Mass customization and guardrails: “You can’t be all things to all people”

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A mass customization ("customer as designer") strategy enables a firm to yield a closer match between its products and customer tastes. A mismatch between those tastes and the delivered product implies a cost to the consumer. Rather than assume all products, other than the one purchased, are immaterial to the customer, we consider the brand-level issue of mismatch between customers’ tastes and the firm’s full range of products. To this end, we permit consumers’ utility to depend upon the distance between their ideal point and the limits of the firm’s product varieties, which we refer to as guardrail products. Mass customization can thus reduce product-specific “fit” costs yet exacerbate brand-level fit. In the absence of brand-level costs, the firm extracts maximal revenue from consumers by offering a range of varieties to cover all possible consumer tastes at a uniform price. We show that considering a brand level cost implies that the firm should limit the breadth of the mass customization region to be less than full market coverage. More interestingly, we also find that the firm should optimally implement a differential pricing scheme for those product varieties at the outer extremes of its product line, catering to those consumers with more extreme tastes. Given that a continuum of prices will likely not be practical in application, we also consider the more pragmatic approach of a set of fixed (non-customized) products to serve customers at the taste extremes, with a corresponding set of (lower) fixed prices. We prove this heuristic solution performs close to optimal.

Key words: Mass customization; product line design; pricing; brand image.

1. Introduction

Product line design involves deciding not only which product varieties to offer within a given range, but also how wide that range should be. Offering a wide range of products may seem like a desirable selling strategy, yet may result in a lack of brand image coherency. Due to the potential negative impact of variety on brand image, in practice managers must be careful not to overextend their product lines. Consider the potential harm that could follow if Porsche, for example, were to extend its line of cars to include a wagon, in an attempt to create a model variant targeting big families. This extreme example highlights the need to bound product line variety, reflecting the notion that “It is rarely possible for a product or brand to be all things to all people” (Levy and Rook 1999).
While brand image concerns may seem somewhat vague or “soft,” it is relatively easy to understand the potential benefits from expanding variety. Increased variety allows a firm to better match diverse consumer tastes with distinct products. The gap between a customer’s ideal product and the firm’s closest product match results in some reduction in the customer’s willingness to pay, and therefore the firm benefits by reducing that gap. Thus, not surprisingly, traditional product line design research in a horizontal differentiation setting focuses on the tradeoff between that gap-reduction benefit and the cost of procuring additional varieties. Per Lancaster (1990), “If there are no economies of scale associated with individual product variants…then it is optimal to produce to everyone’s chosen specification.” This conclusion held because brand image effects are not captured in the traditional product variety literature.

To justify analyzing variety decisions apart from brand image effects, an argument could be made that variety decisions would typically involve carefully vetted in-house designs that reflect a brand’s status quo. The firm’s designers would know, for example, to not suggest a Porsche station wagon. However, as we discovered in discussions with management involved with Nike during the time when its new FlyKnit knitted shoe-upper technology was being developed, variety propagation and brand image become significant concerns when mass customization allows customers to create product design. In particular, we learned that Nike’s new FlyKnit technology has the potential to allow customized shoe uppers. Beyond the simple mix-and-match color choices that Nike’s current NikeID site allows for various shoe regions, FlyKnit could potentially allow customers to design custom patterns at the thread level. And, we were told, the cost implications should be minor, provided the patterns utilized some subset of the standard thread colors within the programmable FlyKnit knitting machines. We learned that a serious concern for Nike was the unbridled range of patterns that customers would produce. While naturally “Christmas sweater” patterns on a Nike shoe might be viewed with disdain by iconic Nike designers, the more significant concern is potential for a negative impact from the resulting wide-ranging set of designs on Nike customers.

In this paper, motivated by the concerns raised by the FlyKnit example, we consider a firm producing a mass-customized product for a market of consumers with a range of ideal points. The standard assumption in the horizontal differentiation literature is that customers care only about the variant they purchase, and ignore all other variants in the firm’s product line. Instead, we permit customers to care not only about the product they purchase, but also to some degree about the extent to which the firm’s other products differ from their respective ideal points. In this way, we capture the notion of brand image directly into each consumer’s willingness-to-pay function. Mass customization can eliminate the product-taste gap for many consumers, which tends
to increase willingness to pay, but an excessive product line width can potentially yield a reduction in all other consumers’ willingness-to-pay, such that a net loss may result. This approach also implies that distinct customers may differently perceive both the value of increasing variety and the associated brand image effect, depending upon where those consumers’ ideal points lie.

A core question we address is: When product aesthetics are customizable and consumers’ tastes are dispersed, to what extent should the firm place limits on customization, and how? Keller (2010) advises that a proper branding strategy should “provide ‘guardrails’ as to appropriate and inappropriate line and category extensions.” In this paper, we employ the term guardrail products to refer to the limits of the firm’s mass customization region. Such guardrails become relevant if the firm wishes to restrict the range of customizable product designs to be a strict subset of consumer tastes. Guardrails are called for when further extending the product line implies brand image repercussions in excess of the incremental varieties’ value to match more far-flung consumer tastes. Our model enables us to explore that tradeoff analytically, to gain a better understanding of the firm’s resulting positioning and pricing decisions.

When considering brand image repercussions, we intuitively expect a firm should utilize guardrail products to optimally limit its mass customization range of product designs, at least if it charges a single price. This range does indeed shrink, but as we will show, only if the brand image cost exceeds a certain threshold. Moreover, we will prove that the optimized region of uniformly priced customized products, which we refer to as the benchmark solution, is not the best the firm can do. We further prove that the firm can increase revenues by augmenting mass customization with differential pricing. We find that by moving the extremes of the customization region inward (i.e., towards less extreme tastes) and charging lower prices for the customized products at the outer regions of the taste spectrum, the firm can increase overall profit by as much as 7%. We also show that some of the customers outside the product portfolio purchase the guardrail products although the guardrails are not their ideal products. From a practical perspective, offering customized products with differential pricing may be hard to implement as it requires infinitely many different prices. Therefore, we propose a heuristic product portfolio that approximates the benefits of differential pricing. In particular, we replace the region of mass customization where the firm charge different prices with finite number of fixed (non-customized) products. We show that our proposed portfolio performs very near optimal when the number of fixed products approaches approximately 20.

The rest of the paper is organized as follows: We provide an overview of the related literature in Section 2. Then, Section 3 introduces the basics of our mathematical model. We solve the firm’s
problem in Section 4. We, next, explore the added benefits of differential pricing in Section 5. Section 6 concludes the paper.

2. Literature Review

Lancaster (1990), in his excellent review of the production differentiation literature, notes: “The fundamental structure of all optimal variety problems, for the individual firm as well as society, is the interplay of two elements in the economy - the existence of a gain from variety and the existence of scale economies of some kind.” Essentially, producing fewer varieties is cheaper than producing many, and therefore even though consumers prefer more variety, the firm may optimally produce less variety. More recent (empirical) research on product variety, reviewed by Chernev (2011), has explored possible consumer-centric disadvantages of variety, putting into question whether more variety is necessarily preferred to less. That research, stemming from the marketing and consumer psychology literature, suggests a potential downside to variety, due to the cost that variety itself can impose upon consumers.

Prior to the relatively recent empirical evidence that variety may impose a cost on consumers, researchers focused solely on the value afforded by variety to satisfy the needs of diverse consumers. In the Hotelling (1929) location model of consumer “taste” or style preferences, the existence of multiple product varieties serves to reduce the mismatch between what the consumer desires and the delivered product. It follows that the firm can increase revenue by increasing product variety and would ideally serve the market via an infinite number of varieties, precisely matching each consumer’s unique taste.

In contrast, when products differ in their quality level, a firm’s resulting vertical differentiation problem can result in an optimal range of product varieties that is less extensive than the range of consumer varieties. Specifically, a monopolist may need to optimally aggregate consumers of disparate types to control for the cannibalization that occurs when otherwise high-paying consumers switch to a low-price product. This aggregation implies a reduction in optimal variety, and has been found to apply both when consumers fall across a continuous spectrum of types (Mussa and Rosen 1978) and across discrete segments (Moorthy 1984). As concluded by Moorthy (1984), “In particular, segments may be aggregated even though there are no economies of scale.” Given the inherent variety limiting nature of the vertical differentiation problem, no supply side costs are required to induce a reduced-variety optimal solution for the firm. Netessine and Taylor (2007) address vertical differentiation for two consumer types while considering EOQ ordering and inventory costs, and explore how those supply-side costs can further induce the firm to offer only one variety.
There is extensive research involving various supply-side constructs that could limit variety with horizontal differentiation. In a circular adaptation of Hotelling’s linear city, Salop (1979) assumed a fixed cost associated with each product variant, thus limiting optimal variety. In a more detailed modeling of costs, de Groote (1994) assumes that varieties are produced via a rotation-cycle manufacturing system, and then explores how the resulting setup and holding costs restrict optimal variety. Green and Krieger (1985) also assume a fixed cost associated with each variety, but model products as a bundle of characteristics as per Lancaster (1975). Gaur and Honhon (2006) follow a similar approach and also permit uncertain consumer preferences. In Jiang et al. (2006), the producer chooses a number of base products (each with fixed cost), each of which can be customized into products of lower quality. In the absence of fixed costs, van Ryzin and Mahajan (1999) show that if consumers follow a multinomial choice model, then variety increases demand variability and hence inventory, thus bounding variety. Desai (2001) models both horizontal and vertical differentiation, but the analysis assumes two discrete segments and two products per firm, in the duopoly extension. Quality is a decision, but to maintain tractability, product locations (i.e., the variety decisions) are fixed a priori.

Dobson and Kalish (1988) pose the firm’s product-line design problem permitting a general number of consumer segments and predefined product varieties, as well as a general assignment of consumer reservation prices to products. Dobson and Yano (2002) include inventory costs and additionally model the contention between varieties for production capacity within a rotation cycle setting (with each variety’s demand decreasing linearly in the production delay). The resulting formulations in both these papers are mixed-integer programs, with fixed costs for each product variety. Given the problem complexity, in both cases the authors develop heuristic solution procedures. A review of product-line design models emphasizing variable manufacturing costs, costs of manufacturing facilities, and engineering design costs, is given in Yano and Dobson (1998).

In contrast with the aforementioned papers, which treat variety restriction stemming from various supply-side factors (namely, either fixed costs, inventory costs, or capacity sharing), we instead consider a potential demand-side variety impediment. When summarizing their empirical study of over 1,400 business units, Kekre and Srinivasan (1990) concluded: “American manufacturing firms may indeed be flexible enough to accommodate product variety without significant effects on costs.” Such flexibility is consistent with the notion of mass customization, i.e., providing a customized product with near mass-market efficiency. In this paper, we will specifically consider a firm that has mass customization capability and can offer a continuous range of product varieties without incurring a fixed cost for each of the (effectively infinite) possible varieties.
Alptekinoğlu and Corbett (2010) weigh the option of using mass customization for make-to-order (MTO) custom items, versus holding inventory for standardized make-to-stock (MTS) items. They permit a general dispersion of consumers over the Hotelling line but assume their reservation price is sufficiently high to ensure that full market coverage is optimal for the firm. They model the firm’s production system as an M/M/1 queue and then weigh lead time and inventory cost considerations to determine the firm’s optimal mix of MTS and MTO products. Earlier papers in the literature on mass customization did not consider the design of a mixed portfolio of customized and standardized products for a single firm, but rather focused on the competition between customized and standardized products (offered by two firms in a duopoly). Alptekinoğlu and Corbett (2008) and Mendelson and Parlaktürk (2008) consider duopoly competition between one firm with mass customization capability and another with mass production capacity (with these technology choices set exogenously). They study under what conditions the two firms can coexist in equilibrium. Alptekinoğlu and Corbett (2008) focus on a case where custom products share a uniform price, and find that firms can coexist in equilibrium even when the mass-producer has higher costs. Mendelson and Parlaktürk (2008) show that equilibrium vanishes in the case where the customizing firm sets differential prices. Earlier papers addressing competition involving mass customization considered symmetric duopoly cases (Syam et al. 2005, Syam and Kumar 2006, Dewan et al. 2003).

