infty} \frac{\sum_{n=1}^{N_A} (r_{A_n})^k y_{A_n}}{R_A - p'} + \gamma^k \) by the choices of \( \gamma^k \) and \( p' \). This is a contradiction because deviating agents increase their revenues (which were zero) by more than \( \epsilon^k \) for large \( k \).

Once we show contradictions for \( \rho_A > \rho_0 + 1 \) and \( \rho_A < \rho_0 + 1 \), we should have that \( V_{me}^{me} = 0 \) for all \( n \in N_{AD} \) when \( |N_{AD}| \geq 2 \).

2. Note that our claim holds by Part 1 if all flexible agents serve the same class. Thus, we only focus on the case where there are flexible agents serving different classes. Let \( V_i^{me} \) be the equilibrium revenue of a flexible agent serving class \( i \in \{A, B\} \). We prove our claim by contradiction. Thus, we suppose \( V_{AF}^{me} \neq V_{BF}^{me} \). When \( V_{AF}^{me} > V_{BF}^{me} \geq 0 \), there must be only one sub-pool, say \( n \), with flexible agents serving class \( A \) by Part 1. We should also have that \( \rho_A > \rho_n^0 + \gamma^k > 0 \) where \( \rho_n^0 = \frac{\sum_{n=1}^{N_A} (r_{A_n}) y_{A_n}}{r_{A_n}} \) by Proposition 1. Consider a deviation where a \( \gamma^k < \delta^k \) fraction of flexible agents serving class \( B \) charge \( p' = (V_{AF}^{me} + V_{BF}^{me})/2 \). By Proposition 1, the revenue of deviating agents should be \( p' \) for large \( k \) since \( \rho_A > \rho_n^0 + \gamma^k > \lim_{k \to \infty} \frac{\sum_{n=1}^{N_A} (r_{A_n}) y_{A_n}}{R_A - p'} + \gamma^k \) as a result of the choices of \( \gamma^k \) and \( p' \). This is a contradiction because deviating agents increase their revenues by more than \( \epsilon^k \) for large \( k \). Similarly, when \( V_{BF}^{me} > V_{AF}^{me} \geq 0 \), a small group of flexible agents serving class-A can improve their revenues. Hence, we should have that \( V_{BF}^{me} = V_{AF}^{me} \).

### Appendix S.4: Extended Proof of Theorem 1

Let \((p_D, p_F; \alpha_D, \alpha_F)\) be a symmetric Market Equilibrium in a marketplace with one customer class and two agent pools. Since \((p_D, p_F; \alpha_D, \alpha_F)\) is an equilibrium, there exists a sequence \((p^k_D, p^k_F; \alpha_D, \alpha_F)\) such that \((p^k_D, p^k_F; \alpha_D, \alpha_F)\) is a \((\epsilon^k, \delta^k)\)-FME where \( \lim_{k \to \infty} \epsilon^k = 0 \), \( \lim_{k \to \infty} \delta^k = 0 \), \( p_D = \lim_{k \to \infty} p^k_D \), and \( p_F = \lim_{k \to \infty} p^k_F \). We let \( V^{sm}_{D}(k) \) and \( V^{sm}_{F}(k) \) be the revenue of a dedicated and a flexible agent, respectively, according to \((p^k_D, p^k_F; \alpha_D, \alpha_F)\). Then, we have that \( V^{sm}_{i}(k) = \lim_{k \to \infty} V^{sm}_{i}(k) \) for all \( i \in \{D, F\} \) by continuity of the revenue functions in Corollary 2.

1 We show that \( V^{sm}_{D} = V^{sm}_{F} = 0 \) by contradiction. Thus, we suppose that either \( V^{sm}_{D} > 0 \) or \( V^{sm}_{F} > 0 \) is true on the contrary and find a contradiction for any possible price pair \((p_D, p_F)\) satisfying either of these conditions. To this end, we follow a case-by-case analysis:
i. \( (R_D - p_D = R_F - p_F) \): We first argue that \( p_D > 0 \) in this case. This holds trivially when \( R_D > R_F \). When \( R_D = R_F, p_D = 0 \) would imply that \( p_F = 0 \), and thus we would have that \( V_{D}^{sm} = V_{F}^{sm} = 0 \). However, we suppose that either \( V_{D}^{sm} > 0 \) or \( V_{F}^{sm} > 0 \) is true. Therefore, we should have \( p_D > 0 \) even when \( R_D = R_F \).

