Mass customization and guardrails: “You can’t be all things to all people”

Eren B. Çil
Lundquist College of Business, University of Oregon, Eugene, Oregon, erencil@uoregon.edu

Michael Pangburn
Lundquist College of Business, University of Oregon, Eugene, Oregon, mpangburn@uoregon.edu

A mass customization strategy enables a firm to match its product designs to unique consumer tastes. In a classic horizontal product-differentiation framework, a consumer’s utility is a decreasing function of the distance between their ideal taste and the taste defined by the most closely aligned product the firm offers. A consumer thus considers the taste mismatch associated with their purchased product, but otherwise the positioning of the firm’s product portfolio (or, “brand image”) is immaterial. In contrast, self-congruency theory suggests that consumers assess how well both the purchased product and its overall brand image match with their ideal taste. Therefore, we incorporate within the consumer utility function both product-specific and brand-level components. Mass customization has the potential to improve taste alignment in regards to a specific purchased product, but at the risk of increasing brand dilution. Absent brand dilution concerns, a firm will optimally serve all consumers’ ideal tastes at a single price. In contrast, by endogenizing dilution costs within the consumer utility model, we prove that a mass-customizing firm optimally uses differential pricing. Moreover, we show that the firm offers reduced prices to consumers with extreme tastes (to stimulate consumer “travel”), with a higher and fixed price being offered to those consumers having more central (mainstream) tastes. Given that a continuous spectrum of prices will likely not be practical in application, we also consider the more pragmatic approach of augmenting the uniformly priced mass customization range with preset (non-customized) outlying designs, which serve customers at the taste extremes. We prove this practical approach performs close to optimal.

Key words: Mass customization; product line design; pricing; brand dilution.

1. Introduction

Mass customization technology can enable a firm to serve unique products to individual customers. For example, Zazzle is an online mass-customizing retailer that supports user-generated designs (printed on common consumer goods such as clothing, mugs, and bags), cost effectively supporting order sizes as small as one unit. Given technological advances such as 3D-printing, this “market of one” customization ideal may soon be feasible in a variety of product categories. Nike’s new FlyKnit knitting process holds similar potential, as we learned from an executive involved in product strategy discussions at Nike at the time of the FlyKnit acquisition. In contrast with the pre-specified shoe configuration options provided by the Nike's 'NikeID' service, FlyKnit could support extreme thread-level customization via user-generated designs. The ability to allow product variety to run rampant via mass customization was viewed as a cause for concern, however, given the strength of Nike's brand name and its emphasis on product design. In this paper, assuming a firm has the mass customization capability to extend its product line variety, we consider the market repercussions (demand, pricing) and the optimal resulting product line.

In spatial location models of product differentiation, a customer's ideal taste is represented as a point on a line (Hotelling 1929). As product line variety increases, the distance between a customer's ideal taste and the closest product decreases, benefiting the customer. The implication is that a firm should optimally offer unbridled variety, given a suitable (e.g., costless) mass customization technology. However, an explosion in variety could potentially dilute the firm's brand image, which in turn could potentially be more significant than the reduction in consumers' taste mismatch. In other words, the extreme of unlimited variety eliminates the issue of taste mismatch, but raises the concern of brand-image dilution. For example, we might expect that a firm possessing a strong brand such as Nike should not necessarily mimic the more design-agnostic approach of a firm such as Zazzle. But then, to what extent should customization be employed, and how should the extent of customization impact pricing?

The standard approach for modeling utility in a spatial context is to assume that a customer's utility hinges only on the product purchased, and not a firm's additional product line variations (Moorthy 1984). That simple assumption is consistent with the principle of independence from irrelevant alternatives (IIA) in choice theory. However, the IIA principle addresses only the ordering of preferences, not what determines their magnitude. Regarding the strength of consumer preferences and their willingness to pay, marketing research has shown that, beyond an individual product's attributes, brand equity is a significant component (Farquhar 1989, Rangaswamy et al. 1993, Thomson et al. 2005). Brand equity is often described as the incremental value a product derives from its brand association (Aaker 1992, Biel 1993, Simon and Sullivan 1993).

A key driver of brand equity for consumer products is brand personality (Keller 1993). Brand personality corresponds to the "human characteristics associated with a brand" (Aaker 1997), which can be "distinguished from the more utilitarian function implied by the tangible, product-related attributes" (Yorkston et al. 2010). Consumers associate personality characteristics with product brands because they perceive products as extensions of themselves—a view that is often promoted in advertising by marketers (Belk 1988, Plummer 2000, Diamantopoulos et al. 2005). A brand's personality yields what consumers consider as the "brand-user image" (Kressmann et al. 2006).

When making a purchase, given that consumers receive only the physical product, one might potentially argue that consumer utility should be a function of only the tangible product attributes. However, consumer psychology research shows that consumers conceptualize products as symbols, specifically with symbology relating to human characteristics and personality (Sirgy 1982, Park et al. 1986, Kleine et al. 1995, Ligas 2000). Self-congruency theory posits that consumers use both products and brands to express and validate their identity (Berger and Heath 2007, Dolich 1969, Malhotra 1988, Onkvisit and Shaw 1987). Brands can imply membership in a group with which a consumer identifies (Braun and Wicklund 1989). The central tenet of self-congruency theory is that consumers value products and brands that align closely with their self-concept, or their idealized version of it. Consumers seek self-image congruence even when not overtly striving to attain status or recognition (Fournier 1998). In such cases, consumers may desire to affirm their self-identity not to others but to themselves (Wicklund and Gollwitzer 1981). Thus, the desire for self-congruency applies for both social and conspicuous products, such as beer and clothing, as well as inconspicuous products, e.g., toiletries such as toothpaste or electric shavers (Ross 1971, Landon 1974, Graeff 1996).

In our model, we represent individual's valuations using the microeconomic notion of consumer utility. Per congruency theory, given that consumers value products and brands that align most closely with their ideal self-image, we assume that utility is decreasing—all other things equal—in the distance between a customer's ideal point and their purchased product, and the firm's product range as a whole. A consumer would thus derive the best-case utility from a firm having a product line focused on their preferred taste niche. (Naturally, a firm might not optimally target such a narrow niche, given that the range of customers it would then attract would tend to be relatively small, even given optimal pricing.). As a side note, such a niche strategy would not necessarily imply that a firm offer only a single product, but rather potentially a set of products all serving the same taste niche. We are not the first to relate the abstract notion of a firm's brand image (or, brand equity) to the level of individuals' valuations. Keller (1993) suggested that brand image could be defined at the level of the individual as "the differential effect of brand knowledge on consumer response." Swait et al. (1993) also translated brand-equity to the level of the individual consumer, suggesting that it corresponds to "the utility a consumer attributes to a bundle consisting of a brand name, product attributes and price." Thus, while we are not the first to suggest individual utility as comprising both product- and brand-level components, to our knowledge we are the first to analytically investigate the resulting implications for managing the firm's product portfolio and pricing.

Specifically, the utility function we employ includes the typical product related term (i.e., ideal point to purchased-product gap) and an additional dilution-related term (i.e., ideal point to brand-level product range distance). With this extension, if the firm offers unbridled variety via mass customization, it can eliminate the "taste mismatch" utility component, but at the same time utility may potentially decrease due to a negative brand dilution effect. Per self-congruency theory, if a brand's personality loses focus due to variety extensions, then some consumers may identify less clearly with the brand. In a vertical differentiation context, Randall and Ulrich (2001) found that the product line bounds (i.e., the highest and lowest quality products) "appear to be useful metrics and seem to be consistent with a theory of how consumers develop impressions of brands." Similarly, we will use the firm's product-range bounds to yield a tractable metric for the dilution effect relating to consumers' distinct ideal points.

The existing literature regarding optimal product line design focuses on supply-side factors that limit variety, such as fixed costs (Salop 1979, Dobson and Yano 2002, Jiang et al. 2006). In de Groote (1994) and Gaur and Honhon (2006), optimal variety is limited to constrained capacity in a rotation-cycle production setting. Alptekinoğlu and Corbett (2010) show that supply lead-times can induce a firm to optimally offer standardized make-to-stock (MTS) items rather than only make-to-order (MTO) products. Starting with a newsvendor-type setting, Huang et al. (2017) explore the potential to use MTO products to both precisely target consumer tastes and reduce demand uncertainty. Alptekinoğlu and Corbett (2008) and Mendelson and Parlaktürk (2008) consider the competition between an MTS and MTO producer, to understand when the two can coexist in equilibrium. Alptekinoğlu and Corbett (2008) show that when custom products share a uniform price, an equilibrium can exist even if the mass-producer has higher costs. Mendelson and Parlaktürk (2008) show that equilibrium vanishes if the customizing firm sets differential prices. Earlier mass customization papers also addressed symmetric duopoly settings (Dewan et al. 2003, Syam et al. 2005, Syam and Kumar 2006).

Although supply-side factors limiting variety can be varied and significant, based on a study of over 1,400 business units Kekre and Srinivasan (1990) suggested: "American manufacturing firms may indeed be flexible enough to accommodate product variety without significant effects on costs." In particular, we focus on demand-side variety implications by considering the firm has the ability to match consumers' distinct tastes via mass customization, at no incremental (fixed) cost (Zipkin 2001). While we focus on horizontal differentiation setting (i.e., with unordered consumer tastes), prior research has shown that demand-side concerns can induce restricted variety within a vertical differentiation setting (i.e., with ordered product qualities) (Mussa and Rosen 1978, Moorthy 1984).

Given that consumers consider not only on their congruency with the product they purchase, but also put some weight on their congruency with the firm's broader set of offerings (i.e., its full product line), the implied brand-image dilution concern can induce a firm to optimally limit variety so as to serve a limited range of consumer tastes. But, as we will also prove, catering to extreme consumer tastes can be optimal even in the presence of dilution costs, below a particular threshold we derive. When full market coverage is not optimal due to taste concerns, multiple managerial questions arise. To what extent should the product range be restricted, and how? And, how should product pricing change in response to a variety restriction, if at all? Should the firm implement differential pricing given that some consumers will have the ability to purchase their ideal product, while others do not? Our analysis sheds light on all these questions.

Our first key result is that we establish that the firm should optimally restrict its product span even when variety is costless. Keller (2010) argues that a proper branding strategy should "provide 'guardrails' as to appropriate and inappropriate line and category extensions." Borrowing Keller's terminology, we refer to the limits on the product range as "guardrails." Our goal is to formally analyze and establish the value of such guardrails from a demand-side perspective. Establishing the optimal existence of guardrails is intuitive, yet highlights an important mass customization concept: limiting variety may be optimal from a demand-side perspective. Dovetailing that result, we prove that the firm should set the price of the customized product so as to permit a strictly positive level of consumer surplus. The firm (optimally) does so as a means to attract those consumers with more extreme tastes, without broadening the product line itself to directly cater to far-flung tastes. We also analyze the extent to which charging variety-specific prices within the mass customization region increases profit, relative to the benchmark of charging a fixed (uniform) price. We show that these variety-specific prices linearly increase towards the middle of the taste spectrum and finally reach a point where the firm leaves no surplus to the consumers with more mainstream tastes.

While combining mass customization with continuously differentiable pricing options is a theoretical ideal, a continuous pricing structure may be impractical. Assuming the firm is restricted to discrete price options, we prove that a strategy of offering lower-priced preset designs (i.e., non-customized) in combination with the firm's mass customization range increases profits, and approaches that theoretical ideal. We show that by using lower-priced preset designs to serve the taste extremes, with a restricted range of mass customization serving more central consumers, the firm improves revenues. As an example of this strategy, consider The North Face's customization options for its popular "Denali" fleece jacket. Even though The North Face allows customers to customize numerous distinct colors for the jacket fleece, zippers, zipper pulls, and logos (yielding over one million varieties), arguably the most extreme-taste fabric options (specifically, camouflage and plaid) for the jacket were not available within the customization program, but only as fixed designs. These pre-vetted camouflage and plaid designs ostensibly may cater to hunter and "hipster" customer segments whose tastes differ from those of mainstream The North Face customers. Our theoretical finding that proves the optimality of using customization to serve more mainstream tastes, with lower-priced fixed designs serving the taste extremes, runs contrary to the conventional view that customization is well suited to serve consumers with unusual tastes.

The rest of the paper is organized as follows. Section 2 introduces the modeling assumptions and analyze the firm's optimal product line design for both a uniform pricing regime and the theoretical ideal of differential pricing. Because differential pricing will not typically be feasible for a firm to implement, in Section 3 we show that by combining a range of mass customized products with outlying preset product, the firm can approach the profits associated with differential pricing. Section 4 generalizes our initial brand-image dilution cost structure to a more general convex form to demonstrate the robustness of our key findings. Section 5 concludes.

2. Portfolio Design and Uniform versus Differential Pricing Models

We consider a monopolist firm that serves customers who are heterogeneous in their tastes. Namely, each consumer is identified by a point z that represents her ideal product. Following the standard Hotelling linear city framework, consumers' ideal points are uniformly spread over the unit interval $[0,1]$. We refer to this linear market as the "*taste spectrum*." Each customer is in the market to purchase at most one unit. For simplicity, we assume customers are small relative to the size of the market, which is denoted by λ.

Customers who decide to purchase a product earn a reward of V and incur a cost t per unit distance between their ideal product and the purchased product. In addition to that mismatch cost which is directly associated with the purchased product, we incorporate the mismatch between a customer's taste and the firm's full product range, which we refer to as the *dilution cost*. We capture this cost by considering the distance from a consumer's ideal product to the range of the firm's product portfolio. In particular, each customer incurs a cost α per unit distance between their ideal taste and the endpoints of the firm's product line. The firm's decisions entail designing its product portfolio and setting prices. Because our emphasis is on understanding the demand-related effects of product variety, we normalize production costs to zero.