In the empirical literature, various advantages and disadvantages relating to product variety have been studied. Empirical research provides evidence that product variety helps to address: (i) the ability of consumers to find products matching their individual preferences (Betancourt and Gautschi 1990, Kahn and Lehmann 1991), (ii) consumer concerns regarding uncertain future tastes and product availability (Kahneman and Snell 1992, Shin and Ariely 2004), (iii) consumers’ enjoyment of their shopping experience (Babin et al. 1994), and (iv) consumers’ post-purchase satisfaction regarding their choices (Botti and Iyengar 2004).

Interestingly, despite such consumer-related variety benefits, several empirical studies have found that variety does not always boost sales. As part of its 1993 report (Kurt Salmon Associates 1993), the Food Marketing Institute conducted a field study in which it reduced SKUs in six product categories (cereal, pet food, salad dressing, spaghetti sauce, toilet tissue, and toothpaste) at 24 test stores. Redundant SKUs were eliminated and shelf space was held constant, with more space now allocated to high market share items. Importantly, the results showed no significant negative impact of SKU reduction on sales. In another series of in-store studies, Dreze et al. (1994) examined a 10% reduction of low volume SKUs in eight test categories over a four month period; across
eight categories, they found that sales increased by 4% for 30 test stores compared with 30 control stores.

Also in the grocery context, Broniarczyk et al. (1998) showed in a retail setting that reductions of up to 54% in slow-selling SKUs had no significant impact on sales. Similarly, Dreze et al. (1994) found a 10% SKU reduction to result in a 4% sales increase, and Boatwright and Nunes (2001) also found that decreasing a grocer’s assortment across nearly all product categories resulted in a notable sales increase. Iyengar and Lepper (2000) performed an in-store experiment in which consumers were presented with distinct gourmet jam assortments. They found consumers to be more likely to purchase when presented with a six-item assortment than with a 24-item assortment. Analogous benefits from variety reduction have been documented in employee’s choosing retirement plan funds (Iyengar et al. 2004, Huberman et al. 2007, Iyengar 2010, Morrin et al. 2008).

Various possible explanations for why increasing variety may inhibit consumer participation and sales have been explored. In his 2004 book The Paradox of Choice, Barry Schwartz wrote: “As the number of choices grows further, the negatives escalate until we become overloaded. At this point, choice no longer liberates, but debilitates.” According to Huberman et al. (2007), a increase in product variety can also exacerbate the uncertainty of consumers’ preferences. Another uncertainty-related issue is that variety may cause consumers to associate a higher probability of post-decisional regret, leading some consumers to refrain from purchasing (Gourville and Soman 2005, Schwartz 2004, Beattie et al. 1994).

In our work, we assume consumers’ preferences are known (deterministic), in which case the uncertainty-related variety issues are moot. And, in contrast with the aforementioned empirical studies of grocery retailers, we focus on a brand-level impact of product line variety. Berger et al. (2007) studied how consumers’ perceptions of a brand’s quality related to the product line breadth. In one experiment, they considered whether the extent of a restaurant’s menu variety influenced brand image. Using a Thai restaurant menu as their baseline, they considered expanding the menu variety to exhibit either “compatible variety (the same options plus five other Thai food options),” or “incompatible variety (i.e., a few Thai options plus five non-Thai options, such as egg rolls).” They found that “participants perceived the brand more favorably (both in terms of expertise and quality) only when the options included in the expanded set were compatible.” In a similar experiment, they provided consumers with information about different bicycle brands, and found that when the expanded product line variety was incompatible or “unfocused” (in this case, adding mountain bikes to a road bike manufacturer’s lineup), brand perceptions decreased.
As concluded by Scheibehenne et al. (2010) in a recent review of the choice overload literature: “To understand the effect that assortment size can have on choice, it will be essential to consider the interaction between the broader context of the structure of assortments beyond the mere number of options available . . . .” Consistent with that message, in this paper a consumer’s utility relates to the product purchased as well as the structure (i.e., the width of the assortment space) of the firm’s product line.

3. Model Basics

We consider a monopolist firm that serves customers who are heterogeneous in their tastes. Namely, each consumer is identified by a point $z$ that represents her ideal product. Following the standard Hotelling linear city framework, consumers’ ideal points are uniformly spread over the unit interval $[0, 1]$. We refer to this linear market as the “taste spectrum.” Each customer is in the market to purchase at most one unit. For simplicity, we assume customers are small relative to the size of the market, which is denoted by $\lambda$.

Customers who decide to purchase a product earn a reward of $V$ and incur a cost $t$ per unit distance between their ideal product and the purchased product. In addition to that mismatch cost which is directly associated with the purchased product, we consider the brand-level issue of mismatch between customers’ tastes and the firm’s full range of products. We refer to this brand-level mismatch cost as brand image cost. We capture this brand image cost considering the distance from a consumer’s ideal product to the range of the firm’s product portfolio. In particular, customers incur a cost $\alpha$ per unit distance between their ideal products and the two end points that define the range of the firm’s product line.

The firm’s decisions entail designing its product portfolio and setting prices. Because our emphasis is on understanding the demand-related effects of product variety, we normalize the production costs to zero. Let us denote the product portfolio as the set $X$, and the corresponding prices by a function $p : X \rightarrow [0, V]$. The utility of a customer located at $z \in [0, 1]$ from product $x \in X$ is, then,

$$U(z, x, p) = V - p(x) - t|z - x| - \alpha(|z - \underline{x}| + |\bar{x} - z|),$$

where $\underline{x} = \min X$ and $\bar{x} = \max X$. We refer to the points $\underline{x}$ and $\bar{x}$, which define the range of the product line, as guardrail products.

Each customer buys the specific product that delivers the highest utility and will make no purchase if all products deliver negative utility. If a customer is faced with multiple options that yield the same net utility, we assume that the customer favors a near product relative to a distant
product, and favors purchasing over not purchasing. We denote customers’ most preferred products by the function \( b: [0, V] \rightarrow X \) for any given price function \( p \) such that \( b(z, p) \equiv \arg \max_{x \in X} U(z, x, p) \) for any \( z \in [0, 1] \). We let the set \( B \) be the set of customers making a purchase, i.e., \( B \equiv \{ z : U(z, b(z, p), p) \geq 0 \} \). We assume \( B \) is an interval and refer to it as the “market coverage” of the firm. The revenue of the firm is, then,

\[
\Pi(X, p, b) = \lambda \int_B p(b(z))dz
\]

for any product portfolio \( X \), price function \( p \), and the corresponding purchasing decisions of the customers.

The existing product portfolio management literature suggests that a revenue-maximizing firm should offer a continuum of products (at a single price \( V \) in this case) to cover the entire market. This result holds because offering mass customization for the entire market allows the firm to extract all of the customers’ surplus. As a first step towards understanding the effects of the brand image cost, we study the traditional mass customization problem as our benchmark model. In particular, we find the optimal range of mass customization \([m, \overline{m}]\) and the optimal corresponding uniform price level, denoted by \( p_u \). In other words, firm’s product portfolio \( X \) is a set in the family of sets \( \mathcal{X}_M = \{[m, \overline{m}] : 0 \leq m \leq \overline{m} \leq 1 \} \), and the price function \( p(x) \) is equal to \( p_u \) for all products in the portfolio \( X \). We refer to this model as Uniform Pricing model and illustrate the firm’s product line and pricing decisions in Figure 1.a.

![Figure 1](image)

**Figure 1** The illustration of the firm’s product portfolio and pricing decisions in: (a) Uniform Pricing, (b) Differential Pricing models.

As noted above, ignoring brand image effects implies that the firm will maximize its sales by simply charging a uniform price \( V \) to all consumers. However, the presence of brand-level variety
effects can cause non-uniform pricing to be optimal, even if variety is costless. For instance, the firm may charge lower prices for the products closer to the ends of the taste spectrum to focus the product portfolio while serving the same portion of the market as it does with uniform pricing. To explore this possibility, we study the firm’s product portfolio decision in a model where the firm can charge different prices for its products. We refer to this model as Differential Pricing model.

In the Differential Pricing model, there are two different categories of mass customization regions. The firm charges a uniform price for the products in the $[m, \bar{m}]$ region, which is referred to as the uniform pricing region. There also exists another mass customization region where the firm charges different prices for the products on each side of the uniform pricing region. Therefore, the product portfolio choice of the firm, $X$, is a set in the family of sets $X_P \equiv \{[x, \bar{m}] \cup [m, \bar{m}] \cup [\bar{m}, \bar{x}] : 0 \leq x \leq m \leq \bar{m} \leq \bar{x} \leq 1\}$. We refer to the $[x, \bar{m}]$ and $[\bar{m}, \bar{x}]$ regions as the differential pricing regions.

We suppose that the firm uses a continuous price function $p(x)$. Furthermore, as in the Uniform Pricing model, we denote the price of the products in the uniform pricing region by $p_u$, i.e., $p(x) = p_u$ for all $x \in [m, \bar{m}]$. Note that the firm can abandon uniform pricing by setting $m = \bar{m}$; or it can implement uniform pricing across the full product line by setting $x = m$ and $\bar{x} = \bar{m}$. Figure 1.b illustrates the firm’s product line and pricing decisions in the Differential Pricing model.

After we formally introduce the models we study, we turn our attention to characterizing the optimal product portfolio and pricing decisions in the next section.

4. The Analysis of the Firm’s Problem

In this section, we analyze the firm’s problem of finding the optimal product portfolio and prices. We begin by focusing on the Uniform Pricing model described in Section 3. In this benchmark model, the firm’s product portfolio decision is limited to $X_M$, so that the firm offers only a continuum of customized products in the interval $[m, \bar{m}]$ with some corresponding uniform price $p_u$. In this case, the most preferred product of a customer whose ideal product is in the range of mass customization will be the product that fits her taste perfectly. Moreover, customers outside the $[m, \bar{m}]$ will prefer the product at the end point that is closest to their ideal products. Therefore, the function representing the most preferred products of customers will be

$$b(z, p_u) = \begin{cases} m & \text{if } z < m \\ z & \text{if } m \leq z \leq \bar{m} \\ \bar{m} & \text{if } z > \bar{m}. \end{cases}$$

The above function shows that the product that delivers the highest utility does not depend on the price. However, customers’ purchasing decisions depend on $p_u$. Note that if $p_u$ is greater than
\[ V - \alpha(\overline{m} - \underline{m}) \], none of the customers make a purchase, so that the firm has to charge a price lower than \( V - \alpha(\overline{m} - \underline{m}) \). For any \( p_u \leq V - \alpha(\overline{m} - \underline{m}) \), all of customers in the interval \([\underline{m}, \overline{m}]\) will purchase but customers outside the mass customization range might not. Specifically, the set of customers making a purchase, \( B \), is the interval \([\underline{b}, \overline{b}]\) where \( \underline{b} = \underline{m} - \left[ V - p_u - \alpha(\overline{m} - \underline{m}) \right] \) and \( \overline{b} = \overline{m} + \left[ V - p_m - \alpha(\overline{m} - \underline{m}) \right] \).