We first want to that dedicated agents are under-utilized according to \( (p_D^k, p_F^k; \alpha_D, \alpha_F) \) since \( \rho < \alpha_D \), and thus their revenue is at most \( (\rho p_D^k) / \alpha_D \). Now, consider a deviation from \( (p_D^k, p_F^k; \alpha_D, \alpha_F) \) where \( y^k < \delta^k \) fraction of dedicated agents deviate and cut their prices by \( \zeta = p_D (1 - \rho / \alpha_D) / 2 \). After this deviation, we have that the revenue of deviating agents is \( p_D^k - \zeta \) for large \( k \) by Proposition 1 since \( \lim_{k \to \infty} y^k < \rho \). Note that \( p_D - \zeta > (\rho p_D^k) / \alpha_D \) for large \( k \) by the choice of \( \zeta \). This deviation improves the revenue of deviating agents by more than \( \epsilon^k \) for large \( k \). Thus, any \( (p_D, p_F) \) satisfying \( R_D - p_D = R_F - p_F \) cannot emerge as an equilibrium price pair.

ii. \( (R_F - p_F > R_D - p_D) \): Let \( \rho_D^0 = \frac{R_F - p_F}{R_D - p_D} \alpha_F \). In this case, we have two sub-cases:

a) \( \rho \leq \rho_D^0 \): By Proposition 1, we have that \( \lim_{k \to \infty} V_{D}^{sm}(k) = 0 \) since \( \rho \leq \rho_D^0 \). Therefore, we should have that \( V_{F}^{sm} > 0 \), which implies that \( p_F > 0 \). Now, consider a deviation from \( (p_D^k, p_F^k; \alpha_D, \alpha_F) \) where \( y^k < \delta^k \) fraction of dedicated agents deviate and charge \( p' = R_D - R_F + p_F / 2 \). After this deviation, we have that the revenue of deviating agents is \( p_D^k - \zeta + p' \) for large \( k \) by Proposition 1 since \( \lim_{k \to \infty} y^k < \rho \) and \( R_D - p' > R_F - p_F \). This deviation improves the revenue of deviating agents by more than \( \epsilon^k \) for large \( k \). Therefore, any \( (p_D, p_F) \) in this sub-case cannot emerge as an equilibrium price pair.

b) \( \rho > \rho_D^0 \): By Proposition 1, we have that \( \lim_{k \to \infty} V_{D}^{sm}(k) = \frac{\rho - \rho_D^0}{\alpha_D} p_D \). Now, consider a deviation from \( (p_D^k, p_F^k; \alpha_D, \alpha_F) \) where \( y^k < \delta^k \) fraction of dedicated agents deviate and decrease their prices by \( \zeta = p_D \frac{(1 - \rho - \rho_D^0 / \alpha_D)}{2} \) (Note that such an \( \zeta > 0 \) exists because \( p_D > R_D - R_F + p_F \geq 0 \) and \( \rho - \rho_D^0 < \rho - \alpha_F < \alpha_D \) as \( \rho \leq \alpha_D + \alpha_F \)). After this deviation, we have that the revenue of deviating agents is \( p_D^k - \zeta + p_F / 2 \) for large \( k \) by Proposition 1 because \( \lim_{k \to \infty} \frac{R_F - p_F}{R_D - p_D} \alpha_F + y^k < \rho_D^0 < \rho \). This deviation improves the revenue of deviating agents by more than \( \epsilon^k \) for large \( k \) since \( p_D - \zeta > \frac{\rho - \rho_D^0}{\alpha_D} p_D \). Therefore, any \( (p_D, p_F) \) in this sub-case cannot emerge as an equilibrium price pair.