We denote the product portfolio as the set X, and the corresponding prices by a function $p : X \to [0, V]$. We assume X is an interval and let $X = [\underline{m}, \overline{m}]$. We refer to the boundary points $\underline{m}$ and $\overline{m}$, which define the range of the product line, as *guardrail products*. The utility of a customer located at $z \in [0,1]$ from product $x \in X$ is, then,

$$U(z,x,p) = V - p(x) - t|z-x| - \alpha\left(|z-\underline{m}| + |\overline{m}-z|\right). \tag{1}$$

This utility structure captures our notion of dilution cost via its last term, $\alpha\left(|z-\underline{m}| + |\overline{m}-z|\right)$. Figure 1 illustrates the structure of this dilution cost function for a given (arbitrary) product portfolio range $[\underline{m}, \overline{m}]$. As can be seen in this figure, that function is convex, continuous, and symmetric around the portfolio midpoint.

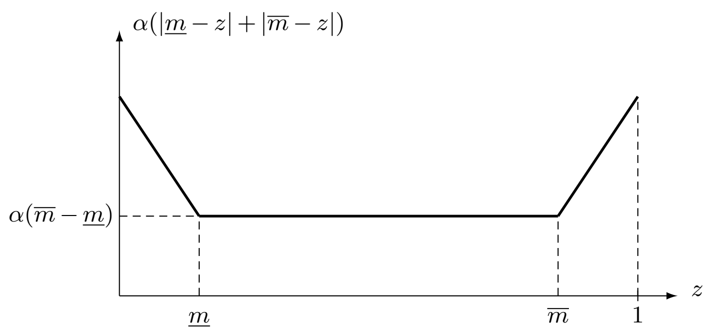


Figure 1 Illustration of the dilution cost.

Each customer buys the specific product that delivers the highest utility and will make no purchase if all products deliver negative utility. If a customer is faced with multiple options that yield the same net utility, we assume that the customer favors a near product relative to a distant product, and favors purchasing over not purchasing. We denote customers' most preferred products by the function $b : [0, V] \to X$ for any given price function p such that $b(z,p) \equiv \arg\max_{x \in X} U(z,x,p)$ for any $z \in [0,1]$. We let the set B be the set of customers making a purchase, i.e., $B \equiv \{z : U(z, b(z,p), p) \geq 0\}$. We assume B is an interval and refer to it as the "*market coverage*" of the firm. Letting $B = [\underline{b}, \overline{b}]$, the revenue of the firm is, then,

$$\Pi(X,p,b) = \lambda \int_{\underline{b}}^{\overline{b}} p(b(z))dz \tag{2}$$

for any product portfolio X, price function p, and the corresponding purchasing decisions of the customers.

The existing product portfolio management literature suggests that a mass customizing firm can extract all consumer surplus by setting a fixed price (equal to consumers' reservation price, V), and delivering a custom product tailored to each individual's taste. In the presence of dilution costs, charging a fixed price may no longer be optimal, but we consider such a *Uniform Pricing* model as a benchmark. Under uniform pricing, the price function $p(x)$ reduces to a fixed price which we denote as p_f for all products in the firm's (mass customized) product range $[\underline{m}, \overline{m}]$. We illustrate the firm's product line and pricing decisions in Figure 2(a). In the figure, we depict the span of the market served by the firm to be less than $[0,1]$, because, as we will analyze, serving less than the full market may be optimal—even if variety is costless. Potentially, due to dilution concerns, the firm might not only optimally want to constrain the range of its product variety, but also charge different prices to distinct consumers. For instance, the firm might charge lower prices for products near the limits of its customization range as a means to attract customers beyond that range. We illustrate such a scenario in Figure 2(b), which we refer to as the *Differential Pricing* model. Under the *Differential Pricing* model, we permit the price function $p(x)$ to be any continuous function, a special case of which is *Uniform Pricing* with $p(x) = p_f, \forall x$. We begin by analyzing how the firm should optimize its portfolio under the *Uniform Pricing* model benchmark, before revisiting the more general *Differential Pricing* model.

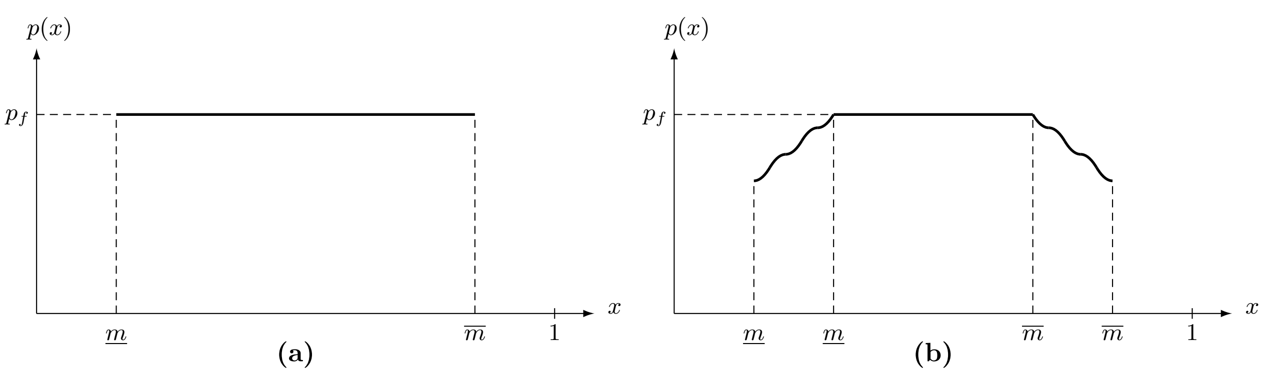


Figure 2 Product portfolio and pricing decisions under: (a) Uniform Pricing, and (b) Differential Pricing.

2.1. Optimal Product Line Design Under Uniform Pricing

Under *Uniform Pricing*, a consumer whose ideal taste lies within the firm's preferred mass customization range $[\underline{m}, \overline{m}]$ will be the product that provides a perfect taste match. Customers outside the $[\underline{m}, \overline{m}]$ will prefer the guardrail product that is closest to their taste. Therefore, in this setting, the function $b(z,p)$ representing consumers' most preferred products is:

$$b(z,p_f) = \begin{cases} \underline{m} & \text{if } z < \underline{m} \\ z & \text{if } \underline{m} \le z \le \overline{m} \\ \overline{m} & \text{if } z > \overline{m}. \end{cases}$$

Given the constant price, consumers' self-selection (choice) does not depend on price, but participation (i.e., a willingness to purchase) does. If p_f is greater than $V - \alpha(\overline{m} - \underline{m})$, no customers will purchase, implying that the firm must charge a price lower than $V - \alpha(\overline{m} - \underline{m})$. For any $p_f \le V - \alpha(\overline{m} - \underline{m})$, all of customers in the interval $[\underline{m}, \overline{m}]$ will purchase but customers outside the mass customization range might not. Specifically, the market coverage B is the interval $[\underline{b}, \overline{b}]$ where $\underline{b} = \underline{m} - [V - p_f - \alpha(\overline{m} - \underline{m})]/t$ and $\overline{b} = \overline{m} + [V - p_f - \alpha(\overline{m} - \underline{m})]/t$. An insight from the horizontal differential and mass customization literature is that a purchasing consumer should buy their ideal product or nothing at all. This insight continues to hold here and implies an optimal price p_f equal to $V - \alpha(\overline{m} - \underline{m})$, as we formally present in the next proposition.

PROPOSITION 1. *Let $X^\circ \equiv [\underline{m}^\circ, \overline{m}^\circ]$ be the optimal product portfolio and B° be the optimal market coverage in the Uniform Pricing model. Then, we have that $B^\circ = X^\circ$ and $p_f^\circ = V - \alpha(\overline{m}^\circ - \underline{m}^\circ)$ where p_f° is the optimal price.*

The above proposition establishes that the firm's pricing decision and customers' purchasing decisions can be expressed in terms of the mass customization bounds $\underline{m}$ and $\overline{m}$. We can thus rewrite the revenue function (2) as

$$\Pi_M(\underline{m}, \overline{m}) = \lambda[V - \alpha(\overline{m} - \underline{m})](\overline{m} - \underline{m}).$$

Then, the firm's problem reduces to $\max_{0 \le \underline{m} \le \overline{m} \le 1} \Pi_M(\underline{m}, \overline{m})$. The following theorem formally presents the optimal decisions of the firm.

THEOREM 1. *Let $X^\circ \equiv [\underline{m}^\circ, \overline{m}^\circ]$ be the optimal product portfolio and Π° be the optimal revenue in the Uniform Pricing Model. It follows that*

$$\overline{m}^\circ - \underline{m}^\circ = \begin{cases} 1 & \text{if } \alpha < V/2 \\ V/(2\alpha) & \text{if } \alpha \ge V/2, \end{cases} \quad \text{and} \quad \Pi^\circ = \begin{cases} \lambda[V - \alpha] & \text{if } \alpha < V/2 \\ \lambda V^2/(4\alpha) & \text{if } \alpha \ge V/2. \end{cases}$$

The above theorem shows that the firm prefers to cover the entire market when α is low; in other words, the firm follows what the traditional mass customization literature prescribes. However, once α exceeds the critical level of $V/2$, the firm narrows the range of its product portfolio and opts to serve less than the entire market. We thus see that the firm optimally limits mass customization as a means to increase revenue, apart from any production costs. Furthermore, the width of the product portfolio shrinks as the brand dilution effect increases. We next analyze the *Differential Pricing* model to explore charging different prices for customized product variations.

2.2. Differential Pricing Model

Under the *Differential Pricing* model, the firm provides a mass customized product range $[\underline{m}, \overline{m}]$ and may charge different prices for its product variations, specified by the price function $p(x), \forall x \in X$. Although we place no restriction on the $p(x)$ function other than it be continuous, we can show that its optimal form is simple. Specifically, $p(x)$ is either constant or linearly increasing function (with slope t) as towards the middle (interior) of the firm's product portfolio. Let $F \equiv [\underline{f}, \overline{f}] \in X$ denote the customization range for which price is constant. Thus, F corresponds to a *fixed price* region whereas $[\underline{m}, \overline{f}]$ and $[\overline{f}, \overline{m}]$ correspond to *differential pricing* regions.

We can show that, as in the benchmark *Uniform Pricing* model, the firm leaves no surplus to consumers whose tastes lie in the *fixed price* region $[\underline{f}, \overline{f}]$, within which the price is constant. In contrast, however, the firm leaves positive surplus to customers outside the *fixed price* region, even though these customers also purchase their ideal product, as illustrated in Figure 3. At the extreme end-points of the differential pricing regions, i.e., $\underline{m}$ and $\overline{m}$, we will show that the firm should optimally leave strictly positive surplus. Moving inwards (towards the interior of the product portfolio) from these extreme points, the optimal price function linearly increases with slope t, meaning $p(x)$ is $p(\underline{m}) + t(x - \underline{m}) \forall x \in [\underline{m}, \underline{f}]$, and equal to $p(\overline{m}) + t(\overline{m} - x) \forall x \in [\overline{f}, \overline{m}]$. This simple linear structure applies because travel costs are linear in t and therefore if the firm attempts to increase price at a rate higher than t (i.e., across the portfolio), then consumers have the flexibility to switch from purchasing their preferred-taste product purchase to a lower-priced product (in which case the firm sacrifices revenue and profit, since the variations are of equivalent cost to the firm). We formally establish these results in the following proposition, and also prove the firm's optimal product portfolio X is symmetric within the market coverage $B \equiv [\underline{b}, \overline{b}]$.

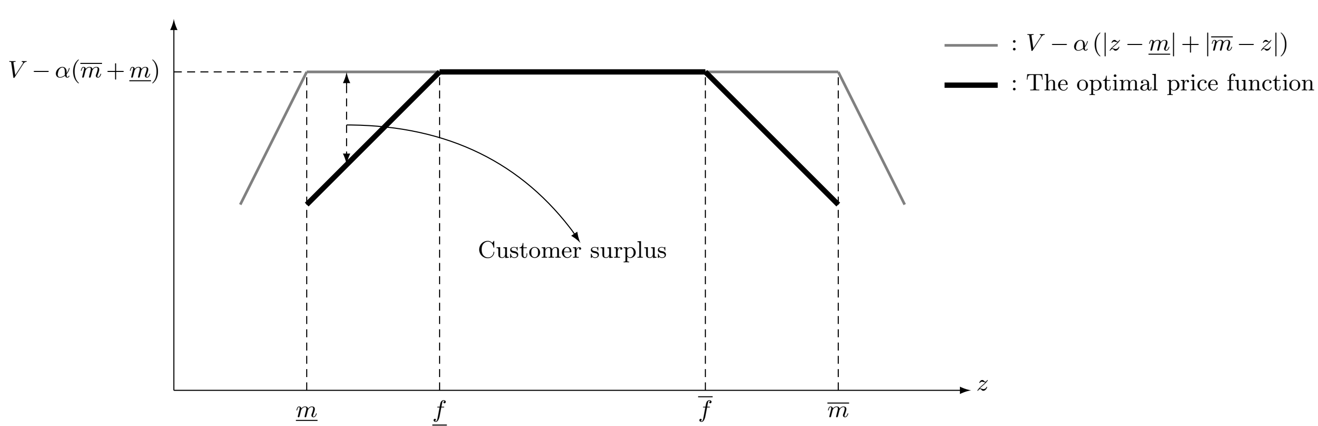


Figure 3 Illustration of the optimal price and non-zero surplus for customers in the differential pricing region.