The literature on mass customization argues that the firm should ensure that customers either buy their ideal product or none at all. We prove, even when incorporating brand image effects into the consumer utility model, this insight continues to hold. As a direct implication of that, the firm has to set its price \( p_u \) equal to \( V - \alpha(\overline{m} - \underline{m}) \). We formally present this result in the following proposition.

**Proposition 1.** Let \( X^* \equiv [\underline{m}^*, \overline{m}^*] \in X_M \) be the optimal product portfolio and \( B^* \) be the optimal market coverage in the Uniform Pricing model. Then, we have that \( B^* = X^* \) and \( p_u^* = V - \alpha(\overline{m}^* - \underline{m}^*) \) where \( p_u^* \) is the optimal price.

The above proposition establishes that the firm’s pricing decision and customers’ purchasing decision can be expressed in terms of the mass customization bounds \( \underline{m} \) and \( \overline{m} \). This helps us to rewrite the revenue function described in (2) as

\[
\Pi_M(\underline{m}, \overline{m}) = \lambda[V - \alpha(\overline{m} - \underline{m})](\overline{m} - \underline{m}).
\]

Then, the firm’s problem reduces to \( \max_{0 \leq \underline{m} \leq \overline{m} \leq 1} \Pi_M(\underline{m}, \overline{m}) \). The following theorem formally presents the optimal decisions of the firm.

**Theorem 1.** Let \( X^* \equiv [\underline{m}^*, \overline{m}^*] \in X_M \) be the optimal product portfolio and \( \Pi^* \) be the optimal revenue in the Uniform Pricing Model. Then, we have that

\[
\overline{m}^* - \underline{m}^* = \begin{cases} 
1 & \text{if } \alpha < V/2 \\
V/(2\alpha) & \text{if } \alpha \geq V/2,
\end{cases}
\text{ and } \Pi^* = \begin{cases} 
\lambda[V - \alpha] & \text{if } \alpha < V/2 \\
\lambda V^2/(4\alpha) & \text{if } \alpha \geq V/2.
\end{cases}
\]

The above theorem shows that the firm prefers to cover the entire market when \( \alpha \) is low; in other words, the firm follows what the traditional mass customization literature prescribes. However, once \( \alpha \) exceeds the critical level of \( V/2 \), the firm narrows the range of its product portfolio and does not serve the entire market. Furthermore, the width of the product portfolio shrinks as the brand image cost increases.

The *Uniform Pricing* model, which permits a uniform price and a continuous range of products in the firm’s portfolio, demonstrates that the firm may optimally limit the range of mass customization even when such customization is costless. We next explore the possibility of charging different prices for the firm’s products. To this end, we study the *Differential Pricing* model introduced in Section 3.
4.1. Differential Pricing Model

We now consider the Differential Pricing model, which generalizes the benchmark model by allowing the firm to offer a continuous range of prices. The firm may offer mass customization in \([x, \bar{x}]\) but charge a uniform price \(p_u\) within \([\underline{m}, \overline{m}]\). The prices for the customized products in the differential pricing regions, \([x, \underline{m}]\) and \([\overline{m}, \bar{x}]\), can be different than \(p_u\). In other words, the price function \(p(x)\) is equal to \(p_u\) for all \(x \in [\underline{m}, \overline{m}]\), but can take any value (less than \(V\)) for \(x \notin [\underline{m}, \overline{m}]\).

We do not have any restriction on the price function \(p(x)\) other than it being a continuous function, yet we are able to show that it has a very simple structure. We show that \(p(x)\) must be linear with slope \(t\) and increasing as the product gets closer to the uniform pricing region. To be specific, \(p(x)\) must be equal to \(p(x) + t(m - x)\) for any \(x \in [x, \underline{m}]\) and \(p(x) + t(x - \overline{m})\) for any \(x \in [\overline{m}, \bar{x}]\). There are two main drivers of this simple structure: First, all customers in the differential pricing region purchase their ideal products under the proposed linear price function, so that the firm has no incentive to further reduce prices. Second, if the price is higher than the proposed linear price for a range products \(x \in (x_1, x_2)\), all customers whose ideal product is in this range will prefer to buy the product with lowest price in \((x_1, x_2)\). As these customers do not buy their ideal products, they incur a positive travel cost, and this creates an opportunity for the firm to increase revenue by moving its prices closer to the proposed linear price function.

The simple structure of the price function simplifies our analysis significantly. We can show that the firm leaves no surplus to the customers whose ideal product is in the uniform pricing region \([\underline{m}, \overline{m}]\) similar to the benchmark model where the firm charges only a uniform price. However, unlike the Uniform Pricing model, the firm chooses to leave some surplus to the customers outside the uniform pricing region, even though the customers purchase their ideal product. The firm gains two major benefits by leaving positive surplus to some of its customers: (i) it serves customers outside the mass customization region, (ii) it narrows the product portfolio, which allows the firm to charge a higher price in the uniform pricing region. We also show that customers at the end points of market coverage obtains zero utility and the firm’s optimal product portfolio is symmetric around the uniform pricing region \([\underline{m}, \overline{m}]\). We formally present these results in the following proposition.

**Proposition 2.** Let \(X^* \equiv [\underline{x}^*, \underline{m}^*] \cup [\overline{m}^*, \overline{x}^*] \cup [\underline{m}^*, \overline{m}^*] \in X_p\) be the optimal product portfolio and \(B^*\) be the optimal market coverage in the Differential Pricing Model. Then, the optimal price function \(p^*\) is

\[
p^*(x) = \begin{cases} 
V - t(m^* - x) - \alpha(x^* - \underline{x}^*) & \text{if } x \in [\underline{x}, \underline{m}], \\
V - \alpha(x^* - \underline{x}^*) & \text{if } x \in [\underline{m}, \overline{m}], \\
V - t(x - \overline{m}^*) - \alpha(x^* - \underline{x}^*) & \text{if } x \in [\overline{m}, \overline{x}],
\end{cases}
\]
and the optimal location of the non-customized products are

\[ m^* - x^* = \frac{t + 2\alpha}{2(t + \alpha)} (m^* - b^*), \quad \bar{x}^* - m^* = \frac{t + 2\alpha}{2(t + \alpha)} (\bar{b}^* - m^*), \]

where \( b^* = \min(B^*) \), \( \bar{b}^* = \max(B^*) \). Furthermore, we have that \( m^* - b^* = \bar{b}^* - m^* \) and the customers’ most preferred products are

\[
b^*(z, p^*) = \begin{cases} 
  x^* & \text{if } z \leq x^* \\
  z & \text{if } x^* \leq z \leq \bar{x}^* \\
  \bar{x}^* & \text{if } z \geq \bar{x}^*.
\end{cases}
\]

Consistent with this result, customers whose ideal tastes lie outside the product portfolio face higher brand-level costs, and thus utility falls off more sharply outside the range \([x^*, \bar{x}^*]\). This implies that the guardrail products (at \( x^* \) and \( \bar{x}^* \)) will not be precisely in the middle of the market segments served by differential pricing (i.e., \([\bar{b}^*, m^*]\) and \([m^*, \bar{b}^*]\), respectively). We illustrate the location of guardrail products and customer surplus in Figure 2.

![Figure 2](Image)

Figure 2 The customer surplus for all customers whose ideal products are in \([\bar{b}, \bar{b}]\).

Similar to our benchmark analysis, Proposition 2 allows us to rewrite the firm’s pricing and product location decisions as a function of the end points of the uniform pricing \( m \) and \( \bar{m} \), and the end points of the market coverage \( \bar{b} \) and \( b \). In fact, due to the symmetry of the firm’s decisions, we can, without loss of generality, assume that the midpoint of the product portfolio is 1/2. Then, we can rewrite the firm’s revenue as a function of \( m \) and \( \bar{b} \) as follows:

\[
\Pi_P(b, m) = 2 \left[ (x^* - b)p^*(x^*) + \int_{x^*}^{m} p^*(x)dz + (1/2 - m)p^*(m) \right],
\]

where the location of the guardrail product, \( x^* \), and the price functions, \( p^*(x) \), are the functions described in Proposition 2. Then, the firm’s problem becomes solving \( \max_{b, m} \Pi_P(b, m) \). It is worth
noting that the firm reverts back to the benchmark model if it sets \( b = m \), i.e., \( \Pi_p(b, b) = \Pi_M(b, 1 - b) \). When the firm uses differential pricing, the guardrail products shift towards the middle of the taste spectrum, which also shifts the uniform pricing region limit \( m \) inward. Such a move towards less extreme tastes allows the firm to charge a higher price in the uniform pricing region, but at the expense of lower prices for the rest of the customized products. We illustrate the firm’s tradeoff in Figure 3 by drawing what each customer pays when \( m = b + \varepsilon \) for some \( \varepsilon > 0 \).

As we formally present in the following theorem, the gains from shrinking the mass customization region outweighs the losses for small values of \( \varepsilon \).

**Theorem 2.** Let \( X^* \equiv [x^*, m^*] \cup [m^*, \overline{m}^*] \cup [\overline{m}^*, x^*] \in \mathcal{X}_p \) be the optimal product portfolio, \( B^* \) be the optimal set of consumers to be served, and \( \Pi^* \) be the optimal revenue of the firm in the Differential Pricing model. Then, denoting \( \ell(B) \) as the length of set \( B \), we have that

\[
\ell(B^*) = \begin{cases} 
\frac{1}{2\alpha} \left[ 1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)} \right] & \text{if } \alpha < \alpha^*_f(t) \\
\frac{1}{2\alpha} \left[ 1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)} \right] & \text{if } \alpha \geq \alpha^*_f(t),
\end{cases}
\]

\[
\frac{\overline{x}^* - x^*}{\ell(B^*)} = \left[ 1 - \frac{2t\alpha}{(t+2\alpha)(3t+2\alpha)} \right], \quad \frac{m^* - \overline{m}^*}{\ell(B^*)} = \left[ \frac{t(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)} \right]
\]

and, \( \Pi^* = \begin{cases} 
\lambda \left[ V - \alpha \left( 1 - \frac{t\alpha}{(t+\alpha)(3t+2\alpha)} \right) \right] & \text{if } \alpha < \alpha^*_f(t) \\
\lambda \frac{V^2}{4\alpha} \left( 1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)} \right) & \text{if } \alpha \geq \alpha^*_f(t),
\end{cases}
\]
where $\alpha^*_f(t)$ solves the equation $\frac{V^2}{2\alpha} \left[ 1 + \frac{\alpha}{(t+\alpha)(3t+4\alpha)} \right] = 1$.

Theorem 2 highlights the crucial role of the brand image cost $\alpha$ while choosing the location of the products. When the firm uses differential pricing, it serves only a portion of the customers with a uniform price and utilizes lower-ranging prices to attract customers outside the product portfolio who do not purchase their ideal products. We also find that the firm might not serve the entire taste spectrum, similar to our results in the benchmark model. The firm serves the entire market only if the brand image cost is less than the threshold $\alpha^*_f(t)$. However, unlike the Uniform Pricing model, the firm never finds it optimal to offer mass customization for the entire taste spectrum. In fact, we show that the firm always offer a smaller range of mass customization in the Differential Pricing than it does in the Uniform Pricing model, as we formally present in Corollary 1 below.