iii. \( (R_D - p_D > R_F - p_F) \): By Corollary 2, we have that \( \lim_{k \to \infty} V_{D}^{sm}(k) = \rho p_D \) and \( \lim_{k \to \infty} V_{F}^{sm}(k) = 0 \). Thus, we should have that \( p_D > 0 \) to make sure \( V_{D}^{sm} > 0 \). Then, as we rigorously show in Part 1.i, a group of dedicated agents can cut their price by a small amount, and they can make sure they will improve their revenue. Thus, any \( (p_D, p_F) \) in this case cannot emerge as an equilibrium price pair.
2. Similar to Part 1, we show our claim by contradiction. Thus, we suppose that either $V_D^{sm} > R_D - R_F$ or $V_F^{sm} > 0$ on the contrary and follow a case-by-case analysis:

   i. $(R_D - p_D = R_F - p_F)$: We first argue that $p_D > R_D - R_F$ in this case. Suppose $p_D = R_D - R_F$, which would imply that $p_F = 0$. Then, we would have that $V_D^{sm} \leq R_D - R_F$ and $V_F^{sm} = 0$. However, we suppose either $V_D^{sm} > R_D - R_F$ or $V_F^{sm} > 0$. Therefore, we should have $p_D > R_D - R_F$, which implies that $p_F > 0$.

   Notice that the utilization of at least one group of agents, say group-$i$ where $i \in \{D, F\}$, should be less than the demand rate over total capacity, i.e. $\rho/(\alpha_D + \alpha_F)$. Now, consider a deviation from $(p_D, p_F; \alpha_D, \alpha_F)$ where $y^k < \delta^k$ fraction of group-$i$ agents deviate and cut their prices by $\zeta = p_i(1 - \rho/(\alpha_D + \alpha_F))/2$. After this deviation, we have that the revenue of deviating agents is $p_i^k - \zeta$ for large $k$ by Proposition 1 since $\lim_{k \to \infty} y^k < \rho$. This deviation improves the revenue of deviating agents by more than $\epsilon^k$ for large $k$ since $p_i - \zeta > p_i \rho/(\alpha_D + \alpha_F)$ by the choice of $\zeta$. Thus, any $(p_D, p_F)$ satisfying $R_D - p_D = R_F - p_F$ cannot emerge as an equilibrium price pair.

   ii. $(R_F - p_F > R_D - p_D)$: Note that we only use the fact that $\rho \leq \alpha_D + \alpha_F$ to show that this case cannot be an equilibrium in Part 1. Therefore, the same proof holds when $\rho = \alpha_D$.

   iii. $(R_D - p_D > R_F - p_F)$: By Proposition 1, we have that $\lim_{k \to \infty} V_D^{sm}(k) = p_D$ and $\lim_{k \to \infty} V_F^{sm}(k) = 0$. Thus, we should have that $p_D > R_D - R_F$ to make sure that $V_D^{sm} > R_D - R_F$. Now, consider a deviation from $(p_D^k, p_F^k; \alpha_D, \alpha_F)$ where $y^k < \delta^k$ fraction of flexible agents deviate and charge $p' = (R_D - R_D + p_D)/2$. After this deviation, we have that the revenue of deviating agents is $p' > 0$ by Proposition 1 since $\lim_{k \to \infty} y^k < \rho$ and $R_F - p' > R_D - p_F$. This deviation improves the revenue of deviating agents by more than $\epsilon^k$ for large $k$. Therefore, any $(p_D, p_F)$ in this sub-case cannot emerge as an equilibrium price pair.

3. Similar to previous claims, we show our claim by contradiction. Thus, we suppose that either $V_D^{sm} \neq R_D - R_F$ or $V_F^{sm} > 0$ on the contrary and follow a case-by-case analysis:

   i. $(R_D - p_D = R_F - p_F)$: As in Part 2.i, any $(p_D, p_F)$ satisfying $R_D - p_D = R_F - p_F$ and $p_F > 0$ cannot emerge as an equilibrium price pair. When $p_F = 0$, we have that $V_F^{sm} = 0$ and $p_D = R_D - R_F$. Thus, we should have that $V_D^{sm} < R_D - R_F$ and $R_D > R_F$ because otherwise we could not satisfy $V_D^{sm} \neq R_D - R_F$ condition. Now, consider a deviation from $(p_D^k, p_F^k; \alpha_D, \alpha_F)$ where $y^k < \delta^k$ fraction of dedicated agents deviate and cut their prices by $\zeta = (p_D - V_D^{sm})/2$. After this deviation, we have that the revenue of deviating agents is $p_D^k - \zeta$ for large $k$ by Proposition 1 since $\lim_{k \to \infty} y^k < \rho$. This deviation improves the revenue of deviating agents by more than $\epsilon^k$ for large $k$ since $p_D - \zeta > V_D^{sm}$ by choice of $\zeta$. Thus any $(p_D, p_F)$ satisfying $R_D - p_D = R_F - p_F$ and $p_F = 0$ can also not emerge as an equilibrium price pair.
ii. \((R_F - p_F > R_D - p_D)\): Note that we only use the fact that \(\rho \leq \alpha_D + \alpha_F\) to show that this case cannot be an equilibrium in Part 1. Therefore, the same proof holds when \(\alpha_D < \rho < \alpha_D + \alpha_F\).

iii. \((R_D - p_D > R_F - p_F)\): By Corollary 2, we have that \(\lim_{k \to \infty} V_{D_{sm}}(k) = p_D\) since \(\rho > \alpha_D\). Now, consider the deviation from \((p_F^D, p_F^D; \alpha_D, \alpha_F)\) where \(y^k < \delta^k\) fraction of dedicated agents increase their price by \(\xi = \min \left\{ \left( (R_D - p_D) - (R_F - p_F), \left(1 - \frac{2p}{\rho} \right) (R_D - p_D) \right) \right\} / 2\) (Note that such an \(\zeta > 0\) exists since \(R_D - p_D > R_F - p_F\) and \(\rho > \alpha_D\)). After this deviation, we have that the revenue of deviating agents is \(p_D^k + \zeta\) for large \(k\) by Proposition 1 since \(R_D - p_D - \zeta > R_F - p_F\) and \(\lim_{k \to \infty} \frac{R_D - p_D}{p_D^k} (\alpha_D - y^k) + y^k < \frac{R_D - p_D}{(R_D - p_D) \alpha_D / p} \alpha_D = \rho\). This deviation improves the revenue of deviating agents by more than \(\epsilon^k\) for large \(k\). Therefore, any \((p_D, p_F)\) in this sub-case cannot emerge as an equilibrium price pair.

4. Similar to previous claims, we show our claim by contradiction. Thus, we suppose that \(V_{D_{sm}} \neq V_{F_{sm}} + R_D - R_F\) on the contrary and follow a case-by-case analysis:

i. \((R_D - p_D = R_F - p_F)\): We have two sub-case in this case: either \(V_{D_{sm}} > V_{F_{sm}} + R_D - R_F\) or \(V_{D_{sm}} < V_{F_{sm}} + R_D - R_F\). When \(V_{D_{sm}} > V_{F_{sm}} + R_D - R_F\), we have that \(p_D > V_{F_{sm}} + R_D - R_F\), which implies that \(p_F > V_{F_{sm}}\). Then, as we show in Part 3.i, a small group of flexible agents cut their prices by \(\zeta = (p_F - V_{F_{sm}})/2\) and improve their revenue by more than \(\epsilon^k\) for large \(k\). Similarly, when \(V_{D_{sm}} < V_{F_{sm}} + R_D - R_F\), we have that \(V_{D_{sm}} < p_F + R_D - R_F\), which implies that \(V_{D_{sm}} < p_D\). Then, a small group of dedicated agents cut their prices by \(\zeta (p_D - V_{D_{sm}})/2\) and improve their revenue by more than \(\epsilon^k\) for large \(k\). Thus, any \((p_D, p_F)\) satisfying \(R_D - p_D = R_F - p_F\) and \(V_{D_{sm}} \neq V_{F_{sm}} + R_D - R_F\) cannot emerge as an equilibrium price pair.

ii. \((R_F - p_F > R_D - p_D)\): Consider the deviation where \(y^k < \delta^k\) fraction of flexible agents increase their price by \(\zeta = \min \left\{ \left( (R_F - p_F) - (R_D - p_D), \left(1 - \frac{2p}{\rho} \right) (R_F - p_F) \right) \right\} / 2\) (Note that such an \(\zeta > 0\) exists since \(R_F - p_F > R_D - p_D\) and \(\rho > \alpha_F\)). After this deviation, we have that the revenue of deviating agents is \(p_F^k + \zeta\) for large \(k\) as in Part 3.iii. This kind of deviation improves the revenue of deviating agents by more than \(\epsilon^k\) for large \(k\). Therefore, any \((p_D, p_F)\) in this sub-case cannot emerge as an equilibrium price pair.

iii. \((R_D - p_D > R_F - p_F)\): Note that we only use the fact that \(\rho > \alpha_D\) to show that this case cannot be an equilibrium in Part 3. Therefore, the same proof holds when \(\rho = 1\).