Proposition 2. *Let $X^* \equiv [\underline{m}^*, \overline{m}^*]$ be the optimal product portfolio and $B^* \equiv [\underline{b}^*, \overline{b}^*]$ be the optimal market coverage in the Differential Pricing Model. Then, the optimal price function p^* is*

$$p^*(x) = \begin{cases} V - t(\underline{f}^* - x) - \alpha(\overline{m}^* - \underline{m}^*) & \text{if } x \in [\underline{m}^*, \underline{f}^*], \\ V - \alpha(\overline{m}^* - \underline{m}^*) & \text{if } x \in [\underline{f}^*, \overline{f}^*], \\ V - t(x - \overline{f}^*) - \alpha(\overline{m}^* - \underline{m}^*) & \text{if } x \in [\overline{f}^*, \overline{m}^*], \end{cases}$$

and the boundaries of the optimal fixed price region $F^* \equiv [\underline{f}^*, \overline{f}^*]$ can be obtained as follows:

$$\underline{f}^* - \underline{m}^* = \frac{t + 2\alpha}{t}(\underline{m}^* - \underline{b}^*), \qquad \overline{m}^* - \overline{f}^* = \frac{t + 2\alpha}{t}(\overline{b}^* - \overline{m}^*),$$

Furthermore, we have that $\underline{m}^* - \underline{b}^* = \overline{b}^* - \overline{m}^*$ and the customers' most preferred products are

$$b^*(z, p^*) = \begin{cases} \underline{m}^* & \text{if } z \le \underline{m}^* \\ z & \text{if } \underline{m}^* \le z \le \overline{m}^* \\ \overline{m}^* & \text{if } z \ge \overline{m}^*. \end{cases}$$

The two primary implications here are that the optimal price function calls for: (i) charging highest prices for product variations that cater to central tastes, with (linearly) decreasing prices for less central varieties, and (ii) providing strictly positive surplus to customers with such outlying tastes, as a means to increase the range of buying consumers while maintaining a more focused portfolio (thus mitigating the negative brand-image related impact that would otherwise result from overly increasing the product range).

The simple structure of the optimal price function simplifies our analysis significantly. As in our *Uniform Pricing* analysis, we can now rewrite the firm's pricing and product location decisions as a function of the portfolio guardrails $\underline{m}$ and $\overline{m}$, and the market coverage endpoints $\underline{b}$ and $\overline{b}$. In fact, due to the symmetry of the firm's decisions, we can, without of loss of generality, assume that the midpoint of the product portfolio is $1/2$. Then, we can rewrite the firm's revenue as a function of $\underline{m}$ and $\underline{b}$ as follows:

$$\Pi_P(\underline{b}, \underline{m}) = 2\left[(\underline{m} - \underline{b})p^*(\underline{m}) + \int_{\underline{m}}^{\underline{f}^*} p^*(z)dz + (1/2 - \underline{f}^*)p^*(\underline{f}^*)\right], \tag{3}$$

where the boundary point of the *fixed price* region, $\underline{f}^*$, and the price function, $p^*(x)$, are as in Proposition 2. Then, the firm's problem requires solving $\max_{\underline{b},\underline{m}} \Pi_P(\underline{b}, \underline{m})$. It is worth noting that the firm reverts back to the *Uniform Pricing* model if it sets $\underline{b} = \underline{m}$, i.e., $\Pi_P(\underline{b}, \underline{b}) = \Pi_M(\underline{b}, 1 - \underline{b})$. Relative to charging a uniform price, under differential pricing the guardrail products shift inward, which also shifts the boundary of the *fixed price* region, $\underline{f}$, inward.

By shifting the guardrails inward and thus focusing its portfolio, the firm can charge a higher price in the *fixed price* region; but, such a shift implies increased “travel” for consumers who lie

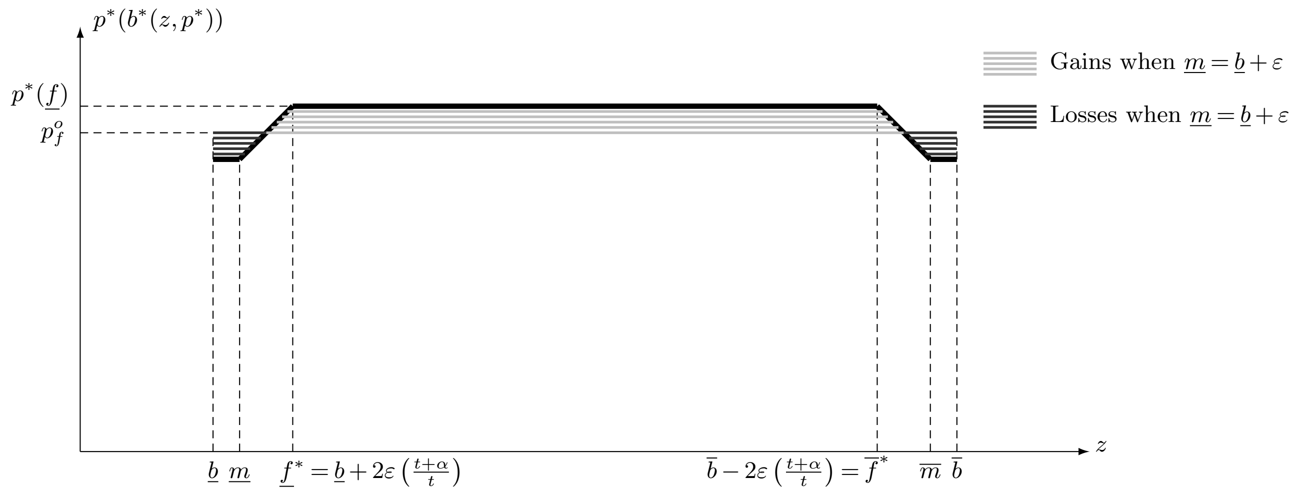


Figure 4 **The illustration of the revenue gains and losses after moving the guardrails inward.** $p^*(b^*(z,p^*))$ **is the price that each customer pays when** $\underline{m} = \underline{b} + \varepsilon$. p_f^o **is the uniform price in the Uniform Pricing model when the product portfolio is** $[\underline{b}, \overline{b}]$.

outside the customization range, and thus attracting such consumers requires some corresponding price concession. Therefore, given such a shift, the (central) fixed price increases while at the same time the firm decreases the prices it charges to consumers with outlying tastes. We illustrate the firm's tradeoff in Figure 4 by drawing what each customer pays when $\underline{m} = \underline{b} + \varepsilon$ for some $\varepsilon > 0$, relative to if the firm charged the uniform price p_f^o over $[\underline{b}, \overline{b}]$. As we formally present in the following theorem, the gains from shrinking the mass customization region outweigh the losses from such a move for small values of ε while keeping the market coverage the same.

Theorem 2. *Let X^* be the optimal product portfolio, F^* be the optimal fixed price region, and B^* be the optimal set of consumers to be served in the Differential Pricing model. Then, denoting the length of set X as $\ell(X)$, we have*

$$\ell(B^*) = \begin{cases} 1 & \text{if } \alpha < \alpha^*_{fc}(t) \\ \frac{V}{2\alpha}\left[1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] & \text{if } \alpha \geq \alpha^*_{fc}(t), \end{cases}$$

$$\frac{\ell(X^*)}{\ell(B^*)} = \left[1 - \frac{2t\alpha}{(t+2\alpha)(3t+2\alpha)}\right], \text{ and } \frac{\ell(F^*)}{\ell(B^*)} = \left[\frac{t(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)}\right],$$

where $\alpha^*_{fc}(t)$ solves the equation $\frac{V}{2\alpha}\left[1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] = 1$ and is greater than $V/2$.

Furthermore, letting Π^* be the optimal revenue of the firm, we obtain

$$\Pi^* = \begin{cases} \lambda\left[V - \alpha\left(1 - \frac{t\alpha}{(t+2\alpha)(3t+2\alpha)}\right)\right] & \text{if } \alpha < \alpha^*_{fc}(t) \\ \lambda\frac{V^2}{4\alpha}\left(1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right) & \text{if } \alpha \geq \alpha^*_{fc}(t). \end{cases}$$

Consistent with our *Uniform Pricing* results, Theorem 2 shows that the firm serves the entire market only if the dilution cost is less than a threshold, i.e., $\alpha^*_{fc}(t)$ in this setting. Despite this similarity in terms of market coverage, our findings in Theorem 2 demonstrates crucial differences between the firm's optimal mass customization and pricing decisions in the *Uniform Pricing* and the *Differential Pricing* models. Specifically, with differential pricing, some degree of customer "travel" at the taste extremes is *always* optimal (even for very low levels of dilution cost), which is not the case under uniform pricing.

It is also noteworthy that if the firm can apply differential pricing, then it will *never* find it optimal to serve all customers with a uniform price. In fact, the firm considerably shrinks the range of fixed-price products, relative to the optimal range of products offered with uniform pricing. The following corollary bounds (from below) the extent to which differential pricing reduces the fixed price region, relative to when the firm adopts uniform pricing. Corollary 1 also shows that the firm optimally serves more customers under the *Differential Pricing* model.

Corollary 1. *Let X° be the optimal product portfolio in the Uniform Pricing model and M^* be the optimal uniform pricing region in the Differential Pricing model. Then, denoting the length of set X as $\ell(X)$, we have that*

$$1 - \frac{\ell(M^*)}{\ell(X^\circ)} \geq \frac{\theta}{1+\theta}, \ \text{where } \theta = \alpha/t.$$

Furthermore, we have that $\ell(B^*) \leq \ell(X^\circ) \leq \ell(X^*)$.

In Figure 5, we illustrate both the reduction in the two models' uniform pricing range and the corresponding lower bound established in Corollary 1. We see that the reduction is as high as 20% even when the ratio of dilution cost to travel cost, α/t, is as low as 0.2. We also see from both Figures 5(a) and 5(b) that there is a small region over which increasing α/t the ratio of the fixed price regions increase (thus lowering $1 - \ell(M^*)/\ell(X^\circ)$), beyond which the established bound tightens.

Summarizing these results, *Differential Pricing* implies two major changes to how the firm optimizes its product line, relative to the *Uniform Pricing* model. First, it significantly shrinks the range of customized products having a fixed price. In fact, as direct implication of Theorem 2, the size of the *fixed price* region (relative to the total market coverage) decreases in α whereas the size of the *differential pricing* region increases in α. In other words, the firm relies on *differential pricing* more as the dilution cost α increases. Second, the firm addresses extreme consumer tastes via lower-priced guardrail products while charging a higher fixed price for the customers with more

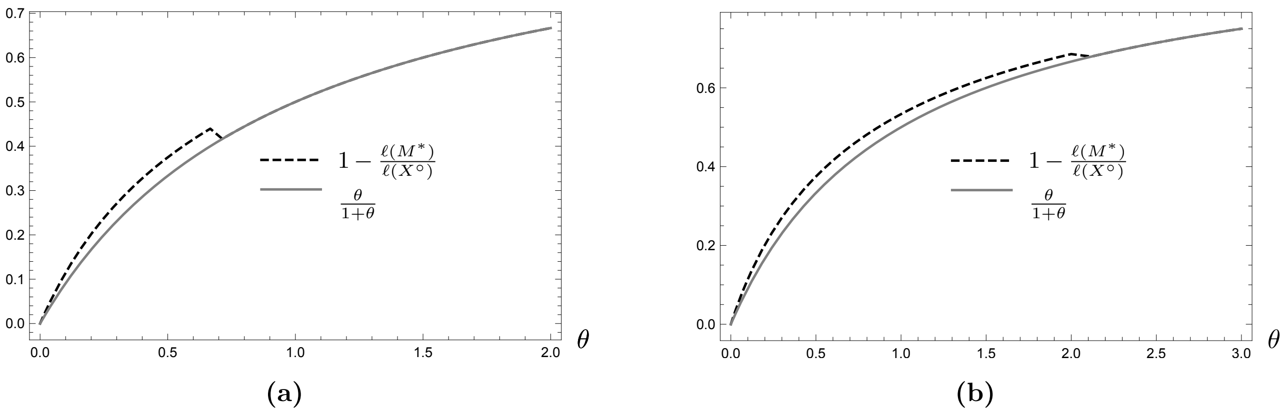


Figure 5 **Reduction in the uniform pricing region from Uniform Pricing model to the Differential Pricing model when $V=1$ and a) $t=3/4$, b) $t=1/4$.**

central taste. As we illustrate in Figure 6, the portion of the customers purchasing a guardrail product (relative to the total market coverage) can be more than 10% for quite a sizable range of dilution costs (i.e., parameter α). In the next section, we analyze a portfolio structure that leverages these insights while maintaining implementation practicality, by avoiding the need for the firm to offer a continuous spectrum of prices—which is required with differential pricing.

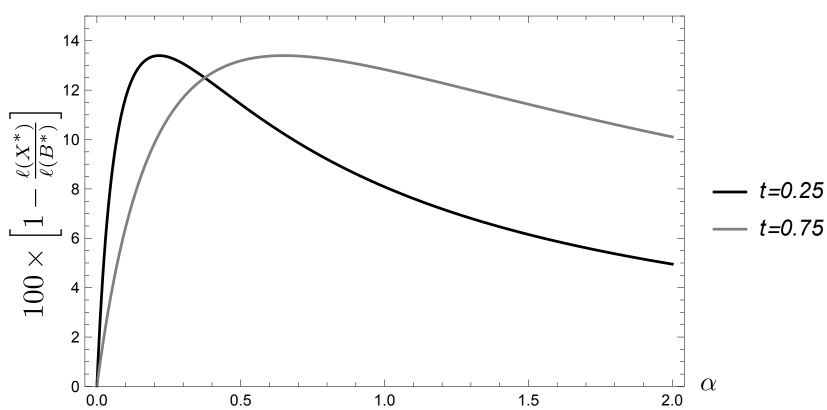


Figure 6 **Percentage of customers purchasing a guardrail product (relative to the total market coverage) when $V=1$ and $t\in\{0.25,0.75\}$.**

3. Mass Customization with Preset Products

A firm might find implementing the optimal differential pricing function problematic or infeasible, because it implies infinitely many prices. Yet, the firm should leverage the key takeaways from the prior section: serve customers with extreme tastes via lower-priced guardrail products, and serve customers with more central tastes with mass customized variants and with a (higher) fixed price. Therefore, we now consider a portfolio where the range of fixed-price customized products is supplemented with a finite number of non-customized outlier products that the firm can leverage to cater to customers with extreme taste, at distinct prices.