**Corollary 1.** Let $X^o \equiv [m^o, \overline{m}] \in X_M$ and $X^* \equiv [\overline{x}^*, m^*] \cup [m^*, \overline{m}] \cup [\overline{m}, \overline{x}^*] \in X_P$ be the optimal product portfolios in the Uniform and Differential Pricing models. Then, we have that $\overline{x}^* - \overline{x}^* \leq \overline{m} - m^o$.

Our analysis of the Differential Pricing model establishes that the brand image cost leads two major adjustments in the firm’s decisions compared to the Uniform Pricing model: (i) the firm optimally implements price discrimination for those consumers with less central tastes, and (ii) the firm attracts some customers who do not purchase their ideal product (in particular, consumers with tastes near the extremes). We next explore the relationship between the brand image cost $\alpha$ and the relative sizes of the differential and the uniform pricing regions, and the extent to which the firm extracts customers whose ideal points lie beyond the bounds of the product portfolio. For a large range of $\alpha$ the firm covers the whole market, so that the relative sizes of these regions are equal to their absolute sizes. Moreover, Theorem 2 showed that their relative sizes did not depend on whether or not the firm served the entire market.

Using the results from Theorem 2, we show that the role of differential pricing becomes more important as $\alpha$ increases. In particular, we show that the size of the differential pricing region (relative to the total market coverage) increases in $\alpha$ whereas the size of the uniform pricing region decreases in $\alpha$. We find that portion of the customers purchasing a non-customized product is not a monotone function of the brand image cost. In particular, it is increasing for any $\alpha < t\sqrt{3}/2$ and decreasing otherwise. As a result of this, the size of the product portfolio is also not a monotone function of $\alpha$. We illustrate this non-monotone effect of $\alpha$ on the firm’s decisions in Figure 4. We also show that total revenues and the firm’s market coverage are both decreasing functions of the brand image cost. We present these results formally in the following proposition.
Proposition 3. Let \( X^* \equiv [x^*, m^*] \cup [m^*, \bar{m}^*] \cup [\bar{m}^*, \bar{x}^*] \in X_p \) be the optimal product portfolio, \( B^* \) be the optimal set of consumers to be served, and \( \Pi^* \) be the optimal revenue of the firm in the Differential Pricing model. Then, we have that

1. The size of the differential pricing region relative to the total market coverage increases by the brand image cost, i.e., \( \frac{m^* - x^*}{\ell(B^*)} \) is increasing in \( \alpha \).

2. The size of the uniform pricing region relative to the total market coverage decreases by the brand image cost, i.e., \( \frac{m^* - m^*}{\ell(B^*)} \) is decreasing in \( \alpha \).

3. The size of the customer segment served outside the product portfolio relative to the total market coverage first increases and then decreases in the brand image cost, i.e., \( \frac{\bar{x}^* - \bar{m}^*}{\ell(B^*)} \) is increasing in \( \alpha \) for any \( \alpha < t\sqrt{3}/2 \) and decreasing in \( \alpha \) for any \( \alpha > t\sqrt{3}/2 \). This also implies that \( \frac{\bar{x}^* - x^*}{\ell(B^*)} \) is decreasing in \( \alpha \) when \( \alpha < t\sqrt{3}/2 \) and increasing otherwise.

4. The optimal revenue and the optimal market coverage both decrease by the brand image cost, i.e., \( \Pi^* \) and \( \ell(B^*) \) are both decreasing in \( \alpha \).

Figure 4  Percentage of customers purchasing a non-customized product when \( V = 1 \) and \( t \in \{0.25, 0.75\} \).

5. Comparison

In the previous section, we studied two different product portfolio decisions of the firm. We showed that when the firm offers only a uniform (single) price, it does not necessarily limit the range of mass customization due to brand image effects. In other words, if brand image effects are limited then the optimal solution structure prescribed by the traditional mass customization literature
applies. On the other hand, if differential pricing is feasible, then the firm never serves all customers via mass customization. It is worth noting that this does not mean that the firm neglects customers who are not offered a customized product. Instead, the firm attracts customers outside the mass customization region by lowering the prices of the customized products that are closer to the ends of the taste spectrum.

Offering differential pricing is, by definition, better than the Uniform Pricing model since the latter one is more restricted. It is still an open question, though, exactly how much benefit the differential pricing yields, given the presence of brand image costs. To answer this question, in this section, we compare the revenues of the firm under the Uniform Pricing and the Differential Pricing models we study in Section 4.

5.1. Differential vs. Uniform Pricing

In this subsection, we measure the benefits from offering differential pricing as the relative improvements in revenue from the Uniform Pricing model to the Differential Pricing model. We define this benefit as a function of \( \alpha \) and \( t \) as follows

\[
\Delta(\alpha, t) = \frac{\Pi^*(\alpha, t)}{\Pi^0(\alpha, t)} - 1, \tag{4}
\]

where \( \Pi^0(\alpha, t) \) and \( \Pi^*(\alpha, t) \) are the optimal profits of the firm in the Uniform Pricing and the Differential Pricing models, respectively.

The structural properties of the firm's profits highly depend on whether it covers the entire market in both the Differential Pricing and the Uniform Pricing models. Thus, the above measure also depends on the market coverage decision of the firm. As we discuss before, the firm covers the entire market when the brand image cost \( \alpha \) is low but it may not serve all customers for larger values of \( \alpha \). The following lemma shows that the firm optimally serves a larger portion of the market in the Differential Pricing model.

**Lemma 1.** Let \( B^0 \) and \( B^* \) be the optimal set of consumers to be served in the Uniform Pricing and the Differential Pricing models. Then, letting \( \ell(B) \) be the length of any set \( B \), we have that

\[
\ell(B^0) \leq \ell(B^*).
\]

Using the above lemma, we conclude that there exists three different cases to consider while studying \( \Delta(\alpha, t) \):

*Case-1:* The firm serves the entire market under both models.

*Case-2:* The firm serves the entire market only under Differential Pricing model.
Case-3: The firm does not serve the entire market under both models

When Case-1 holds, the firm serves the entire market under both models regardless of the brand image cost $\alpha$. In other words, the firm disregards the brand image cost while choosing how much market segment it serves in Case-1. This also implies that the firm does not change the size of its product portfolio as the brand image cost increases when it is limited to mass customization. On the other hand, the firm is more responsive to the changes in the brand image cost under Differential Pricing model because it alters the size of the product portfolio as $\alpha$ increases even for low level of brand image cost. As a result of that we show that the benefit from employing differential pricing increases as the brand image cost $\alpha$ increases when Case-1 holds. In cases 2 and 3, the firm does not serve the entire market in the Uniform Pricing model, and thus its market coverage decision depends on the brand image cost when it employs uniform pricing. As the firm starts to take $\alpha$ into account while making product portfolio decision in the Uniform Pricing model, we show that the incremental benefits to be gained from offering differential pricing are not always monotonically increasing. We show that the structure of the benefit from differential pricing depends on when the firm stops covering the entire market in the Differential Pricing model. In Theorem 2, we describe the brand image cost level $\alpha_{fc}(t)$ at which the firm stops covering the entire market. The benefits from differential pricing $\Delta(\alpha, t)$ monotonically increases by $\alpha$ in Case-2 if the Differential Pricing model stops covering the entire market at a level of $\alpha$ that is less than $t\sqrt{3}/2$, i.e., if $\alpha_{fc}(t) < t\sqrt{3}/2$. Otherwise, $\Delta(\alpha, t)$ achieves its maximum when Case-2 holds and decreases by $\alpha$ when Case-3 holds.

We formally present these results in the following proposition and illustrate them in Figure 5.

**Proposition 4.** Let $\Delta(\alpha, t)$ be the benefits from offering differential pricing as described in (4) and $\alpha_{fc}(t)$ be the level of brand-level mismatch up to which the firm serves the entire market under Differential Pricing model, which is as described in Theorem 2. Contrasting the Uniform Pricing and Differential Pricing models’ respective optimal coverage regions and profits, we find:

1. The firm serves the entire market in both models for all $\alpha < V/2$. Furthermore, $\Delta(\alpha, t)$ is increasing in $\alpha$ for any $\alpha < V/2$
2. The firm serves the entire market only in the Differential Pricing model for all $V/2 \leq \alpha < \alpha_{fc}(t)$. Furthermore, $\Delta(\alpha, t)$ is an increasing function of $\alpha$ for any $V/2 \leq \alpha < \alpha_{fc}(t)$ if $\alpha_{fc}(t) \geq t\sqrt{3}/2$. Otherwise, $\Delta(\alpha, t)$ first increases and then decreases by $\alpha$ in the $[V/2, \alpha_{fc}(t))$ interval.
3. The firm does not serve the entire market in both models for all $\alpha > \alpha_{fc}(t)$. Furthermore, $\Delta(\alpha, t)$ is an decreasing function of $\alpha$ for any $\alpha > \alpha_{fc}(t)$ if $\alpha_{fc}(t) < t\sqrt{3}/2$. Otherwise, $\Delta(\alpha, t)$ increases by $\alpha$ in the $[\alpha_{fc}(t), t\sqrt{3}/2)$ interval and then decreases by $\alpha$. 
Case-1: BOTH pricing schemes cover the entire market.

Case-2: ONLY differential pricing covers the entire market.

Case-3: NEITHER pricing schemes covers the entire market.

\[ \alpha \Delta(\alpha, t) \]

when \( \alpha_{fc}(t) < t \sqrt{3}/2 \)

when \( \alpha_{fc}(t) \geq t \sqrt{3}/2 \)

Figure 5  The relative improvement in revenue from the Uniform Pricing model to Differential Pricing model as a function of the brand image cost \( \alpha \) for a given distance cost \( t \).

The above proposition shows the structure of the benefits stemming from the differential pricing, for a given distance cost \( t \). It shows that the maximum benefits cannot be achieved for low levels of brand image cost where the firm serves the entire market under both pricing policies. We next explore how and when the firm earns the highest benefits from guardrail products. We let \( \Delta_{\text{max}}(t) \) be the highest benefit from differential pricing for any given distance cost \( t \), i.e. \( \Delta_{\text{max}}(t) = \max_{0<\alpha} \Delta(\alpha, t) \). As one might expect, the benefits from differential pricing can be small for low levels of the distance cost \( t \) since the solutions of the Differential Pricing and the Uniform Pricing models become identical as \( t \) approaches to zero. However, we are still able to show that \( \Delta_{\text{max}}(t) \) has a lower bound which depends on the ratio of the travel cost \( t \) to the customer valuation \( V \). We also show that if the distance cost is greater than a critical level \( \bar{t} \approx 2V/3 \), then \( \Delta_{\text{max}}(t) \) is achieved when the firm limits its market coverage under both pricing policies (i.e., when Case-3 occurs) as illustrated by the dashed line in Figure 5. Furthermore, \( \Delta_{\text{max}}(t) \) can be as much as 7.2% when the distance cost \( t \) is greater than the critical level \( \bar{t} \). The value of brand-level mismatch achieving the 7.2% improvement is \( t \sqrt{3}/2 \). We formally present these results in the following theorem.

**Theorem 3.** Let \( \Delta_{\text{max}}(t) \) be the highest benefit from differential pricing for any given distance cost \( t \) and \( \hat{\alpha}^\theta(t) \) be the level of brand-level mismatch up to which the firm serves the entire market under differential pricing, i.e., \( \Delta_{\text{max}}(t) = \max_{0<\alpha} \Delta(\alpha, t) \).

1. \( \Delta_{\text{max}}(t) \) is bounded below by \( \frac{\theta}{2(1+\theta)(1+\theta)} \) where \( \theta = t/V \).