5. Similar to previous claims, we show our claim by contradiction. Thus, we suppose that either \(V_{D_{sm}} < R_D\) or \(V_{F_{sm}} < R_F\) on the contrary and follow a case-by-case analysis:

i. \((R_D - p_D = R_F - p_F)\): By Proposition 1, we have that \(V_{D_{sm}} = p_D\) and \(V_{F_{sm}} = p_F\). Thus, we should have that \(p_i < R_i\) for all \(i \in \{D, F\}\). Now, consider the deviation where \(y^k < \delta^k\) fraction
of group-i agents increase their price by $\zeta = \min \left\{ \left( R_i - p_i \right), \left( 1 - \frac{\alpha_D + \alpha_F}{\rho} \right) \left( R_i - p_i \right) \right\} / 2$. After this deviation, we have that the revenue of deviating agents is $p_i^k + \zeta$ for large $k$ by Proposition 1 since

$$\lim_{k \to \infty} \frac{R_i - p_i^k}{R_i - p_i - \zeta} \left( (\alpha_D + \alpha_F - y^k) + y^k \right) = \frac{(R_i - p_i)(\alpha_D + \alpha_F)}{(R_i - p_i)(\alpha_D + \alpha_F) / \rho} = \rho.$$ 

This kind of deviation improves the revenue of deviating agents by more than $\epsilon^k$ for large $k$. Therefore, any $(p_D, p_F)$ in this sub-case cannot emerge as an equilibrium price pair.

ii. $(R_D - p_D \neq R_F - p_F)$: Note that we only use the fact that $\rho > \alpha_i$ for $i \in \{D, F\}$ to show that these cases cannot be an equilibrium in Part 4. Therefore, the same proof holds when $\rho > \alpha_D + \alpha_F$.

**Non-symmetric equilibria:** As we show, in Proposition 4, that any non-symmetric equilibrium results in zero revenue for agents, the possibility of non-symmetric equilibrium does not affect our results for the revenue of dedicated agents in parts 1 and 2 (also 3 and 4 when $R_D = R_F$) and for the revenue of flexible agents in parts 1-4 because we do not exclude the possibility of zero revenue in these cases. In the remaining cases, we can show that there is not any non-symmetric equilibrium as follows: Suppose there is a non-symmetric equilibrium, where dedicated agents charge different prices, when $\rho > \alpha_D$ and $R_D > R_F$. By Proposition 4, we should have that all of the dedicated agents earn zero in the equilibrium. However, a small group of dedicated agents can guarantee a strictly positive revenue by charging a very low price $\zeta$ since $\rho > \alpha_D$.

Similarly, suppose there is a non-symmetric equilibrium, where flexible agents charge different prices, when $\rho > 1$. By Proposition 4, we should have that all of the flexible agents earn zero in the equilibrium. However, a small group of flexible agents can guarantee a strictly positive revenue by charging a very low price $\zeta$ since $\rho > 1$.

**S.4.1. Existence of the equilibrium:**

We prove the existence of the equilibrium by constructing one for each of the following three cases assuming $R_D \geq R_F$:

**Case-1 ($\rho < \alpha_D$):** We show that $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ is a $(\epsilon^k, \delta^k)$-FME where $\tilde{p}_D^k = \tilde{p}_F^k = 0$, and $\epsilon^k$ and $\delta^k$ go to zero as $k \to \infty$. To prove this claim by contradiction, we suppose that $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ is not $(\epsilon^k, \delta^k)$-FME for $k > K$ for some $K$. Then, at least one group of agents must have a profitable deviation. Suppose, a $y^k < \delta^k$ fraction of dedicated agents improve their revenues by increasing

$^3$If the lowest price among the dedicated agents is zero, then the deviating agents can choose $\zeta = (1 - \alpha_D / \rho)(R_D) / 2$. Otherwise, it must be the case that the the lowest price among the dedicated agents is greater than $R_D - R_F$. Then, the deviating agents can choose $\zeta = (R_D - R_F) / 2$.