3.1. Presets Model

We now consider a product portfolio structure such that the firm offers N discrete products lying outside each end of the mass customization region. We employ the results of Theorem 2 to define the *fixed-price region* (with mass customization) $F^* \equiv [\underline{f}^*, \overline{f}^*]$ and the guardrail locations $\underline{m}^*$ and $\overline{m}^*$. The product portfolio is thus $\{x_1, \ldots, x_N\} \cup [\underline{f}^*, \overline{f}^*] \cup \{x_{N+1}, \ldots, x_{2N}\}$, where x_i denotes the location of preset product-i for any $i \in \{1, 2N\}$ and $\{x_1, x_{2N}\} = \{\underline{m}^*, \overline{m}^*\}$. The rationale for this portfolio is that the firm defines a discrete number of preset products between the guardrail locations and the mass customization region, rather than attempt to implement a continuum of prices and products. The mass customization region's fixed price is $V - \alpha(\overline{m}^* - \underline{m}^*)$, and the price of preset product-i is $p^*(x_i)$. We refer to this portfolio structure as the *Presets* model. We will show that as the number of such preset products grows, the resulting profits converge in the limit to the optimal revenue of the *Differential Pricing* model. We also assess the gains relative to the *Uniform Pricing* model.

Given the Presets model structure, for the special case of $N = 1$ the only discrete products (i.e., products outside the mass customization region) are those located precisely at the guardrail locations $\underline{m}^*$ and $\overline{m}^*$, with corresponding prices $p^*(\underline{m}^*)$ and $p^*(\overline{m}^*)$. For $N > 1$, however, the firm must determine the optimal (revenue maximizing) locations of the preset products located in-between the guardrails and the mass customization region. We next show that the firm should evenly distribute the preset products between the guardrails and the uniform pricing region when $N > 1$.

PROPOSITION 3. *Let $\{\tilde{x}_1, \ldots, \tilde{x}_{2N}\}$ be the optimal locations of non-customized products in the Presets model for any given $N > 1$. Then, we have that*

$$\tilde{x}_1 = \underline{m}^* \text{ and } \tilde{x}_{i+1} - \tilde{x}_i = (\underline{f}^* - \underline{m}^*)/N \text{ for all } 1 \le i < N.$$

We also have that $\tilde{x}_i + \tilde{x}_{2N-i+1} = 1$ for all $1 \le i \le N$. In other words, the firm distributes the preset products uniformly between the guardrails and the mass customization price region. Furthermore, customers' most preferred products are

$$\tilde{b}(z) = \begin{cases} \tilde{x}_1 & \text{if } z < \tilde{x}_2 \\ \tilde{x}_i & \text{if } \tilde{x}_i \le z < \tilde{x}_{i+1},\ 1 < i < N \\ \tilde{x}_N & \text{if } \tilde{x}_N \le z < \underline{f}^* \\ z & \text{if } \underline{f}^* \le z \le 1/2, \end{cases}$$

for any $z \le 1/2$, and $\tilde{b}(z) = \tilde{b}(1-z)$ for any $z > 1/2$.

Using the above result, we express the firm's revenue in the *Presets* model as:

$$\tilde{\Pi}(N) = 2\left[(\tilde{x}_1 - \underline{b}^*)p^*(\underline{m}^*) + (\underline{f}^* - \underline{m}^*)\left(\sum_{i=1}^{N} p^*(\tilde{x}_i)/N\right) + (1/2 - \underline{f}^*)p^*(\underline{f}^*)\right],$$

where $\tilde{x}_i$ is the optimal location of the preset product for any given $N > 1$ as described in Proposition 3, and $\{\underline{b}^*, \underline{m}^*, \underline{f}^*\}$ corresponds to the optimal portfolio structure from the *Differential Pricing* model. As we formally show in the following theorem, $\tilde{\Pi}(N)$ increases and converges to the optimal revenue under the *Differential Pricing* model as the number of preset products increases.

Lemma 1. *Let $\tilde{\Pi}(N)$ be the firm's optimal revenue when it offers N preset products on each side of the fixed price region. Then, we have that $\tilde{\Pi}(N)$ is increasing in N and $\lim_{N\to\infty} \tilde{\Pi}(N) = \Pi^*$.*

This result supports our choice to utilize the *Differential Pricing* model's results to define the price function and the mass customization region and guardrail locations. A more complex heuristic that allows more flexibility in these decisions should likewise converge to the *Differential Pricing* model's optimal solution, but Lemma 1 proves that despite restricting these decisions, the *Presets* model converges to the optimum nonetheless. This suggests there is little value to be gained from considering alternative assumptions. We next turn our attention to assessing the incremental gains the firm realizes by offering even a relatively small number of preset products.

3.2. Presets vs Uniform Pricing

The portfolio structure of the *Presets* model can be viewed as a generalization of the *Uniform Pricing* model, as it permits a fixed-price mass customization region in conjunction with ($2N$) outlying products. Starting with $N = 1$ and for larger values of N in particular, we wish to assess the profit contribution from this portfolio structure extension. For a given value of N, and parameters α, t, we therefore denote the incremental profit gain as $\Delta(\alpha, t, N)$, where

$$\Delta(\alpha, t, N) = 100 \times \left(\frac{\tilde{\Pi}(\alpha, t, N)}{\Pi^\circ(\alpha, t)} - 1\right). \tag{4}$$

The following lemma formally shows the ability to supplement a fixed-price mass customization region with discrete outlying products adds value.

Lemma 2. *Let $\Delta(\alpha, t, N)$ be the incremental profit gain from preset products as described in (4). Then, we have that $\Delta(\alpha, t, N) > 0$ for any $\alpha > 0$ and $N \geq 1$.*

Moreover, we know from Lemma 1, that for large N the *Presets* model converges to the optimal *Differential Pricing* solution. The question we want to address here is whether the firm can realize much of its potential profit gain by offering only a few preset products.

We first focus on settings for which $\alpha < V/2$, with relatively low brand image effects. In the *Uniform Pricing* model, the firm then serves the entire market using mass customization, effectively ignoring any potential brand image repercussions. In contrast, the firm is more responsive to the dilution cost parameter α if the *Presets* model applies because the firm then alters its locations for the guardrails and the size of the fixed price range as α increases—even for very low α values. As a result, in the following proposition we show that the benefit from offering presets increases as the dilution cost α increases, as long as α is less than $V/2$ and $N \geq 2$.

PROPOSITION 4. *Let $\Delta(\alpha, t, N)$ be the incremental profit gain from preset products as described in (4). Then, we have that $\Delta(\alpha, t, N)$ is increasing in α for any $\alpha < V/2$ and $N \geq 2$.*

When $N = 1$, it is possible for the gains from presets to be declining in α but we also observe that such non-monotone behavior arises only when the benefit itself is negligibly small, e.g., when $\Delta(\alpha, t, 1)$ is less than 0.3%. Given that Proposition 4 addresses cases where $\alpha < V/2$, let us now consider the incremental value of allowing preset products when brand-image effects are more significant (i.e., at higher α values).

For $\alpha > V/2$, the firm does not serve the entire market in the *Uniform Pricing* model, and the optimal extent of market coverage depends on the dilution cost. Because the firm thus starts to take α into account, we might expect the incremental gains from allowing outlier products as in the *Presets* model to be lower. In fact, we numerically observe that once α passes a critcal threshold (greater than 1/2), the incremental gains from having the preset products are decreasing—yet remain strictly positive.

Figure 7 illustrates the results of numeric experiments in which we consider values numbers of discrete preset products, as defined by the parameter N, where for we such setting we vary the dilution cost parameter α between 0 and 1. In Figure 7(a), we assume a travel cost of $t = V/4$, whereas a higher travel cost of $t = 3V/4$ applies in Figure 7(b). In all cases, we witness a consistent trend: $\Delta(\alpha, t, N)$ first increases and then decreases after reaching its peak value. We also observe that when the travel cost is low as in Figure 7(a), the gains from offering non-customized products peak before the firm stops covering the entire taste spectrum in the *Presets* model, which occurs at the threshold level we denote as $\alpha^*_{fc}(t)$. On the other hand, when the travel cost is high, the highest benefit from offering non-customized products (compared to the *Uniform Pricing* model) is achieved after the firm starts to partially cover the market. It is also worth noting that the firm can obtain sizable benefits from offering non-customized products even by adding around 10 ($N = 5$) non-customized products, which is quite practical to implement.

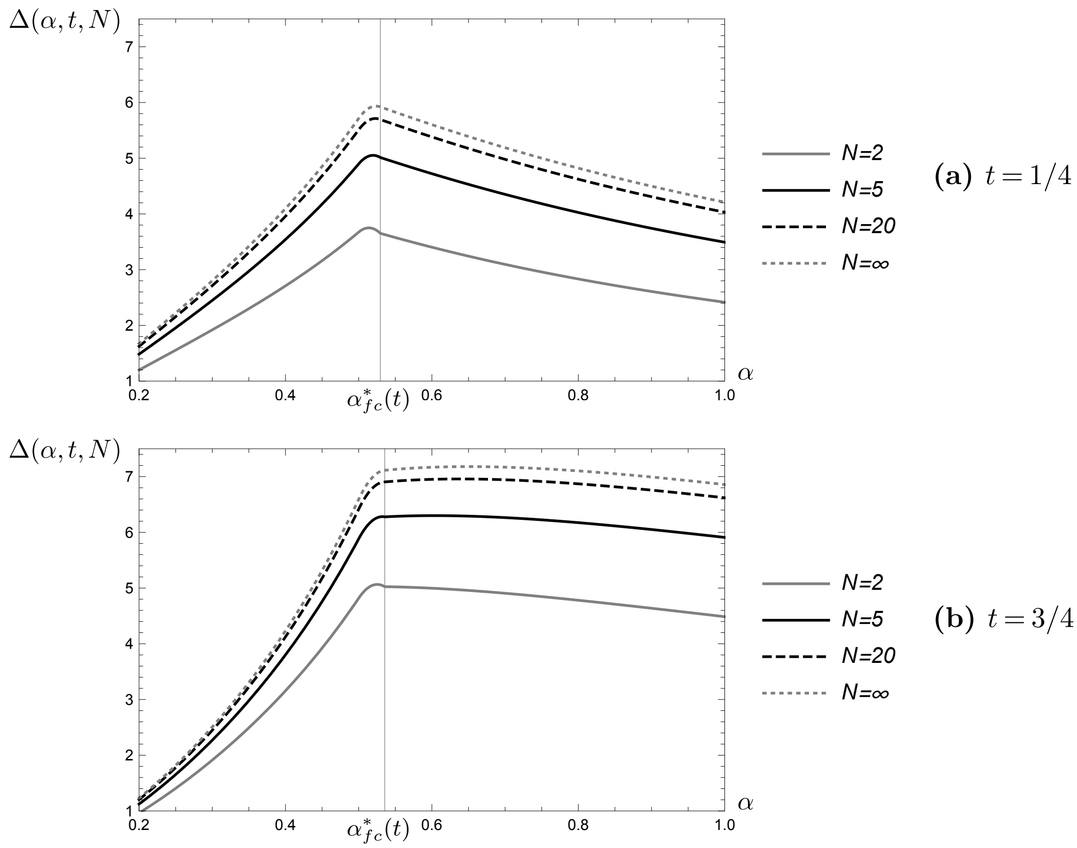


Figure 7 **Relative improvement in revenue from the Uniform Pricing model to the Presets model when** $V = 1$.

Despite the complex nature of the *Presets* model's profit function, we can analytically establish the limiting behavior of $\Delta(\alpha,t,N)$ as the number of preset products approaches to infinity. Furthermore, we show that if the travel cost t lies below a critical level, then the benefit from offering non-customized products achieves its maximum when the firm covers the entire taste spectrum in the *Presets* model. On the other hand, for high levels of the travel cost the firm obtains the highest gains from non-customized products after it begins partially serving the market in the *Presets* model. We formally present these results in the following proposition.

Proposition 5. *Let $\Delta(\alpha,t,N)$ be the incremental profit gain from offering differential pricing as described in (4) and $\Delta^*(\alpha,t)$ be the limit of $\Delta(\alpha,t,N)$ as $N \to \infty$. Then, we find:*

1. *$\Delta^*(\alpha,t)$ is increasing in α for any $\alpha < V/2$.*

2. *There exists a critical level $\hat{\alpha}(t)$ such that $\Delta^*(\alpha,t)$ is increasing in α if $\alpha < \hat{\alpha}(t)$ and decreasing in α otherwise. Furthermore, we have that $\hat{\alpha}(t) \le \alpha^*_{fc}(t)$ if and only if $t \le \frac{4V}{3+2\sqrt{3}}$.*

The values $t = 1/4$ and $t = 3/4$ from Figures 7(a) and 7(b), respectively, lie above and below the threshold $t \le (4/(3+2\sqrt{3})) \approx 0.62$ from Proposition 5. Therefore, the proposition implies that the $N \to \infty$ curve in Figure 7(a) should reach its maximum for some $\alpha < \alpha^*_{fc}(t)$, whereas the $N \to \infty$

curve within Figure 7(b) should reach its maximum for $\alpha > \alpha^*_{fc}(t)$. The figure clearly illustrates this result. This implies that, for travel costs below the threshold, a firm can realize maximal gains from preset products even at moderate levels of the brand-image effects (specifically, with $\alpha < \alpha^*_{fc}(t)$).