2. \( \Delta_{\text{max}}(t) = \frac{1}{7+4\sqrt{3}} \) and the highest benefit is achieved when \( \alpha = \sqrt{3}/2 \) for any \( t \geq \frac{4}{3+2\sqrt{3}} \sqrt{3}/2 \).
Theorem 3 proves that a 7.2% improvement in firm’s revenue is achieved when the distance cost \( t \) is sufficiently high, and identifies a lower bound on the firm’s benefit from the differential pricing. As we numerically show in Figure 6, the lower bound provides a reasonable approximation for the exact benefits of differential pricing, especially when the distance cost \( t \) is low.

Figure 6  The highest benefit from differential pricing for low values of travel cost \( t \) when \( V = 1 \).

In this subsection, we establish the possible gains of differential pricing under the assumption that the firm has the ability to offer and communicate infinitely many prices. In practice, this might not be a feasible option. Therefore, in the next subsection, we consider a heuristic portfolio with a finite number of differentially priced products.

5.2. Application of the Differential Pricing

In this subsection, we consider a heuristic model where the firm offers a finite number of products on each side of the optimal uniform pricing region \([\overline{m}^*, \underline{m}^*] \), as described in Theorem 2. We refer to these as fixed products and assume there are \( N > 1 \) such fixed products on each side of the uniform pricing region. Therefore, the product portfolio of the firm is \( \{x_1, \ldots, x_N\} \cup [\overline{m}^*, \underline{m}^*] \cup \{x_{N+1}, \ldots, x_{2N}\} \), where \( x_i \) denotes the locations of fixed product-\( i \) for any \( 1 \leq i \leq 2N \). We suppose that the firm follows its optimal strategy from the Differential Pricing model to determine the price function and locations of the guardrails and uniform pricing region, i.e., the price function is \( p^*(x) \) with \( \{x_1, x_{2N}\} = \{\underline{x}^*, \overline{x}^*\} \). For any \( N > 1 \), the firm sets the locations of all in-between products to maximize its revenue. We next show that the firm should optimally distribute the in-between products uniformly between the guardrails and the uniform pricing region.
Proposition 5. Let \( \{\tilde{x}_1, \ldots, \tilde{x}_{2N}\} \) be the optimal locations of in-between products in the heuristic model for any given \( N > 0 \). Then, we have that

\[
\tilde{x}_1 = x^* \quad \text{and} \quad \tilde{x}_{i+1} - \tilde{x}_i = (m^* - x^*) / N \quad \text{for all} \quad 1 \leq i < N.
\]

We also have that \( \tilde{x}_i + \tilde{x}_{2N-i+1} = 1 \) for all \( 1 \leq i \leq N \). In other words, the firm distributes the in-between products uniformly between the guardrails and the uniform pricing region. Furthermore, customers’ most preferred products are

\[
\tilde{b}(z) = \begin{cases} 
  x^* & \text{if } z \leq \tilde{x}_2 \\
  x_i^* & \text{if } \tilde{x}_i \leq z \leq \tilde{x}_{i+1}, \quad 1 < i < N, \quad \text{and} \quad z < m^* \\
  z & \text{if } m^* \leq z \leq 1/2,
\end{cases}
\]

for any \( z \leq 1/2 \), and \( \tilde{b}(z) = \tilde{b}(1-z) \) for any \( z > 1/2 \).

Using the above result, we can write the firm’s revenue under the heuristic portfolio as follows:

\[
\tilde{\Pi}(N) = 2 \left[ (\tilde{x}_2 - \tilde{b}^*)p^*(x^*) + (m^* - x^*) \sum_{i=2}^{N} p^*(\tilde{x}_i)/N + (1/2 - m^*)p^*(m^*) \right],
\]

where \( \tilde{x}_i \) is the optimal locations of the in-between products for any given \( N > 0 \) as described in Proposition 5, and \( \{\tilde{b}^*, x^*, m^*, \tilde{b}\} \) are the optimal portfolio structure from the Differential Pricing model. As we formally show in the following theorem, \( \tilde{\Pi}(N) \) converges to the optimal revenue under the Differential Pricing model as the number of in-between products approaches to infinity.

Theorem 4. Let \( \tilde{\Pi}(N) \) be the firm’s optimal revenue when it offers \( N \) in-between products on each side of the uniform pricing region. Then, we have that \( \lim_{N \to \infty} \tilde{\Pi}(N) = \Pi^* \).

As the revenue of the firm under the heuristic model converges to the optimal revenue under the Differential Pricing model, the firm achieves the benefits of differential pricing we discussed in the previous subsection. We illustrate the relative improvement in revenue from the Uniform Pricing model to the heuristic model, denoted by \( \tilde{\Delta}(\alpha, t) \), in Figure 7. We consider two different travel cost \( t \) values in this numerical study: equal to \( 3V/4 \) and \( V/4 \), respectively. When \( t \) is high, the highest benefit from the heuristic model (compared to the Uniform Pricing model) is achieved when the firm partially covers the market, which corresponds Case-3 in the previous subsection. On the other hand, the benefits from the heuristic model peak when the firm serves the entire taste spectrum. Figure 7 shows that the revenue improvements by the heuristic model converges to the improvement levels by the Differential Pricing model as the number of fixed products \( N \) increases.
It also shows that the firm captures most of the benefits of differential pricing by introducing around 20 fixed products, which is quite practical to implement.

In our heuristic model, we suppose that the price function, the locations of the guardrail products, and the boundaries of the uniform pricing region are fixed values determined based on the optimal strategy from the *Differential Pricing* model. One may envision a model where these fixed values are also decision variables. Although this richer model would generate a higher revenue than the heuristic model we consider, we observe that its solution would be overly complicated and must converge to the solution of the *Differential Pricing* model. Therefore, solving such a richer model will be unnecessary because the solution of the simple heuristic model we propose also converge to the solution of the *Differential Pricing* model.
6. Conclusion

The classic product-line design problem was based on striking the optimal balance between operational costs and the benefits of variety. If costless, variety would imply an unlimited array of products. In practice, however, more variety is not always preferable to less. As put by Broniarczyk and Hoyer (2005) “For years there has been a strong belief among retailers that having more assortment is always better . . . . Rather, having an optimal amount of assortment (which may not be the largest) is more critical.” In this paper, we leverage the standard Hotelling (1929) framework for horizontal product differentiation, but specifically consider a mass-customized product context in which consumers’ willingness to pay depends on the variety purchased as well as the relative range of other varieties (i.e., the full product line) offered by the firm.

If variety is unchecked, mass customization is a double-edge sword. The prospect of matching tastes with products appeals to individual consumers, but the resulting range of implied products can be less appealing to those same consumers. This characteristics helps explain why individuals, when surveyed, show a significant interest in ordering mass-customized products, yet so few mass-customized products have been a market success. In a 2013 survey of over 1,000 online shoppers, Bain & Company found that while fewer than 10% of ordered customized products, between 25% and 30% were interested in doing so.

When optimizing the firm’s product line, we show that considering a brand-level cost implies that the firm should limit the breadth of the mass customization region, even if there are no associated operational costs. We also find that the firm should optimally implement a differential pricing scheme for those product varieties at the outer extremes of its product line, catering to those consumers with more extreme tastes. More specifically, the optimal pricing entails charging a higher price within the central mass customized region, with continuously (and linearly) declining prices at the extremes of the product space. We also note that the firm offers a more restricted range of mass customized products under differential pricing than under uniform pricing. While we assume a uniform dispersion for consumer tastes, if consumer tastes follow a normal distribution (or other distribution implying more common centralized tastes), then the aforementioned result should follow even more strongly. Given that a continuum of prices will likely not be practical in application, we also consider the more pragmatic approach of a set of fixed (non-customized) products to serve customers at the taste extremes, with a corresponding set of (lower) fixed prices. We also derive a profit bound on the performance of this heuristic solution, and prove it performs close to optimal.
To help frame these theoretical results with a practical application, consider the mass customization example of the Denali Jacket offered by The North Face. At the time of this writing, The North Face offers over 1.9 million customized variations of the Denali jacket, using a customer-specific set of color variations for the fleece, taslan, zippers, zipper pulls, and logos (with 21, 14, 17, 17, and 23 color options for each, respectively). As should be expected, and can be seen by viewing a selection of previous customers’ color designs on the website, distinct consumers order starkly different variations. Viewing this product as one that exemplifies our prototypical firm’s mass customized product, let us revisit our main results.

First, let us consider whether The North Face follows the result that the mass customized region should be limited, excluding patterns pertaining to the most extreme of consumer tastes. Arguably, this is evidently the case, as the pattern options for the customized Denali jackets include only solid color patterns, yet The North Face offers non-custom Denali jackets with pattern fabrics including camouflage and mineral prints. Our model results also suggest that the fixed (non-customized) patterns addressing the more extreme tastes should have lower price points than the customized versions. While we prove this result purely from a price-discrimination perspective, it could also follow from cost considerations, if customization is not costless for the firm. In the case of the Denali jacket, The North Face prices its non-customized versions at fifty dollars less than the customized versions. Some of those standard versions appear to be popular color variations that regularly stock items at The North Face retailers. Keller’s (2010) “guardrail products” concept is that a firm needs to think strategically, from a branding perspective, about limiting the extent of its product variety. Particularly if mass customization permits unbridled customization, the intentional and proper positioning of those guardrails becomes important. The Nike’s Flyknit technology can potentially permit an essentially unlimited degree of “thread level” customization and Nike indeed offers some pattern knits with numerous blended colors for a startling effect. But, as in the case of The North Face, the firm offers such pattern knits as fixed (non-customized) products, at a lower price point that the customized variants. Naturally, a valid argument could be made that competition could be a primary motive, apart from the possible price discrimination and cost motivations we have explored in this paper, for a firm to offer discounted versions of its non-customized product versions. We would not dispute that point, but this paper adds to the literature by showing that such a product positioning and pricing structure can result even in the absence of competition (and traditional operations costs), given endogenous brand-level implications resulting from variety propagation.
References


**Appendix A: Proofs in Section 4**

**A.1. Proof of Proposition 1**

For the Uniform Pricing case, we employ proof-by-contradiction to establish that the optimal market coverage $B^o$ and the range of mass customization $[m^o, \bar{m}^o]$ coincide. To this end, suppose $B^o \neq [m^o, \bar{m}^o]$. Then, we should have that $\bar{b}^o < m^o \leq \bar{m}^o < \bar{b}^o$ where $\bar{b}^o = \min(B^o)$ and $\bar{b}^o = \max(B^o)$.

Now, letting $\varepsilon < \min\{m^o - \bar{b}^o, \bar{b}^o - \bar{m}^o\}$, consider an alternative mass customization region $[m^', \bar{m}^'] \equiv [m^o - \varepsilon, \bar{m}^o + \varepsilon]$ with an alternative price function $p'(x) = p^o_m + t\varepsilon$ for all $x \in [m', \bar{m}']$. Then, we have that

$$U(\bar{b}^o, m^o - \varepsilon, p) = V - p'_m - t(m^o - \bar{b}^o - \varepsilon) - \alpha(m^o - \bar{b}^o - \varepsilon + \bar{m}^o - \bar{b}^o + \varepsilon)$$

$$= U(\bar{b}^o, \bar{m}^o, p^o) \geq 0,$$

similarly we have that $U(\bar{b}^o, \bar{m}^o, p') = U(\bar{b}^o, \bar{m}^o - \varepsilon, p')$. This means that the firm’s market coverage under the alternative portfolio is at least as much as before while increasing its price. Therefore, the firm improves its revenues by using the alternative product portfolio since it charges a higher price. This contradicts with the optimality of $[m^o, \bar{m}^o]$. Hence, we should have that $\bar{b}^o = m^o$. 
A.2. Proof of Theorem 1

By labelling the size of mass customization region $\overline{m} - m$ as $\mu$ for any given $\overline{m} \leq m$ and ignoring the constant $\lambda$, we can rewrite the firm’s problem as:

$$\max_{\mu \leq 1} \pi(\mu) \equiv [V - \alpha \mu]\mu.$$  

Note that $\pi'(\mu) = V - 2\alpha \mu$, so that $\pi'(\mu) \geq 0$ for any $\mu \leq V/(2\alpha)$ and $\pi'(\mu) < 0$ otherwise. Therefore, the optimal solution of the above problem is

$$\mu^* = \begin{cases} 1 & \text{if } \alpha < V/2 \\ V/(2\alpha) & \text{if } \alpha \geq V/2, \end{cases}$$

which leads to the optimal product portfolio stated in the theorem.