$^4$If the price charged by the dedicated agents is less than $R_D - R_F$, then deviating agents can choose $\zeta = (1 - 1 / \rho)(R_D) / 2$. Otherwise, they choose $\zeta$ as the half of the price charged by the dedicated agents.
their prices to $p'$. Notice that they must increase their prices by more than $\epsilon^k$, i.e., $p' > \epsilon^k$, in order to have a profitable deviation. However, the revenue of deviating agents would be zero for large $k$ after such a deviation by Proposition 1 because $\lim_{k \to \infty} \frac{(R_D - \tilde{p}_D^k)(\alpha_D - y^k)}{(R_D - p')} \geq \alpha_D > \rho$. Similarly, flexible agents cannot improve their revenues by increasing their prices. Thus, $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ is a $(\epsilon^k, \delta^k)$-FME as $k \to \infty$.

**Case-2** ($\alpha_D \leq \rho < \alpha_D + \alpha_F$): In this case, we show that $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ is a $(\epsilon^k, \delta^k)$-FME where $\tilde{p}_D^k = R_D - R_F - \epsilon^k/2$, $\tilde{p}_F^k = 0$, and $\epsilon^k$ and $\delta^k$ go to zero as $k \to \infty$. We first want to note that the revenue of the dedicated agents according to $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ is $\tilde{p}_D^k$ by Proposition 1 since $\rho \geq \alpha_D$.

Similar to the above case, suppose, on the contrary, that $y^k < \delta^k$ fraction of dedicated agents improve their revenues by increasing their prices to $p'$. Notice that $p'$ must be greater than $\tilde{p}_D^k + \epsilon^k$ to be a profitable deviation. However, the revenue of deviating agents would be zero for large $k$ after such a deviation by Proposition 1 because $\lim_{k \to \infty} \frac{(R_D - \tilde{p}_D^k)(\alpha_D - y^k) + (R_F - \tilde{p}_F^k)\alpha_F}{(R_D - p')} \geq \alpha_D + \alpha_F > \rho$. Similarly, flexible agents cannot improve their revenues by increasing their prices. Thus, $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ is a $(\epsilon^k, \delta^k)$-FME as $k \to \infty$.

**Case-3** ($\rho \geq \alpha_D + \alpha_F$): Consider the agents strategy $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ with $\tilde{p}_D^k = R_D - (1 - a)\epsilon^k$ and $\tilde{p}_F^k = R_F - a\epsilon^k$ where $1/2 > a > (\bar{a} - 1)/|2\bar{a} - 1|$. The revenue of the dedicated agents according to $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$ is $\tilde{p}_D^k$ by Proposition 1, so that they cannot improve their revenues by more than $\epsilon^k$. Moreover, the revenue of the flexible agents according to $(\tilde{p}_D^k, \tilde{p}_F^k; \alpha_D, \alpha_F)$, say $\tilde{V}_F^k$, is greater than or equal to $(R_F - a\epsilon^k) \left(1 - \frac{\alpha_F}{a\epsilon^k} \left(\frac{\epsilon^k(1 - 2a)}{a\epsilon^k}\right)\right)$ by Corollary 2. After some algebra, we can show that $\tilde{V}_F^k + \epsilon^k > R_F + \epsilon^k[1 - \bar{a} + a(2\bar{a} - 1)] > R_F$, where the last inequality holds by the choice of $a$ (If $\bar{a} \geq 1/2$, then $1 - \bar{a} + a(2\bar{a} - 1) > 0$ since $a > (\bar{a} - 1)/(2\bar{a} - 1)$. Otherwise, $1 - \bar{a} + a(2\bar{a} - 1) > 0$ since $a < 1/2$). As the flexible agents cannot charge more than $R_F$, this implies that the flexible agents also cannot improve their revenues by more than $\epsilon^k$. 