4. A More General Dilution Cost

In the previous two sections, we study the firm's product portfolio and pricing problems under the assumption that customers incur a dilution cost based on the distance between their ideal products and the endpoints of the product portfolio. Incorporating the brand-image effect has led to the understanding that a manager should reduce the prices of products nearest the taste extremes (to stimulate consumer "travel"), and use mass customization specifically to cater to consumer with more central tastes. We also show that augmenting the uniformly priced mass customization range with preset outlying designs improve the firm's revenues. The goal of this section is to demonstrate the robustness of these managerial insights by considering a more general functional form of the dilution cost function. We denote the dilution cost function as $C(z,X)$, where $z \in [0,1]$ denotes a consumer's ideal taste, and X is the firm's product portfolio. Given the generalized form $C(z,X)$, in the next subsection, we analyze the optimal portfolio price structure. Subsequently, in the following subsection, we study the firm's optimal product portfolio. We will establish that, optimally, the firm continues to price such that the customization range is strictly smaller than the resulting market coverage. Finally, in analogous fashion to our approach in the prior section, we assess the potential value to be gained from establishing fixed guardrail products as well as other discretized (preset) products.

As we illustrated in Figure 1, our original dilution cost function is convex, continuous, and symmetric around the portfolio midpoint. Analogously, we assume the function $C(z,X)$ is also continuous, convex, and symmetric, as can be seen in Figure 8.

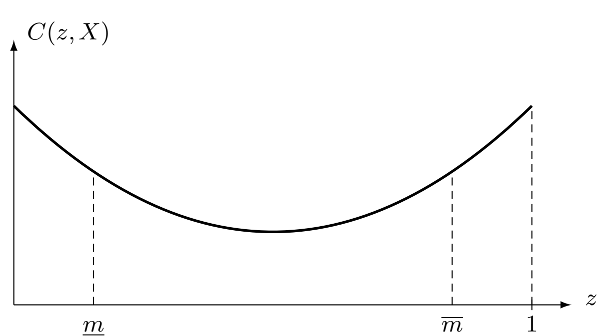


Figure 8 Illustration of the generalized convex cost $C(z,X)$.

4.1. Optimal Pricing Structure

Under the more general cost function $C(z,X)$, we first show that our earlier findings on optimal pricing continue to hold. In particular, Proposition 6 shows that the optimal price function should be either a linear function increasing towards middle of the product portfolio or equal to $V - C(z,X)$, which leaves zero surplus to the customers purchasing at that price.

PROPOSITION 6. *Let $X^\star \equiv [\underline{m}^\star, \overline{m}^\star]$ be the optimal product portfolio and $B^\star \equiv [\underline{b}^\star, \overline{b}^\star]$ be the optimal market coverage when the dilution cost is $C(z,X^\star)$. Then, the optimal price function $p^\star$ is*

$$p^\star(x) = \min\{p^\star(\underline{m}^\star) + t(x - \underline{m}^\star), V - C(x,X^\star), p^\star(\overline{m}^\star) + t(\overline{m}^\star - x)\} \text{ for all } x \in X^\star,$$

where $p^\star(\underline{m}^\star) = V - C(\underline{b}^\star, X^\star) - t(\underline{m}^\star - \underline{b}^\star)$ and $p^\star(\overline{m}^\star) = V - C(\overline{b}^\star, X^\star) - t(\overline{b}^\star - \overline{m}^\star)$.

Furthermore, customers' most preferred products are $b^\star(z,p^\star) = \begin{cases} \underline{m}^\star & \text{if } z \le \underline{m}^\star \\ z & \text{if } \underline{m}^\star \le z \le \overline{m}^\star \\ \overline{m}^\star & \text{if } z \ge \overline{m}^\star. \end{cases}$

Similar to our previous results, there are two main drivers of this simple structure: First, if the price is higher than the proposed linear price for a range products $x \in (x_1, x_2) \in X^\star$, the firm has an incentive to move its prices closer to the proposed linear price function, and thus avoid consumers located in (x_1, x_2) from purchasing the lowest-priced product in (x_1, x_2). Second, the firm does not find it profitable to charge less than the proposed price function because all customers inside the firm's product portfolio purchase their ideal products when the firm employs the proposed price function. It is also important to note, as leveraged in the proof of Proposition 6, that the convexity of the dilution cost function plays a crucial role in establishing that customers inside the product portfolio purchase their ideal products.

The above proposition derives a simple optimal price structure akin to that in Proposition 2. However, in contrast with Section 2, this simple structure is not sufficient to yield an analytically tractable profit function, due to the implicit nature of the cost function $C(z,X)$. Furthermore, Proposition 6 does not prove the symmetry of the optimal product portfolio $X^\star$ inside the optimal market coverage $B^\star$. Lacking a guarantee of symmetry in $X^\star$, we cannot simplify the profit function by reducing it to only two variables. Due to these complications, we will focus our attention to a specific functional form for the dilution cost in order to check the robustness of our two remaining major results.

4.2. The Optimality of Serving Customers Outside the Mass Customization Region

In the previous subsection, we prove that the firm's optimal pricing function follows a similar structure as we obtain under our original dilution cost. We now analyze the firm's pricing and portfolio selection problem in more detail to study whether the firm's market coverage and the mass customization region differ. To this end, we assume that the dilution cost that a customer located at z incurs is

$$\hat{C}(z,X) = \alpha\left[(\underline{m}-z)^2 + (\overline{m}-z)^2\right]$$

for any given product portfolio $X = [\underline{m}, \overline{m}]$. Note that the above cost function is convex and symmetric around $z = (\overline{m} + \underline{m})/2$. As we mentioned before, we focus on a specific functional form for the dilution cost function to ensure that the firm's problem is analytically tractable.

Having this explicit functional form for $\hat{C}(z,X)$, we can prove the symmetry of the optimal portfolio inside the optimal market coverage. We formally present this result in Proposition 7. Its proof relies heavily on the functional form of $\hat{C}(z,X)$ and thus cannot be replicated for an implicit cost function.

PROPOSITION 7. *Let $X^\star \equiv [\underline{m}^\star, \overline{m}^\star]$ be the optimal product portfolio and $B^\star \equiv [\underline{b}^\star, \overline{b}^\star]$ be the optimal market coverage when the dilution cost is $\hat{C}(z, X^\star)$. Then, we have that*

$$\underline{m}^\star - \underline{b}^\star = \overline{b}^\star - \overline{m}^\star.$$

Similar to our previous analysis, the firm's profit becomes a function of the guardrail products and the endpoints of the market coverage, based on the simple structure proven in Proposition 6. Furthermore, due to the symmetry of the firm's optimal portfolio proven in Proposition 7, we can without loss of generality assume that the midpoint of the product portfolio is 1/2. Then, we can rewrite the firm's revenue as a function of $\underline{m}$ and $\underline{b}$ as follows:

$$\Pi_C(\underline{b},\underline{m}) = \begin{cases} 2\left[(\underline{m}-\underline{b})p^\star(\underline{m}) + \int_{\underline{m}}^{f^\star(\underline{b},\underline{m})} p^\star(\underline{m}) + t(z-\underline{m})dz + \int_{f^\star(\underline{b},\underline{m})}^{1/2} V - \hat{C}(z,X)dz\right] & \text{if } f^\star(\underline{b},\underline{m}) < 1/2, \\ 2\left[(\underline{m}-\underline{b})p^\star(\underline{m}) + \int_{\underline{m}}^{1/2} p^\star(\underline{m}) + t(z-\underline{m})dz\right] & \text{if } f^\star(\underline{b},\underline{m}) \geq 1/2, \end{cases} \tag{5}$$

where the price function, $p^\star(\underline{m})$, is the function described in Proposition 6 and

$$f^\star(\underline{b},\underline{m}) \equiv \max\{z : V - \hat{C}(z,X) = p^\star(\underline{m}) + t(z-\underline{m})\}.$$

As the cost function $\hat{C}(z,X)$ is convex, $p^\star(\underline{m}) + t(z-\underline{m})$ is less than $V - \hat{C}(z,X)$ for all $x \in [\underline{m}, f^\star(\underline{b},\underline{m})]$. Therefore, the optimal price becomes a linear function for all $x \leq \min\{1/2, f^\star(\underline{b},\underline{m})\}$ and equal to $V - \hat{C}(z,X)$ for all $x \in (\min\{1/2, f^\star(\underline{b},\underline{m})\}, 1/2]$. In fact, the optimal price is linear for all products when $f^\star(\underline{b},\underline{m}) \geq 1/2$ as we illustrate in Figure 9.

Once we write the firm's profit as above, the firm's problem becomes solving $max_{\underline{b},\underline{m}}\Pi_C(\underline{b},\underline{m})$. As one might expect, firm's problem under the cost function $\hat{C}(z,X)$ is more complicated than

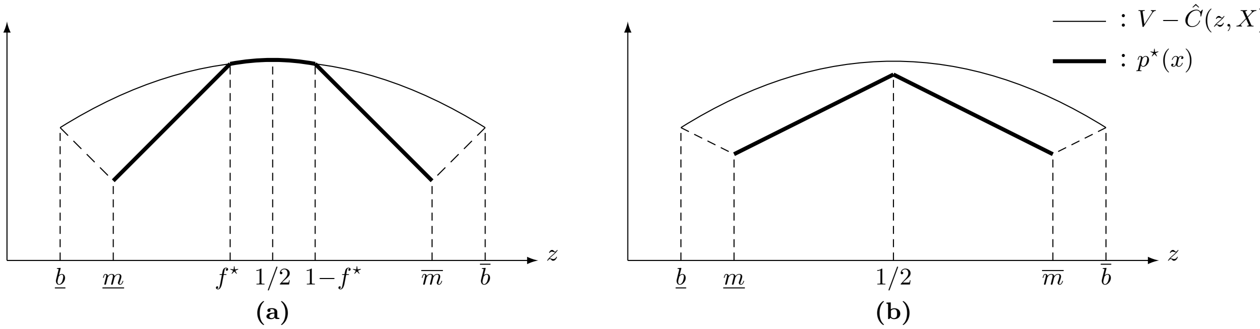


Figure 9 **Illustration of the optimal price function $p^\star(x)$ when: (a) $f^\star(\underline{b},\underline{m})<1/2$, and (b) $f^\star(\underline{b},\underline{m})\geq 1/2$.**

the optimization problem in Section 2. Thus, it is not possible to obtain closed form expressions similar to our previous findings. However, we can still show that the firm optimally sets a mass customization region that is strictly smaller than the market coverage, and thus the firm serves customers outside its product portfolio, except for a very specific case. Similar to our findings in Theorem 2, the firm can reduce brand-image related costs by limiting its mass customization region, and thus charging higher prices to customers located closer to the middle of the taste spectrum. Although such a move comes at the expense of lower prices for customers with extreme tastes, the gains from shrinking the mass customization region outweigh the losses at $\underline{m}=\underline{b}$ unless the market coverage endpoint $\underline{b}$ is set such that $f^*(\underline{b},\underline{b})=1/2$. In that particular case of $f^*(\underline{b},\underline{b})=1/2$ (which we show below requires a specific value of α to occur), the mass customization range coincides with the market coverage. Otherwise, it is optimal to induce some customer "travel," with the mass-customization range being strictly less than the range of purchasing customers. We formally present this result in the following theorem.

Theorem 3. *Let $X^\star\equiv[\underline{m}^\star,\overline{m}^\star]$ be the optimal product portfolio, and $B^\star\equiv[\underline{b}^\star,\overline{b}^\star]$ be the optimal set of consumers to be served when the dilution cost is $\hat{C}(z,X^\star)$. If $\alpha\neq\max\left\{t,\frac{5t^2}{2V}\right\}$, then we have that*

$$\overline{m}^\star-\underline{m}^\star<\overline{b}^\star-\underline{b}^\star.$$

Hence, the firm optimally serves customers outside its product portfolio, leaving non-zero surplus to customers buying the guardrail products. Furthermore, if $\alpha=\max\left\{t,\frac{5t^2}{2V}\right\}$, then we have that

$$\overline{m}^\star-\underline{m}^\star=\overline{b}^\star-\underline{b}^\star.$$

Theorem 3 shows that the firm optimally serves customers who are outside its product portfolio via guardrail products under the cost function $\hat{C}(z,X)$. This result establishes that our findings about the firm's optimal product portfolio are not driven by the dilution cost function we consider in Section 2. To shed more light on the implications of Theorem 3, we carry out a numerical study to illustrate the portion of customers served by the guardrail products, i.e., $1-(1-2\underline{m}^\star)/(1-2\underline{b}^\star)$, as the dilution cost parameter α changes. In this numerical study, we normalize the reward V to 1 and consider three different levels of travel cost t: 0.2, 0.5, and 0.8. Figure 10 presents the optimal percentage of customers purchasing the guardrail products, for values of the dilution cost parameter α ranging from 0 to 2. These results show that firm optimally serves a significant percentage of its customers via the guardrails. In fact, we see that in some cases corresponding to larger values of α, the majority of purchasing customers optimally opt to purchase the guardrail products.