A.3. Supplementary Results for the Proof of Proposition 2

**Lemma 2.** All customers whose ideal tastes are in $[x, \overline{x}]$ will purchase their ideal products under the linear price function

$$p_L(x, X) = \begin{cases} V - t(m - x) - \alpha(\overline{x} - x) & \text{if } x \in [x, m], \\ V - \alpha(\overline{x} - x) & \text{if } x \in [m, \overline{m}], \\ V - t(x - m) - \alpha(\overline{x} - x) & \text{if } x \in [\overline{m}, \overline{x}], \end{cases}$$

and the product portfolio $X = [x, m] \cup [m, \overline{m}] \cup [\overline{m}, \overline{x}]$.

**Proof:** To prove our claim, we show that the utility of the customer at $z$ from the product at $z$ is greater than her utility from any other products, i.e. $U(z, z, \hat{p}) \geq U(z, x, \hat{p})$ for any $x \in X$.

First, note that $U(z, x, \hat{p}) \leq 0$ for any $z \in [m, \overline{m}]$ and $x \in X$. Moreover, any customer in $[m, \overline{m}]$ who buys a product matching their ideal taste gains zero utility under the pricing regime $p_L(x, X)$.

Furthermore, if we consider customers in $[x, m]$, we have that

$$U(z, z, \hat{p}) = V - \hat{p}(z, X) - \alpha(\overline{x} - x)$$

$$= t(m - z) \geq t(m - x) - t(z - x) = U(z, x, \hat{p}) \text{ for any } x < z,$$

$$U(z, z, \hat{p}) > t(m - x) - t(z - x) = U(z, x, \hat{p}) \text{ for any } z < x \leq m.$$  

Therefore, given the differential pricing customers in $[x, m]$ will not prefer any product in the differential pricing region other than their ideal product. Furthermore, they do not prefer any product in the uniform pricing region because those products gives negative utility to the customers in $[x, m]$. Thus, all customers whose ideal products are in $[x, m]$ buy their ideal products.

We can similarly show that all customers whose ideal products are in $[\overline{m}, \overline{x}]$ also buy their ideal products.

**Lemma 3.** Let $X^* = [x^*, m^*] \cup [m^*, \overline{m}] \cup [\overline{m}, \overline{x}] \in X_p$ be the optimal product portfolio, $p^*(x)$ be the optimal price function, and $b^*(z, p^*)$ be the customers’ most preferred products in the Differential Pricing Model. Then, we have that $b^*(z, p^*) = x^*$ for all $z < x^*$ and $b^*(z, p^*) = \overline{x}$ for all $z > \overline{x}$. In other words, the customers outside product portfolio should purchase a guardrail product if they make a purchase under the optimal portfolio and the optimal price function.
Proof: We prove our claim by contradiction. Therefore, we suppose \( b^*(z,p^*) = x' > z^* \) for some \( z < z^* \). Then, we should have that all \( b^*(z,p^*) = x' \) for all \( z < x' \) (even for customers whose ideal tastes are \( z \in [x^*, x'] \)). This also implies that \( p^*(x) > p^*(x') + t(x' - x) \) for \( x \in [z^*, x'] \). Now, consider an alternative price function \( p'(x) \) such that \( p'(x) = p^*(x') + t(x' - x) \) for \( x \in [z^*, x'] \) and \( p'(x) = p^*(x) \) otherwise. Under this alternative price, all customers outside \( x^* \) buy the guardrail product at \( z^* \), and customers whose ideal tastes are in \( z \in [x^*, x'] \) buy their ideal product (proof of this is very similar to the proof of Lemma 2). The market coverage stays the same by the construction of the price function \( p'(x) \). Thus, the alternative price function improves the revenues of the firm, which contradicts with the optimality of \( p^*(x) \).

A.4. Proof of Proposition 2

For any given optimal portfolio \( X^* \) and optimal price function \( p^*(x) \), consider the price function \( p_L(x, X_L) \), as described in Lemma 2, where \( X_L = [x^*, m_L] \cup [m_L, x^*] \cup [m_L, x^*] \cup [x^*, x'] \) with \( (m_L - z^*) = [V - \alpha(z^* - x^*) - p^*(x^*)]t \) and \( (x^* - m_L) = [V - \alpha(x^* - x^*) - p^*(x^*)]t \).

Linear price:

We first prove that \( p^*(x) \) must be equal to \( p_L(x, X_L) \) and thus \( X^* = X_L \). Note that market coverages under both price functions are the same because customers outside the product portfolio buy the guardrail products by Lemma 3 and the construction of \( p_L(x, X_L) \).

Under the optimal price function \( p^*(x) \), customers should pay less than \( V - \alpha(x^* - z^*) \) because otherwise her utility would be negative. Moreover, a customer whose ideal taste is \( z \in [x^*, m^*] \) should not pay more than \( p^*(x) + t(z - x^*) \) because otherwise she can improve her utility by purchasing the guardrail product at \( z^* \). Similarly, a customer whose ideal taste is \( z \in [m^*, x^*] \) should not pay more than \( p^*(x) + t(z^* - x^*) \). Therefore, we should have that

\[
p^*(b^*(z,p^*)) \leq \begin{cases} 
\min\{V - \alpha(z^* - x^*), p^*(z) + t(z - x^*)\} & \text{if } z \in [x^*, m^*] \\
V - \alpha(x^* - x^*) & \text{if } z \in [m^*, x^*] = p_L(z, X_L) \\
\min\{V - \alpha(x^* - x^*), p^*(x) + t(x^* - z)\} & \text{if } z \in [m^*, x^*]
\end{cases}
\]

By Lemma 2, we have that all customers whose ideal tastes are in \( [x^*, x^*] \) will purchase their ideal products under \( p_L(x, X_L) \). Therefore, as the market coverages are the same under both \( p_L(x, X_L) \) and \( p^*(x) \), the above inequality implies that the firm’s revenue under \( p_L(x, X_L) \) is an upper bound for the optimal revenue, which proves that \( p^*(x) \) must be equal to \( p_L(x, X_L) \).

No surplus at the boundaries:

We next prove by contradiction that customers at the boundaries of the optimal market coverage earn zero utility. Suppose the utility of customers at the left market boundary, \( U(b^*, x^*, p^*) \), is \( u \) (the proof is almost the same for the right boundary). Note that \( u = 0 \) trivially holds if \( B^* \subset [0, 1] \). Thus, we focus on the case where \( B^* = [0, 1] \) and suppose \( u > 0 \) to the contrary that \( u = 0 \). Then, letting \( \varepsilon < u/t \), we construct an alternative product portfolio \( X' = [x^*, m^*-\varepsilon] \cup [m^*-\varepsilon, m^*] \cup [m^*, x^*] \) with the price function

\[
p'(x) = \begin{cases} 
p^*(x) + \varepsilon t & \text{if } x \in [x^*, m^*-\varepsilon] \\
p^*(x) + \varepsilon t - t(x - m^*) & \text{if } x \in [m^*-\varepsilon, m^*] \\
p'(x) & \text{if } x > m^*.
\end{cases}
\]
In this alternative strategy, the firm increases the prices of the products on the left side of the uniform pricing region in order to extract some of the positive surplus from the customer at the boundary. The firm also expands the uniform pricing region towards the left end of the taste spectrum. As long as \( \varepsilon \) is small enough, all customers inside the product portfolio purchase their ideal taste, and the customers outside \( X' \) buy the guardrail products, like they do under the optimal portfolio and price function. Furthermore, the firm continues to serve the entire spectrum under the new strategy. As the firm charges higher prices for the products in \([x^*, m^*]\) while keeping the price for the rest unchanged, the firm must increase its revenues, which contradicts with optimality. Hence, we should have that \( u = 0 \).

**The structure of the product portfolio**

Using the fact that the utility of customers at the market boundaries is zero, we have that

\[
U(b^*, x^*, p^*) = 0 \Rightarrow V - p^*(x^*) - t(x^* - b^*) - \alpha(x^* + x^* - 2b^*) = 0
\]

\[
\Rightarrow 2(t + \alpha)(m^* - x^*) - (t + 2\alpha)(m^* - b^*) = 0
\]

\[
\Rightarrow m^* - x^* = \frac{t + 2\alpha}{2(t + \alpha)}(m^* - b^*).
\]

Similarly, we have that \( x^* - m^* = \frac{t + 2\alpha}{2(t + \alpha)}(b^* - m^*) \).

**Symmetry of the product portfolio:**

Our proof ends by showing that the product portfolio is symmetric around the uniform pricing region. We, again, suppose on the contrary that \( X^* \) is not symmetric. Then, we should have that \( p^*(x^*) \neq p^*(x^*) \). Without loss of generality, assume \( p^*(x^*) > p^*(x^*) \), which also implies that \( m^* - x^* < x^* - m^* \) and \( x^* - b^* < b^* - x^* \).

Now, consider an alternative product portfolio \( X' \) and the price function \( p'(x) \) such that

\[
X' = [x', m'] \cup [m', m'] \cup [m', x'], \quad \text{and}
\]

\[
p'(x) = \begin{cases} 
    p^*(x) - 2\varepsilon(t + \alpha) & \text{if } x \in [x', m'] \\
    p^*(x) - 2\varepsilon(t + \alpha) + t(x - m^*) & \text{if } x \in [m', m'] \\
    p^*(x) & \text{if } x \in [m', m'] \\
    p^*(x) + 2\varepsilon(t + \alpha) - t(x - m^*) & \text{if } x \in [m', m'] \\
    p^*(x) + 2\varepsilon(t + \alpha) & \text{if } x \in [m', x'], 
\end{cases}
\]

where \((x', m', m', x') = (x^* + \varepsilon, m^* + \frac{t + 2\alpha}{t} \varepsilon, m^* + \frac{t + 2\alpha}{t} \varepsilon, x^* + \varepsilon)\).

Notice that the firm moves the entire product portfolio to the right in order to alleviate the asymmetry. While doing this, it has to reduce the price of the product on the left differential pricing region and increase the prices on the right side. The changes in the price matches each other but the price increases apply to a larger area because \( p^*(x^*) > p^*(x^*) \).