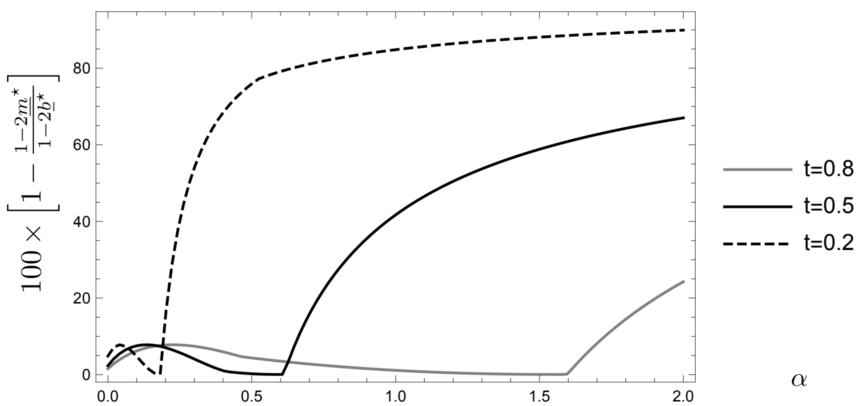


Figure 10 **Percentage of customers purchasing a guardrail product (relative to the total market coverage) when the dilution cost is $\hat{C}(z,X)$, $V=1$, and $t \in \{0.2, 0.5, 0.8\}$.**

4.3. Presets Model and Comparison

Having established that the optimal pricing and portfolio structures are not limited to the cost function we consider in Section 2, we now assess, under a family of convex cost functions, the relative value of establishing outlying discrete products in addition to a range of mass customized products.

We will again refer to these discrete outlying products as "preset" designs, including those designs defining the guardrail locations $\underline{m}$ and $\overline{m}$. As in Section 3, we propose augmenting the mass customization region via non-customized products as a practical implementation of the optimal pricing structure—which would entail the difficulty of communicating a continuum (i.e., infinitely many) prices to customers.

At the risk of some abuse in nomenclature, we again refer to this portfolio structure as the *Presets* model, but here the associated pricing follows from the function $p^\star(x)$ in Proposition 6. Here, we assume the firm will add $N \geq 1$ preset products on each side of the mass customization region, where the customized products are offered at a fixed price. We again denote the location of the preset product-i by x_i for any $1 \leq i \leq 2N$, and the mass customization region by the interval $[\underline{f}, \overline{f}]$. We suppose that the product portfolio is symmetric around the market midpoint and that the preset products are spread uniformly between the two guardrail products (x_1 and x_{2N}) and the mass customization region. Therefore, the product portfolio of the firm is $\{x_1, x_1 + \zeta, \ldots, x_1 + (N-1)\zeta\} \cup [\underline{f}, 1 - \underline{f}] \cup \{1 - x_1 - (N-1)\zeta, \ldots, 1 - x_1 - \zeta, 1 - x_1\}$ where $\zeta \equiv (\underline{f} - x_1)/N$.

Given such a product portfolio, there are three decisions to optimize: the boundary of the market coverage $\underline{b}$, the location of the first preset product x_1, and the boundary of the mass customization region $\underline{f}$. Once the market boundary and the first preset design are decided, we can use Proposition 6 to obtain the prices of the preset products and the fixed price for the customized products. Namely, the price of the preset product-i is $p^\star(x_i)$, and $p^\star(\underline{f})$ applies within the customization range. Then, the firm's problem is to maximize:

$$\tilde{\Pi}^\star(\alpha, t, N) \equiv \max_{0 \leq \underline{b} \leq x_1 \leq \underline{f} \leq 1/2} \; 2\left[(x_1 - \underline{b})p^\star(x_1) + (\underline{f} - x_1)\left(\sum_{i=1}^{N} p^\star(x_i)/N\right) + (1/2 - \underline{f})p^\star(\underline{f})\right]$$

By computing $\tilde{\Pi}^\star(\alpha, t, N)$, and by also evaluating the performance of the uniform pricing regime under the dilution cost $C(z, X)$, which we denote as $\Pi_u(\alpha, t)$, we can again assess the gains from offering preset products, as we did in the prior section. (But now for a more general convex $C(z, X)$, in contrast with the earlier piecewise linear function.) Denoting the incremental gain from presets in this setting as $\Delta_C(\alpha, t, N)$, we have:

$$\Delta_C(\alpha, t, N) = 100 \times \left(\frac{\tilde{\Pi}^\star(\alpha, t, N)}{\Pi_u(\alpha, t)} - 1\right).$$

Under *Uniform Pricing* we can without loss of generality assume the firm's product portfolio is symmetric around the market midpoint. Moreover, given any product portfolio $X = [\underline{m}, 1 - \underline{m}]$ and market coverage $B = [\underline{b}, 1 - \underline{b}]$, the optimal resulting price is $p^\star(\underline{m})$, because that price level implies zero surplus for customers at the ends of the market coverage range. Thus, we have that

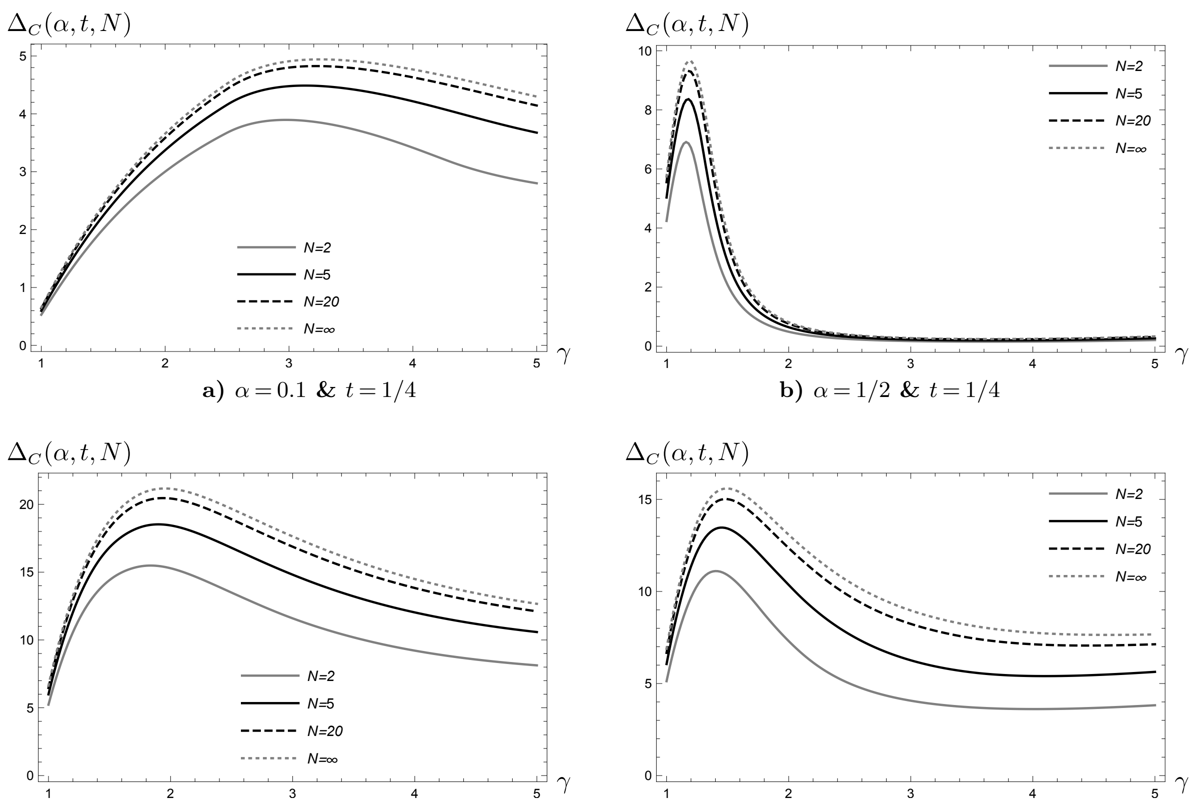
$$\Pi_u(\alpha, t) = \max_{\underline{b}, \underline{m}} [1 - 2\underline{b}] p^\star(\underline{m}).$$

As we consider a heuristic product portfolio to compute $\tilde{\Pi}^\star(\alpha, t, N)$, we expect that the optimal product portfolio under the cost function $C(z, X)$ would be different. Furthermore, the optimal portfolio would lead to a higher (or at least an equal) revenue for the firm. On the other hand, we optimally solve the *Uniform Pricing* model. Hence, it is important to note that the above profit-gap measure $\Delta_C(\alpha, t, N)$, while stemming from our *Presets* model analysis, also provides a lower bound on the incremental revenue gains (relative to the *Uniform Pricing* policy) from offering the optimal mass customization region with differential pricing.

Due to the general form of the dilution cost $C(z,X)$, optimizing $\tilde{\Pi}^{\star}(\alpha,t,N)$ for the firm's product portfolio and pricing problem in the *Presets* model is analytically intractable. Thus, we turn to numerical analysis to assess the benefits from augmenting customized products with non-customized ones. We also focus on a family of dilution cost functions with a shape parameter γ that controls the curvature of the convex function. Specifically, we suppose that the dilution cost is

$$C_{\gamma}(z,X) \equiv \alpha\left(|\underline{m}-z|^{\gamma}+|\overline{m}-z|^{\gamma}\right).$$

Notice that if $\gamma=1$ then this function reduces to the original form of the dilution cost. We normalize the reward V to 1 and consider two different levels of travel cost t and two different levels of dilution cost parameter α. We present the gains from offering non-customized products when the firm supplements its customized products with 4, 10, and 40 non-customized products in Figure 11.



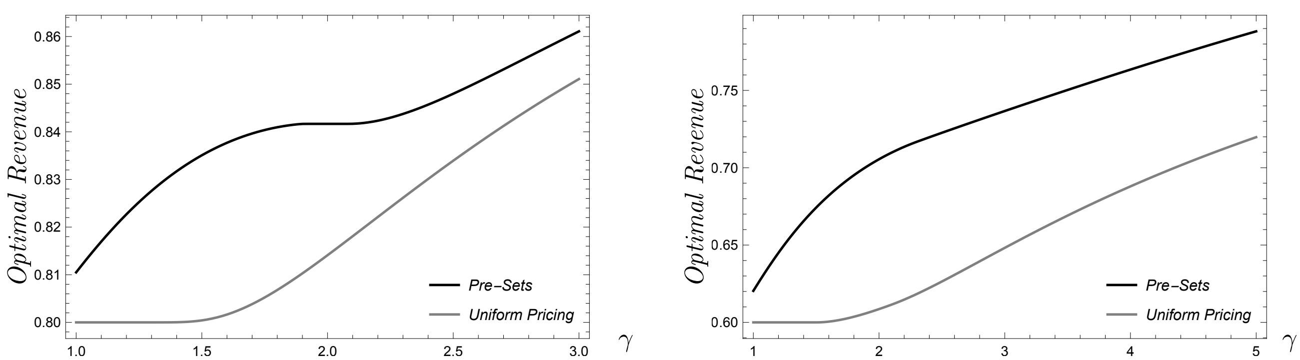
a) $\alpha=0.1$ **&** $t=1/4$ **b)** $\alpha=1/2$ **&** $t=1/4$

c) $\alpha=1/2$ **&** $t=3/4$ **d)** $\alpha=1$ **&** $t=3/4$

Figure 11 **The relative improvement in revenue from the Uniform Pricing model to the Presets model when the dilution cost is** $C_{\gamma}(z,X)$ **and** $V=1$**.**

As the results within Figure 11 illustrate, the firm obtains considerable benefits from additional non-customized products under the more general dilution cost function. In fact, our numerical study reveals that the firm's benefits from non-customized products can increase as the shape parameter γ increases. Furthermore, the above graph illustrates that the firm's profit under our heuristic approach quickly converges, as $N \to \infty$ to the point where the firm offers a connected product line.

We also observe that the benefits from non-customized products do not *always* increase as the dilution cost function becomes more convex. The main driver of this non-monotone behavior is that for low levels of γ, the firm's revenue in the *Uniform Pricing* model increases in γ at a slower rate compared to the *Presets* model because the firm reacts to changes in the curvature of the dilution cost function more actively when it uses preset products. Hence, we observe that the benefits from offering non-customized products increases when the curvature of the dilution cost is low. In fact, the firm's revenue may change minimally over a range of γ when it is limited to uniform pricing, as Figure 12 illustrates. As the dilution cost function becomes more convex, the firm begins making significant updates to its optimal decisions even when it is limited to uniform pricing. Therefore, the incremental benefits from offering presets declines once γ exceeds a critical level.



a) $\alpha = 0.2$ **&** $t = 1/4$ **b)** $\alpha = 0.4$ **&** $t = 3/4$

Figure 12 The firm's optimal revenue in the Uniform Pricing and the Presets models when the dilution cost is $C_\gamma(z, X)$ **and** $V = 1$**.**

Our numerical study also demonstrates patterns similar to Section 3. As one may expect, the benefits from augmenting the customized products with presets increases as the number of non-customized products increases. Furthermore, an increase in the travel cost also improves the firm's gains from non-customized products. Finally, similar to our previous findings, the dilution cost parameter α has a non-monotone impact on the benefits from offering non-customized products.

5. Conclusion

Within the original Hotelling (1929) horizontal differentiation setting, eliminating all mismatches between the firm's products and consumers' unique tastes is the theoretical profit-maximizing ideal. Lancaster (1990) suggested in his review of product variety literature that: "If there are no economies of scale associated with individual product variants... then it is optimal to produce to everyone's chosen specification." Mass customization (the ability to tailor designs to precisely match consumers' tastes, at little-to-no added cost) have therefore been viewed as the Holy Grail of horizontal differentiation (Zipkin 2001).

The classic Hotelling (1929) horizontal differentiation framework considers the taste mismatch only between a customer's taste and their *purchased* product. In this paper, we have extended that framework by augmenting the consumer utility model to consider the potential for consumers' (ideal) taste mismatches between both their purchased products and the firm's (full) product line. Studies of consumer behavior provide evidence that consumers desire not only a taste "fit" with the product they purchase, but also with the firm's product line as a whole. For example, in an experiment using a Thai restaurant menu, Berger et al. (2007) considered expanding the menu variety to exhibit either "compatible variety (the same options plus five other Thai food options)," or "incompatible variety (i.e., a few Thai options plus five non-Thai options, such as egg rolls)." In the former case, they found that "participants perceived the brand more favorably...." In a similar experiment, they provided consumers with information about different bicycle brands, and found that when the expanded product line variety was broadened (by adding mountain bikes to a road bike manufacturer's lineup), brand perceptions decreased. Consistent with this experimental evidence, we see that in practice firms offer designs that are carefully vetted (e.g., employing professional designers and focus groups). As an example, Nike's new FlyKnit manufacturing process, which originated from sweater-knitting with complex designs, could potentially allow customers to design intricate custom patterns at the thread level. We were informed that Nike has no plans to offer such an unrestricted degree of customization, even if technologically feasible. The expressed concern related to the sentiment that consumers don't simply purchase a pair of sneakers, they buy the Nike brand as well. In sum, a plethora of unfocused designs risks brand-dilution losses.

We thus see that mass customization may be a double-edge sword. The adage "more variety is strictly preferred to less" would seem to imply that an unlimited variety of products should be optimal for a firm with mass customization capability. Yet, the evidence suggests that variety extensions should be weighed carefully, especially given taste and image-dilution concerns (e.g., for branded products). Catering to a diverse set of tastes may result in a firm having an unfocused image which (some) consumers may identify less closely with. This may explain why, even though viable applications of mass customization technology exist (e.g., Zazzle), successful applications to established firms with strong brands are few (Piller 2004). In the words of Levy and Rook (1999), "It is rarely possible for a product or brand to be all things to all people."

In our model, given that brand-dilution effects are considered endogenously, interdependent with the firm's product line decisions, there are conditions under which we prove that the firm optimally restricts variety–even when costless (from a production standpoint). We also find that the firm should optimally implement a differential pricing scheme for those product varieties at the outer extremes of its product line, catering to those consumers with more extreme tastes. More specifically, the optimal pricing entails charging a higher price within the central mass customized region, with continuously and linearly declining prices at the extremes of the product space. We also establish that, relative to using a simple uniform pricing scheme, implementing differential pricing leads to a reduction in the optimal extent of the mass customization product range. Given that a continuum of prices will likely not be practical in application, we also consider the more pragmatic approach of a set of fixed (non-customized) products to serve customers at the taste extremes, with a corresponding set of (lower) fixed prices. We also derive a profit bound on the performance of this heuristic solution, and prove it performs close to optimal. We have also supported the robustness of our results by considering two convex forms for the function characterizing the dilution cost effect: we begin with a simple piecewise-linear structure, but then subsequently show our key insights continue to hold for a more general convex functional form.

To help frame these theoretical results with a practical application, let us revisit Denali Jacket (by The North Face) example from the introduction in light of our key findings. Our results suggest that the mass customized region should be restricted, and augmented via lower-priced fixed designs to cater to non-mainstream consumer tastes. Arguably, The North Face is following precisely such a strategy, as its customizable fleece patterns include only solid color patterns, while you can purchase preset (non-customizable) Denali jackets with patterned fabrics (e.g., camouflage). Moreover, our results suggest that the preset designs catering to outlier tastes should be priced lower, which is also reflected in the current prices charged by The North Face for its custom versus non-custom Denali Jacket versions. While, naturally, cost considerations and competition are alternative forces that can drive such price reductions, we establish this result purely from a price-discrimination perspective for a single firm.

References

Aaker, David A. 1992. The value of brand equity. *Journal of Business Strategy* **13**(4) 27–32. doi:10.1108/eb039503.

Aaker, J. L. 1997. Dimensions of brand personality. *Journal of Marketing Research* **34**(3) 347–356.

Alptekinoğlu, A., C. J. Corbett. 2008. Mass customization vs. mass production: Variety and price competition. *Manufacturing & Service Operations Management* **10**(2) 204–217.

Alptekinoğlu, A., C. J. Corbett. 2010. Leadtime-variety tradeoff in product differentiation. *Manufacturing & Service Operations Management* **12**(4) 569–582.

Belk, Russell W. 1988. Possessions and the extended self. *Journal of consumer research* **15**(2) 139–168.

Berger, J., M. Draganska, I. Simonson. 2007. The influence of product variety on brand perception and choice. *Marketing Science* **26**(4) 460–472.

Berger, J., C. Heath. 2007. Where consumers diverge from others: Identity signaling and product domains. *Journal of Consumer Research* **34**(2) 121–134.

Biel, A. L. 1993. Converting image into equity. *Brand equity and advertising: Advertising's role in building strong brands* 67–82.

Braun, Ottmar L, Robert A Wicklund. 1989. Psychological antecedents of conspicuous consumption. *Journal of Economic psychology* **10**(2) 161–187.

de Groote, X. 1994. Flexibility and marketing/manufacturing coordination. *International Journal of Production Economics* **36**(2) 153–167.

Dewan, R., B. Jing, A. Seidmann. 2003. Product customization and price competition on the internet. *Management Science* **49**(8) 1055–1070.

Diamantopoulos, A., G. Smith, I. Grime. 2005. The impact of brand extensions on brand personality: experimental evidence. *European Journal of Marketing* **39**(1/2) 129–149.

Dobson, G., C. A. Yano. 2002. Product offering, pricing, and make-to-stock/make-to-order decisions with shared capacity. *Production and Operations Management* **11**(3) 293–312.

Dolich, I. J. 1969. Congruence relationships between self images and product brands. *Journal of Marketing Research* **6**(1) 80–84.

Farquhar, P. H. 1989. Managing brand equity. *Marketing research* **1**(3).

Fournier, Susan. 1998. Consumers and their brands: Developing relationship theory in consumer research. *Journal of consumer research* **24**(4) 343–373.

Gaur, V., D. Honhon. 2006. Assortment planning and inventory decisions under a locational choice model. *Management Science* **52**(10) 1528–1543.

Graeff, Timothy R. 1996. Image congruence effects on product evaluations: The role of self-monitoring and public/private consumption. *Psychology & Marketing* **13**(5) 481–499.

Hotelling, H. 1929. Stability in competition. *The Economic Journal* **39**(153) 41–57.

Huang, T., C. Liang, J. Wang. 2017. The value of “bespoke”: Demand learning, preference learning, and customer behavior. *Management Science* Forthcoming.

Jiang, K., H. L. Lee, R. W. Seifert. 2006. Satisfying customer preferences via mass customization and mass production. *IIE Transactions* **38**(1) 25–38.

Kekre, S., K. Srinivasan. 1990. Broader product line: A necessity to achieve success? *Management Science* **36**(10) 1216–1232.

Keller, K. L. 1993. Conceptualizing, measuring, and managing customer-based brand equity. *The Journal of Marketing* **57**(1) 1–22.

Keller, K. L. 2010. *The new branding imperatives*. Cambridge, MA: Marketing Science Institute.

Kleine, S. S., R. E. Kleine, C. T. Allen. 1995. How is a possession “me” or “not me”? characterizing types and an antecedent of material possession attachment. *Journal of Consumer Research* **22**(3) 327–343.

Kressmann, F., M. J. Sirgy, A. Herrmann, F. Huber, S. Huber, D. Lee. 2006. Direct and indirect effects of self-image congruence on brand loyalty. *Journal of Business Research* **59**(9) 955–964.

Lancaster, K. 1990. The economics of product variety: A survey. *Marketing Science* **9**(3) 189–206.

Landon, E. L. 1974. Self concept, ideal self concept, and consumer purchase intentions. *Journal of consumer research* **1**(2) 44–51.

Levy, S. J., D. W. Rook. 1999. *Brands, Consumers, Symbols and Research: Sidney J Levy on Marketing*. Sage Publications.

Ligas, M. 2000. People, products, and pursuits: Exploring the relationship between consumer goals and product meanings. *Psychology & Marketing* **17**(11) 983–1003.

Malhotra, N. K. 1988. Self concept and product choice: An integrated perspective. *Journal of Economic Psychology* **9**(1) 1–28.

Mendelson, H., A. K. Parlaktürk. 2008. Product-line competition: Customization vs. proliferation. *Management Science* **54**(12) 2039–2053.

Moorthy, K. S. 1984. Market segmentation, self-selection, and product line design. *Marketing Science* **3**(4) 288–307.

Mussa, M., S. Rosen. 1978. Monopoly and product quality. *Journal of Economic theory* **18**(2) 301–317.

Onkvisit, S., J. Shaw. 1987. Self-concept and image congruence: Some research and managerial implications. *Journal of Consumer Marketing* **4**(1) 13–23.

Park, C. W., B. J. Jaworski, S. J. Maclnnis. 1986. Strategic brand concept-image management. *The Journal of Marketing* 135–145.

Piller, F. T. 2004. Mass customization: Reflections on the state of the concept. *International Journal of Flexible Manufacturing Systems* **16**(4) 313–334.

Plummer, J. T. 2000. How personality makes a difference. *Journal of advertising research* **40**(6) 79–83.

Randall, T., K. Ulrich. 2001. Product variety, supply chain structure, and firm performance: Analysis of the us bicycle industry. *Management Science* **47**(12) 1588–1604.

Rangaswamy, A., Raymond R. B., T. A. Oliva. 1993. Brand equity and the extendibility of brand names. *International Journal of Research in marketing* **10**(1) 61–75.

Ross, I. 1971. Self-concept and brand preference. *The Journal of Business* **44**(1) 38–50.

Salop, S. C. 1979. Monopolistic competition with outside goods. *The Bell Journal of Economics* **10**(1) 141–156.

Simon, C. J., M. W. Sullivan. 1993. The measurement and determinants of brand equity: A financial approach. *Marketing science* **12**(1) 28–52.

Sirgy, M. J. 1982. Self-concept in consumer behavior: A critical review. *Journal of consumer research* **9**(3) 287–300.

Swait, J., T. Erdem, J. Louviere, C. Dubelaar. 1993. The equalization price: A measure of consumer-perceived brand equity. *International Journal of Research in Marketing* **10**(1) 23–45.

Syam, N. B., N. Kumar. 2006. On customized goods, standard goods, and competition. *Marketing Science* **25**(5) 525–537.

Syam, N. B., R. Ruan, J. D. Hess. 2005. Customized products: A competitive analysis. *Marketing Science* **24**(4) 569–584.

Thomson, M., D. J. MacInnis, C. W. Park. 2005. The ties that bind: Measuring the strength of consumers' emotional attachments to brands. *Journal of consumer psychology* **15**(1) 77–91.

Wicklund, R. A., P. M. Gollwitzer. 1981. Symbolic self-completion, attempted influence, and self-deprecation. *Basic and Applied Social Psychology* **2**(2) 89–114.

Yorkston, E. A., J. C. Nunes, S. Matta. 2010. The malleable brand: The role of implicit theories in evaluating brand extensions. *Journal of Marketing* **74**(1) 80–93.

Zipkin, P. 2001. The limits of mass customization. *Sloan Management Review* **42**(3) 81–87.

Appendix A: Proofs in Section 2

A.1. Proof of Proposition 1

For the Uniform Pricing case, we employ proof-by-contradiction to establish that the optimal market coverage B° and the range of mass customization $[\underline{m}^\circ, \overline{m}^\circ]$ coincide. To this end, suppose $B^o \neq [\underline{m}^\circ, \overline{m}^\circ]$. Then, we should have that $\underline{b}^\circ < \underline{m}^\circ \leq \overline{m}^\circ < \overline{b}^\circ$ where $\underline{b}^\circ = min(B^\circ)$ and $\overline{b}^\circ = max(B^\circ)$.

Now, letting $\varepsilon < \min\{\underline{m}^\circ - \underline{b}^\circ, \overline{b}^\circ - \overline{m}^\circ\}$, consider an alternative mass customization region $[\underline{m}', \overline{m}'] \equiv [\underline{m}^\circ - \varepsilon, \overline{m}^\circ + \varepsilon]$ with an alternative price function $p'(x) = p_f^o + t\varepsilon$ for all $x \in [\underline{m}', \overline{m}']$. Then, we have that

$$U(\underline{b}^\circ, \underline{m}^\circ - \varepsilon, p) = V - p_f^o - t\varepsilon - t(\underline{m}^\circ - \underline{b}^\circ - \varepsilon) - \alpha(\underline{m}^\circ - \underline{b}^\circ - \varepsilon + \overline{m}^\circ - \underline{b}^\circ + \varepsilon)$$

$$= U(\underline{b}^{\circ}, \underline{m}^{\circ}, p^{o}) \geq 0,$$

similarly we have that $U(\overline{b}^{\circ}, \overline{m}', p') = U(\overline{b}^{\circ}, \overline{m}^{\circ} - \varepsilon, p^{o})$. This means that the firm's market coverage under the alternative portfolio is at least as much as before while increasing its price. Therefore, the firm improves its revenues by using the alternative product portfolio since it charges a higher price. This contradicts with the optimality of $[\underline{m}^{\circ}, \overline{m}^{\circ}]$. Hence, we should have that $\underline{b}^{\circ} = \underline{m}^{\circ}$.