We illustrate the revenues of the firm under \( X^* \) and \( X' \) in Figure 8. The revenue that the firm loses under the alternative portfolio is the sum of areas \( L_1, L_2, L_3, L_4 \) whereas the firm’s revenue gain by following \( X' \) is the sum of \( G_1, G_2, G_3, G_4 \). Notice that \( L_2 = G_2 \) and \( L_4 = G_4 \). The firms net gain from the alternative portfolio is

\[
G_1 + G_3 - (L_1 + L_3) = \varepsilon(t + 2\alpha) \left[ (b^* - x^*) - (x^* - b^*) - \varepsilon \right]
\]
Denoting the size of the total market coverage \( \tilde{b} - \tilde{b} \) as \( \tau \) and the ratio between the mass customization region and the total coverage \( \frac{m - m}{\tilde{b} - \tilde{b}} = \rho \), we can rewrite the firm's problem as:

\[
\max_{\rho, \tau \leq 1} \pi(\tau, \rho) = (1 - \rho)\tau p_\rho(\tau, \rho) + \rho \tau p_d(\tau, \rho) + 2 \int_0^{\ell_d(\tau, \rho)} t z \, dz,
\]

where \( p_\rho(\tau, \rho) = V - \alpha \tau \frac{2\alpha + (1 + \rho) t}{2(\alpha + \tau) t} \), \( p_d(\tau, \rho) = V - \tau \frac{4\alpha^2 + 4\alpha t + (1 - \rho) \rho}{4(\alpha + \tau)} \), and \( \ell_d(\tau, \rho) = \frac{\tau + 2\alpha}{4(\alpha + \tau)(1 - \rho) \tau} \). Note that \( p_\rho \) is the uniform price, \( p_d \) is the price of the guardrails, and \( \ell_d \) is the size of the differential pricing region on each side of the uniform pricing region. We obtain these function by rewriting the optimal decisions in Proposition 4 as functions of \( \tau \) and \( \rho \).

Taking the derivative of the above profit function with respect to \( \rho \) and \( \tau \) yields

\[
\frac{\partial \pi(\tau, \rho)}{\partial \rho} = \frac{\tau^2 [t(3t + 4\alpha) - (4\alpha^2 + 8t \alpha + 3t^2) \rho]}{8(t + \alpha)^2}
\]

\[
\frac{\partial \pi(\tau, \rho)}{\partial \tau} = V - \tau \frac{(16\alpha^3 + 4\alpha^2(7 + \rho^2) + 8\alpha(2 - (1 - \rho) \rho)t^2 + 3(1 - \rho)^2t^3)}{8(t + \alpha)^2}
\]

A.5. Proof of Theorem 2

Denoting the size of the total market coverage \( \tilde{b} - \tilde{b} \) as \( \tau \) and the ratio between the mass customization region and the total coverage \( \frac{m - m}{\tilde{b} - \tilde{b}} = \rho \), we rewrite the firm's problem as:

\[
\pi(\tau, \rho) = (1 - \rho)\tau p_\rho(\tau, \rho) + \rho \tau p_d(\tau, \rho) + 2 \int_0^{\ell_d(\tau, \rho)} t z \, dz,
\]

where \( p_\rho(\tau, \rho) = V - \alpha \tau \frac{2\alpha + (1 + \rho) t}{2(\alpha + \tau) t} \), \( p_d(\tau, \rho) = V - \tau \frac{4\alpha^2 + 4\alpha t + (1 - \rho) \rho}{4(\alpha + \tau)} \), and \( \ell_d(\tau, \rho) = \frac{\tau + 2\alpha}{4(\alpha + \tau)(1 - \rho) \tau} \). Note that \( p_\rho \) is the uniform price, \( p_d \) is the price of the guardrails, and \( \ell_d \) is the size of the differential pricing region on each side of the uniform pricing region. We obtain these function by rewriting the optimal decisions in Proposition 4 as functions of \( \tau \) and \( \rho \).

Taking the derivative of the above profit function with respect to \( \rho \) and \( \tau \) yields

\[
\frac{\partial \pi(\tau, \rho)}{\partial \rho} = \frac{\tau^2 [t(3t + 4\alpha) - (4\alpha^2 + 8t \alpha + 3t^2) \rho]}{8(t + \alpha)^2}
\]

\[
\frac{\partial \pi(\tau, \rho)}{\partial \tau} = V - \tau \frac{(16\alpha^3 + 4\alpha^2(7 + \rho^2) + 8\alpha(2 - (1 - \rho) \rho)t^2 + 3(1 - \rho)^2t^3)}{8(t + \alpha)^2}
\]
Note that \( \frac{\partial \pi(\tau, \rho)}{\partial \rho} > 0 \) for any \( \rho < \frac{t(3t + 4\alpha)}{(t+2\alpha)(3t+2\alpha)} \) and \( \frac{\partial \pi(\tau, \rho)}{\partial \tau} \leq 0 \) otherwise. Therefore, the optimal \( \rho \) is \( \rho^* \equiv \frac{t(3t + 4\alpha)}{(t+2\alpha)(3t+2\alpha)} \). Furthermore, we have that

\[
\frac{\partial \pi(\tau, \rho)}{\partial \tau} \bigg|_{\rho=\rho^*} = V - \tau \frac{2\alpha(t + \alpha)(3t + 4\alpha)}{(t+2\alpha)(3t+2\alpha)}.
\]

Using the above equation, we have \( \frac{\partial \pi(\tau, \rho)}{\partial \tau} \bigg|_{\rho=\rho^*} > 0 \) for any \( \tau < \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} \) and \( \frac{\partial \pi(\tau, \rho)}{\partial \tau} \bigg|_{\rho=\rho^*} \leq 0 \) otherwise. Therefore, the optimal size of the market coverage is

\[
\tau^* = \begin{cases} 
1 & \text{if } \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} > 1 \\
\frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} & \text{if } \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} \leq 1,
\end{cases}
\]

which leads to the optimal solution stated in the theorem because \( \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} \) can be rewritten as \( \frac{V}{2\alpha} \left[ 1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)} \right] \). Furthermore, \( \frac{V}{2\alpha} \left[ 1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)} \right] > 1 \) if and only if \( \alpha < \alpha_j^*(t) \) because \( \frac{V}{2\alpha} \left[ 1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)} \right] \) is a decreasing function of \( \alpha \).

Using \( \tau^* \) and \( \rho^* \), the size of the product portfolio relative to the market coverage is

\[
\frac{\pi^* - \pi^*}{\ell(B^*)} = \frac{2m^* - m^*}{\ell(B^*)} + \frac{m^* - m^*}{\ell(B^*)} = \frac{t + 2\alpha}{t + \alpha} \left( \frac{m^* - b^*}{\ell(B^*)} \right) + \frac{t - 2\alpha}{t + \alpha} \left( 1 - \rho^* \right) + \rho^* = \frac{t + 2\alpha}{t + \alpha} + \frac{t - 2\alpha}{t + \alpha} \rho^* = 1 - \frac{2\alpha}{(t + \alpha)(3t + 2\alpha)}.
\]

Finally, plugging in \( \tau^* \) and \( \rho^* \) to \( \pi(\tau, \rho) \), we obtain the optimal profit function stated in the theorem.

### A.6. Proof of Corollary 1

First, note that the result holds true trivially when \( \alpha \leq V/2 \) because \( m^* - m^* = 1 \) when \( \alpha \leq V/2 \) by Theorem 1. Now, we consider the case where \( V/2 < \alpha \leq \alpha_j^*(t) \). Then, by Theorems 1 and 2, we have that

\[
(m^* - m^*) - (\pi^* - \pi^*) = \frac{V}{2\alpha} - \left[ 1 - \frac{2\alpha}{(t + 2\alpha)(3t + 2\alpha)} \right] \geq 0,
\]

where the inequality holds true because we have that \( V/(2\alpha) \geq 1 - \frac{t\alpha}{(t+2\alpha)(3t+2\alpha)} \) for \( \alpha \leq \alpha_j^*(t) \). Finally, when \( \alpha > \alpha_j^*(t) \), Theorems 1 and 2 imply that

\[
(m^* - m^*) - (\pi^* - \pi^*) = \frac{V}{2\alpha} - \frac{V}{2\alpha} \left[ 1 - \frac{t\alpha}{(t+\alpha)(3t+4\alpha)} \right] \geq 0.
\]

### A.7. Proof of Proposition 3

1. By Theorem 2, we have that \( \frac{m^* - m^*}{\ell(B^*)} = 1/2 \left[ \frac{\tau^* - \pi^*}{\ell(B^*)} - \frac{m^* - m^*}{\ell(B^*)} \right] = \frac{\alpha}{3t+2\alpha} \cdot \frac{m^* - m^*}{\ell(B^*)} \) is decreasing in \( \alpha \) because its derivative with respect to \( \alpha \) is \( \frac{3t}{(3t+2\alpha)^2} > 0 \) for all \( \alpha \).

2. By Theorem 2, we have that \( \frac{m^* - m^*}{\ell(B^*)} = \frac{t(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)} \cdot \frac{m^* - m^*}{\ell(B^*)} \) is decreasing in \( \alpha \) because its derivative with respect to \( \alpha \) is \( -\frac{4t(4\alpha^2 + 6\alpha + 3\alpha^2)}{(t+2\alpha)^2(3t+2\alpha)^2} < 0 \) for all \( \alpha \).

3. By Theorem 2, we have that \( \frac{\pi^* - \pi^*}{\ell(B^*)} = 1/2 \left[ 1 - \frac{\tau^* - \pi^*}{\ell(B^*)} \right] = \frac{t\alpha}{(t+2\alpha)(3t+2\alpha)} \). The derivative of \( \frac{\pi^* - \pi^*}{\ell(B^*)} \) with respect to \( \alpha \) is \( \frac{2(3\alpha^2 - 4\alpha^2)}{(t+2\alpha)^2(3t+2\alpha)^2} \). Therefore \( \frac{\pi^* - \pi^*}{\ell(B^*)} \) is increasing in \( \alpha \) for any \( \alpha \leq t\sqrt{3}/2 \) and decreasing in \( \alpha \) for any \( \alpha \geq t\sqrt{3}/2 \).
4. By Theorem 2, the derivative of the optimal revenue with respect to \( \alpha \) is

\[
\frac{\partial \Pi^*}{\partial \alpha} = \begin{cases} 
\lambda \left[ \frac{2\alpha V^2 + t(4\alpha + 2t) + 1}{(2\alpha + 2t)(3\alpha + 4t)^2} \right] & \text{if } \alpha < \alpha_f^c(t), \\
-\lambda V^2 \left[ \frac{16\alpha^4 + 64\alpha^3 t + 80\alpha^2 t^2 + 42\alpha t + 9t^4}{4\alpha^2(\alpha + t)^2(4\alpha + 3t)^2} \right] & \text{if } \alpha \geq \alpha_f^c(t),
\end{cases}
\]

which is less than zero in both cases. Therefore, the optimal revenue is decreasing in \( \alpha \). Similarly, the derivative of the optimal market coverage with respect to \( \alpha \) is

\[
\frac{\partial \ell(B^*)}{\partial \alpha} = \begin{cases} 
0 & \text{if } \alpha < \alpha_f^c(t), \\
-\lambda (\alpha_f^c)^2 \left[ \frac{16\alpha^4 + 64\alpha^3 t + 80\alpha^2 t^2 + 42\alpha t + 9t^4}{2\alpha^2(\alpha + t)^2(4\alpha + 3t)^2} \right] & \text{if } \alpha \geq \alpha_f^c(t),
\end{cases}
\]

which is less than zero when \( \alpha \geq \alpha_f^c(t) \). Therefore, the optimal market coverage is decreasing in \( \alpha \).

Appendix B: Proofs in Section 5

B.1. Proof of Lemma 1

By Theorems 1 and 2, we have that

\[
\ell(B^*) = \begin{cases} 
1 & \text{if } \alpha < V/2, \\
V/(2\alpha) & \text{if } \alpha \geq V/2,
\end{cases}
\]

and

\[
\ell(B^*) = \begin{cases} 
1 & \text{if } \alpha < \alpha_f^c(t), \\
\frac{V}{2\alpha} \left[ 1 + \frac{t\alpha}{(t+\alpha)(3\alpha + 4t)} \right] & \text{if } \alpha \geq \alpha_f^c(t).
\end{cases}
\]

We also have that \( \alpha_f^c(t) > V/2 \) because \( 1 + \frac{t\alpha}{(t+\alpha)(3\alpha + 4t)} > 2\alpha/V \) for any \( \alpha < V/2 \). Therefore, the difference between the market coverage in both pricing schemes is

\[
\ell(B^*) - \ell(B^*) = \begin{cases} 
0 & \text{if } \alpha < V/2, \\
1 - \frac{V}{2\alpha} & \text{if } V/2 \leq \alpha < \alpha_f^c(t), \\
\frac{V}{2\alpha} \left[ \frac{t\alpha}{(t+\alpha)(3\alpha + 4t)} \right] & \text{if } \alpha \geq \alpha_f^c(t),
\end{cases}
\]

which implies that \( \ell(B^*) - \ell(B^*) \geq 0 \).

B.2. Proof of Proposition 4

1. For any \( \alpha < V/2 \), we have that \( \Delta(\alpha, t) = \frac{\alpha t^2}{(t+2\alpha)(3\alpha + 4t)(V-\alpha)} \) and \( \frac{\partial \Delta(\alpha, t)}{\partial \alpha} = \frac{\alpha t^2 + 8V t + 3t^2 (2V - \alpha)}{(t+2\alpha)^2 (3\alpha + 2t)^2 (V-\alpha)^2} \). Then, \( \Delta(\alpha, t) \) is increasing in \( \alpha \) for any \( \alpha < V/2 \) because \( \frac{\partial \Delta(\alpha, t)}{\partial \alpha} > 0 \) when \( \alpha < V/2 \).

2. For any \( V/2 \leq \alpha < \alpha_f^c(t) \), we have that \( \Delta(\alpha, t) = \frac{4\alpha (V-\alpha) (t+3\alpha + 4t)}{(t+2\alpha)(3\alpha + 4t)} - 1 \). Furthermore, we have that

\[
\frac{\partial^2 \Delta(\alpha, t)}{\partial \alpha^2} = -\frac{8(64\alpha^6 + 384\alpha^5 t + 912\alpha^4 t^2 + 1036\alpha^3 t^3 + 612\alpha^2 t^4 + 189\alpha t^5 + 27t^6)}{V^2(t+2\alpha)^3(3\alpha + 2t)^3(2V - \alpha)} < 0
\]

\[
\frac{\partial \Delta(\alpha, t)}{\partial \alpha} \bigg|_{\alpha = V/2} = \frac{t(9t^2 + 8Vt + V^2)}{(t+V)^2(3t+V)^2} > 0
\]

\[
\frac{\partial \Delta(\alpha, t)}{\partial \alpha} \bigg|_{\alpha = \alpha_f^c(t)} = \frac{t(3t^2 - 4\alpha_f^c(t)^2)}{(t + \alpha_f^c(t))^2(3t + 4\alpha_f^c(t))^2}.
\]

The first equation above shows that \( \Delta(\alpha, t) \) is concave in \( \alpha \) when \( V/2 \leq \alpha < \alpha_f^c(t) \). In other words, \( \frac{\partial \Delta(\alpha, t)}{\partial \alpha} \) is decreasing in \( \alpha \). The second equation shows that \( \Delta(\alpha, t) \) is increasing at \( \alpha = V/2 \). Combining these two with the third equation above, we have that \( \frac{\partial \Delta(\alpha, t)}{\partial \alpha} > 0 \) for any \( V/2 \leq \alpha < \alpha_f^c(t) \) when \( \alpha_f^c(t) \leq t\sqrt{3}/2 \). If \( \alpha_f^c(t) > t\sqrt{3}/2 \), there must be an \( \hat{\alpha} \in (V/2, \alpha_f^c(t)) \) such that \( \Delta(\alpha, t) \) is increasing in \( \alpha \) up to \( \hat{\alpha} \) and decreasing afterwards.
3. For any \( \alpha \geq \alpha^*_c(t) \), we have that \( \Delta(\alpha, t) = \frac{\alpha^t}{(\alpha + t)(4\alpha + 3t)} \) and \( \frac{\partial \Delta(\alpha, t)}{\partial \alpha} = \frac{t(3\alpha^2 - 4\alpha^2)}{(t + \alpha)^2(3t + 4\alpha)^2} \). Then, we have that \( \frac{\partial \Delta(\alpha, t)}{\partial \alpha} < 0 \) for any \( \alpha \geq \alpha^*_c(t) \) when \( \alpha^*_c(t) > t\sqrt{3}/2 \). On the other hand, \( \Delta(\alpha, t) \) is maximized at \( \alpha = t\sqrt{3}/2 \) when \( \alpha^*_c(t) \leq t\sqrt{3}/2 \).

B.3. Proof of Theorem 3

1. Note that \( \Delta_{\text{max}}(t) \geq \Delta(V/2, t) = \frac{4V}{3(t + \theta)(3t + 4\theta)} \). Furthermore, by substituting \( t \) with \( \theta V \), we obtain that \( \Delta_{\text{max}}(t) \geq \frac{\theta}{2(1 + \theta)(1 + 3\theta)} \).

2. Let \( \mu(\alpha) = \frac{V}{2\alpha} \left[ 1 + \frac{t\alpha}{(t + \alpha)(3t + 4\alpha)} \right] \). Then, we have that \( \mu(\alpha) \) is decreasing in \( \alpha \) and by the definition of \( \mu(\alpha^*_c(t)) = 1 \). Using these two observations, we have that

\[
\alpha^*_c(t) \leq t\sqrt{3}/2 \Leftrightarrow \mu(t\sqrt{3}/2) < 1 \Leftrightarrow \left( \frac{2}{\sqrt{3}} - 1 \right) \frac{4V}{t} < 1 \Leftrightarrow t > \frac{4V}{3 + 2\sqrt{3}}. \]

The above inequality implies that \( \Delta_{\text{max}}(t) \) is achieved when both pricing schemes partially cover the market. By Proposition 4, \( \Delta_{\text{max}}(t) = \Delta(t\sqrt{3}/2, t) = \frac{1}{7 + 4\sqrt{3}} \).

B.4. Proof of Proposition 5

For notational convenience, we let \( \delta_i = \tilde{x}_{i+1} - \tilde{x}_i \) for \( 1 \leq i < N \) and \( \delta_N = m^* - \tilde{x}_N \). Then, we have that \( p^*(\tilde{x}_i) = p^*(z^*) - t \sum_{i=1}^{i-1} \delta_j \) and \( \sum_{i=1}^{N} \delta_i = m^* - \tilde{x}^* \).

Note that firm’s revenue from customers outside the product portfolio (i.e., those in the \([\tilde{z}^*, x^*]\) interval) does not depend on the locations of the fixed products because the location of the guardrail products are fixed. Similarly, firm’s revenue from customers in the uniform pricing region (i.e., those in the \([m^*, \overline{m}]\) interval) also does not depend on the locations of the fixed products because the uniform pricing region is fixed. Therefore, we can focus our attention on the revenue from customers in the \([x^*, m^*]\) region.

Under any given portfolio of in-between products, customers whose ideal products are between \( \tilde{x}_i \) and \( \tilde{x}_{i+1} \) purchase the product at \( \tilde{x}_i \) because

\[
U(z, \tilde{x}_i, p^*) = V - p(\tilde{x}_i) - t(z - \tilde{x}_i) - \alpha(\tilde{x}^* - z) = t(m^* - z) > t(m^* - \tilde{x}_{i+1}) > V - p(\tilde{x}_{i+1}) - t(\tilde{x}_{i+1} - z) - \alpha(\tilde{x}^* - z) = U(z, \tilde{x}_{i+1}, p^*),
\]

for any \( \tilde{x}_i < z < \tilde{x}_{i+1} \). Therefore, we can write the firm’s revenue in the \([x^*, m^*]\) region as

\[
\pi_{\delta}(\delta) = \delta_1 p^*(z^*) + \sum_{i=2}^{N} p^*(\tilde{x}_i) \delta_i = p^*(z^*) \sum_{i=1}^{N} \delta_i - t \sum_{i=2}^{N} \delta_i \sum_{j=1}^{i-1} \delta_j = p^*(z^*) [m^* - x^*] - t \sum_{i=2}^{N} \delta_i \sum_{j=1}^{i-1} \delta_j,
\]

where \( \delta = \{\delta_1, \ldots, \delta_N\} \). Then, the firm solves

\[
\max_{\delta} \pi_{\delta}(\delta) \quad \text{s.t.} \quad \sum_{i=1}^{N} \delta_i = m^* - x^*.
\]

First order conditions for the above problem are

\[
\frac{\partial \pi_{\delta}(\delta)}{\partial \delta_i} + \gamma = - \sum_{j=1}^{i-1} \delta_j - \sum_{j=i+1}^{N} \delta_j + \gamma = 0 \text{ for all } 0 \leq i \leq N,
\]
\[
\sum_{i=1}^{N} \delta_i = m^* - x^*,
\]
where \( \gamma \) is the Lagrangian multiplier for the equality constraint. Using the first set of equations, we have that \( \delta_l = \delta_m \) for any \( l, m \in \{1, \ldots, N\} \). Then, we have that \( \delta_i = (m^* - x^*)/N \) by the last equation.

B.5. Proof of Theorem 4

We first consider the case where the firm serves the entire market, i.e., \( \alpha < \alpha^*_f(t) \). Using the fact that the fixed products are uniformly distributed, the firm’s revenue under the heuristic model is

\[
\tilde{\Pi}(N) = \lambda \left[ V - \frac{\alpha \left( 8\alpha^2 N + 2\alpha^2 (13Nt + t) + \alpha (27N + 1)t^2 + 9Nt^3 \right)}{N(t + 2\alpha)(3t + 2\alpha)^2} \right]
\]

\[
= \lambda \left[ V - \alpha \left( 1 - \frac{\alpha t((3N-1)t + 2\alpha(N-1))}{N(t + 2\alpha)(3t + 2\alpha)^2} \right) \right]
\]

\[
= \lambda \left[ V - \alpha \left( 1 - \frac{\alpha t}{(t + 2\alpha)(3t + 2\alpha)} + \frac{t\alpha}{N(3t + 2\alpha)^2} \right) \right] = \Pi^* - \lambda \frac{t\alpha^2}{N(3t + 2\alpha)^2}.
\]

Thus, \( \lim_{N \to \infty} \tilde{\Pi}(N) = \Pi^* \) since \( \lim_{N \to \infty} \frac{t\alpha^2}{N(3t + 2\alpha)^2} = 0 \).

When \( \alpha \geq \alpha^*_f(t) \), we can, similarly, write the firm’s revenue under the heuristic model as

\[
\tilde{\Pi}(N) = \Pi^* - \lambda V^2 \frac{t(t + 2\alpha)^2}{4N(t + \alpha)^2(3t + 4\alpha)^2},
\]

which also implies that \( \lim_{N \to \infty} \tilde{\Pi}(N) = \Pi^* \) since \( \lim_{N \to \infty} \frac{t(t + 2\alpha)^2}{4N(t + \alpha)^2(3t + 4\alpha)^2} = 0 \).