A.2. Proof of Theorem 1

By labelling the size of mass customization region $\overline{m} - \underline{m}$ as μ for any given $\overline{m} \leq \underline{m}$ and ignoring the constant λ, we can rewrite the firm's problem as:

$$\max_{\mu \leq 1} \ \pi(\mu) \equiv [V - \alpha\mu]\mu.$$

Note that $\pi'(\mu) = V - 2\alpha\mu$, so that $\pi'(\mu) \geq 0$ for any $\mu \leq V/(2\alpha)$ and $\pi'(\mu) < 0$ otherwise. Therefore, the optimal solution of the above problem is

$$\mu^{o} = \begin{cases} 1 & \textit{if } \alpha < V/2 \\ V/(2\alpha) & \textit{if } \alpha \geq V/2, \end{cases}$$

which leads to the optimal product portfolio stated in the theorem.

A.3. Supplementary Results for the Proof of Proposition 2

Lemma 3. *For any given product portfolio $X = [\underline{m}, \overline{m}]$ and the market coverage $B = [\underline{b}, \overline{b}]$, consider the linear price function*

$$p_L(x, X, B) = \min\{V - \alpha(\overline{m} - \underline{m}), p_\ell(x), p_r(x)\},$$

where $p_\ell(x) = V - \underline{u} - \alpha(\overline{m} - \underline{m}) + t(x - \underline{m})$, $p_r(x) = V - \overline{u} - \alpha(\overline{m} - \underline{m}) + t(\overline{m} - x)$, and $\underline{u}$ and $\overline{u}$ are chosen such that the customers at the market coverage boundaries obtain zero utility from the guardrail products, i.e, $U(\underline{b}, \underline{m}, p_\ell) = 0$ and $U(\overline{b}, \overline{m}, p_r) = 0$.

Under the price function $p_L(x, X, B)$, all customers whose ideal tastes are inside the product portfolio X will purchase their ideal products. Furthermore, the market coverage is B under $p_L(x, X, B)$.

Proof: Let $\underline{f} = \{x \in X : p_\ell(x) = V - \alpha(\overline{m} - \underline{m})\}$ and $\overline{f} = \{x \in X : p_r(x) = V - \alpha(\overline{m} - \underline{m})\}$. Note that it is possible to have $\underline{f} > \overline{f}$ but in this prove, we focus on the case where $\underline{f} \leq \overline{f}$. The proof is almost the same when $\underline{f} > \overline{f}$, in fact with less sub-cases to consider.

To prove our claim about the purchasing behavior of customers in X, we show that the utility of the customer at z from the product at z is greater than her utility from any other products, i.e. $U(z, z, \hat{p}) \geq U(z, x, \hat{p})$ for any $x \in X$.

First, note that $U(z, x, p_L) \leq 0$ for any $z \in [\underline{f}, \overline{f}]$ and $x \in X$. Moreover, any customer in $[\underline{m}, \overline{m}]$ who buys a product matching their ideal taste gains zero utility under the pricing regime $p_L(x, X, B)$.

Furthermore, if we consider customers in $[\underline{m}, \underline{f}]$, we have that

$$U(z, z, p_L) = V - p_\ell(z) - \alpha(\overline{m} - \underline{m})$$

$$= \underline{u} - t(z-\underline{m}) \geq \underline{u} - t(x-\underline{m}) - t(z-x) = U(z,x,p_L) \text{ for any } x < z,$$
$$U(z,z,p_L) > \underline{u} - t(x-\underline{m}) - t(x-z) = U(z,x,p_L) \text{ for any } z < x \leq \underline{f}.$$

Therefore, under $p_L(x,X,B)$, customers in $[\underline{m},\underline{f}]$ do not prefer any product in the differential pricing region other than their ideal product. Furthermore, they do not prefer any product in the fixed price region because those products give negative utility to customers in $[\underline{m},\underline{f}]$. Thus, all customers whose ideal products are in $[\underline{m},\underline{f}]$ buy their ideal products.

We can similarly show that all customers whose ideal products are in $[\overline{f},\overline{m}]$ also buy their ideal products.

Finally, our claim about the market coverage is a direct implication of the choices of $\underline{u}$ and $\overline{u}$.

Lemma 4. *Let $X^* \equiv [\underline{m}^*, \overline{m}^*]$ be the optimal product portfolio, $p^*(x)$ be the optimal price function, and $b^*(z,p^*)$ be the customers' most preferred products in the Differential Pricing Model. Then, we have that $b^*(z,p^*) = \underline{m}^*$ for all $z < \underline{m}^*$ and $b^*(z,p^*) = \overline{m}^*$ for all $z > \overline{m}^*$. In other words, the customers outside product portfolio should purchase a guardrail product if they make a purchase under the optimal portfolio and the optimal price function.*

Proof: We prove our claim by contradiction. Therefore, we suppose $b^*(z,p^*) = x' > \underline{m}^*$ for some $z < \underline{m}^*$. Then, we should have that all $b^*(z,p^*) = x'$ for all $z < x'$ (even for customers whose ideal tastes are $z \in [\underline{m}^*, x']$). This also implies that $p^*(x) > p^*(x') + t(x'-x)$ for $x \in [\underline{m}^*, x']$. Now, consider an alternative price function $p'(x)$ such that $p'(x) = p^*(x') + t(x'-x)$ for $x \in [\underline{m}^*, x']$ and $p'(x) = p^*(x)$ otherwise. Under this alternative price, all customers outside X^* buy the guardrail product at $\underline{m}^*$, and customers whose ideal tastes are in $z \in [\underline{m}^*, x']$ buy their ideal product (proof of this is very similar to the proof of Lemma 3). The market coverage stays the same by the construction of the price function $p'(x)$. Thus, the alternative price function improves the revenues of the firm, which contradicts with the optimality of $p^*(x)$.

A.4. Proof of Proposition 2

For any given optimal portfolio X^* and the optimal market coverage B^*, consider the price function $p_L(x,X^*,B^*)$, as described in Lemma 3.

Linear price:

We first prove that $p^*(x)$ must be equal to $p_L(x,X^*,B^*)$. Note that market coverages under both price functions are the same by the construction of $p_L(x,X^*,B^*)$.

Under the optimal price function $p^*(x)$, customers should pay less than $V - \alpha(\overline{m}^* - \underline{m}^*)$ because otherwise their utility would be negative. Moreover, customers located inside X^* should not pay more than $p^*(\underline{m}) + t(z - \underline{m}^*)$ because otherwise they can improve their utility by purchasing the guardrail product at $\underline{m}^*$. Similarly, they should not pay more than $p^*(\overline{m}) + t(\overline{m}^* - z)$. Therefore, for any $z \in X^*$, we should have that

$$p^*(b^*(z,p^*)) \leq \min\{V - \alpha(\overline{m}^* - \underline{m}^*), p^*(\underline{m}) + t(z - \underline{m}^*), p^*(\overline{m}) + t(\overline{m}^* - z)\} \leq p_L(z,X^*,B^*).$$

The last inequality holds because $p_L(x,X^*,B^*)$ leaves zero surplus for the customers at the market boundaries whereas $p^*(x)$ might leave non-zero surplus to those customers. Therefore, we should have that $p_L(\underline{m}^*,X^*,B^*) \geq p^*(\underline{m})$ and $p_L(\overline{m}^*,X^*,B^*) \geq p^*(\overline{m})$.

By Lemma 3, we have that all customers whose ideal tastes are in $[\underline{m}^*, \overline{m}^*]$ will purchase their ideal products under $p_L(x, X^*, B^*)$. Furthermore, Lemma 4 shows that customers outside X^* purchase the guardrail products under $p^*(x)$. Similarly, customers outside X^* purchase the guardrail products under $p_L(x, X^*, B^*)$ by construction. Then, as the market coverages are the same under both $p_L(x, X^*, B^*)$ and $p^*(x)$, the above inequality implies that the firm's revenue under $p_L(x, X^*, B^*)$ is an upper bound for the optimal revenue, which proves that $p^*(x)$ must be equal to $p_L(x, X^*, B^*)$. Thus, we can write the optimal price function as:

$$p^*(x) = \min\{V - \alpha(\overline{m}^* - \underline{m}^*), p^*_\ell(x), p^*_r(x)\}, \tag{6}$$

where $p^*_\ell(x) = V - \underline{u}^* - \alpha(\overline{m}^* - \underline{m}^*) + t(x - \underline{m}^*)$, $p^*_r(x) = V - \overline{u}^* - \alpha(\overline{m}^* - \underline{m}^*) + t(\overline{m}^* - x)$, $\underline{u}^* = (t + 2\alpha)(\underline{m}^* - \underline{b}^*)$, $\overline{u}^* = (t + 2\alpha)(\overline{b}^* - \overline{m}^*)$, $\underline{b}^* = \min(B)$, and $\overline{b}^* = \max(B)$.

Existence of the uniform pricing region:

Let $\underline{f}^* = \{x : p^*_\ell(x) = V - \alpha(\overline{m}^* - \underline{m}^*)\}$ and $\overline{f}^* = \{x : p^*_r(x) = V - \alpha(\overline{m}^* - \underline{m}^*)\}$, where the price functions $p^*_\ell(x)$ and $p^*_r(x)$ are as defined in Equation 6.

We now show that $\underline{f}^* \leq \overline{f}^*$, proving the existence of the $[\underline{f}^*, \overline{f}^*]$ interval within which the firm charges a fixed price of $V - \alpha(\overline{m}^* - \underline{m}^*)$. This also implies that the utility of customers inside the $[\underline{f}^*, \overline{f}^*]$ interval is zero. We prove this claim by contradiction, so that we suppose $\underline{f}^* > \overline{f}^*$. Then, there must exist a product x^c at which the price functions $p^*_\ell(x^c)$ and $p^*_r(x^c)$ intersect at a level lower than $V - \alpha(\overline{m}^* - \underline{m}^*)$, i.e., $p^*_\ell(x^c) = p^*_r(x^c) < V - \alpha(\overline{m}^* - \underline{m}^*)$. Now, consider an alternative product portfolio $X^a = [\underline{m}^* - \varepsilon, \overline{m}^* + \varepsilon]$ with $0 < \varepsilon < [V - \alpha(\overline{m}^* - \underline{m}^*) - p^*(x^c)]/t$ and price function

$$p^a(x) = \min\{V - \alpha(\overline{m}^* - \underline{m}^* + 2\varepsilon), p^a_\ell(x), p^a_r(x)\},$$

where $p^a_\ell(x) = V - \underline{u}^a - \alpha(\overline{m}^* - \underline{m}^* + 2\varepsilon) + t(x - \underline{m}^* - \varepsilon)$, $p^*_r(x) = V - \overline{u}^a - \alpha(\overline{m}^* - \underline{m}^* + 2\varepsilon) + t(\overline{m}^* - x - \varepsilon)$, $\underline{u}^a = (t + 2\alpha)(\underline{m}^* - \underline{b}^* - \varepsilon)$, $\overline{u}^a = (t + 2\alpha)(\overline{b}^* - \overline{m}^* - \varepsilon)$.

The above price function is constructed to ensure that the market coverage is still B^* and all customers in X^a buys their ideal product (by Lemma 3). Furthermore, we have that all customers pay a higher price under $p^a(x)$ because $p^a(x) = p^*(x) + 2\varepsilon t$ for any $x \in [\underline{m}^*, \overline{m}^*]$, $p^a(\underline{m}^* - \varepsilon) = p^*(\underline{m}^*) + \varepsilon t$, and $p^a(\overline{m}^* - \varepsilon) = p^*(\overline{m}^*) + \varepsilon t$. Therefore, the alternative portfolio and the price function improves the revenues of the firm , which contradicts with the optimality of X^* and $p^*(x)$. Thus, we must have that $\underline{f}^* \leq \overline{f}^*$.

The structure of the product portfolio

Using the relations that $p^*_\ell(\underline{f}^*) = V - \alpha(\overline{m}^* - \underline{m}^*)$ and $\underline{u}^* = (t + 2\alpha)(\underline{m}^* - \underline{b}^*)$, we have that

$$\begin{aligned}
&V - \underline{u}^* - \alpha(\overline{m}^* - \underline{m}^*) + t(\underline{f}^* - \underline{m}^*) = V - \alpha(\overline{m}^* - \underline{m}^*) \\
&\Rightarrow \underline{u}^* = t(\underline{f}^* - \underline{m}^*) \\
&\Rightarrow \underline{m}^* - \underline{b}^* = \frac{t}{t + 2\alpha}(\underline{f}^* - \underline{m}^*).
\end{aligned}$$

Similarly, we have that $\overline{b}^* - \overline{m}^* = \frac{t}{t+2\alpha}(\overline{m}^* - \overline{m}^*)$.

Symmetry of the product portfolio:

Our proof ends by showing that the product portfolio is symmetric around the uniform pricing region $[\underline{f}^*, \overline{f}^*]$. We prove this claim by contradiction, so that we suppose on the contrary that X^* is not symmetric. Then, we should have that $p^*(\underline{m}^*) \neq p^*(\overline{m}^*)$. Without loss of generality, assume $p^*(\underline{m}^*) > p^*(\overline{m}^*)$, which also implies that $\underline{f}^* - \underline{m}^* < \overline{m}^* - \overline{f}^*$ and $\underline{m}^* - \underline{b}^* < \overline{b}^* - \overline{m}^*$. Now, consider an alternative product portfolio X' and the price function $p'(x)$ such that

$$X' = [\underline{m}', \overline{m}'],\ \textit{and}$$

$$p'(x) = \begin{cases} p^*(x) - 2\varepsilon(t+\alpha) & \textit{if } x \in [\underline{m}', \underline{f}^*] \\ p^*(x) - 2\varepsilon(t+\alpha) + t(x - \underline{f}^*) & \textit{if } x \in [\underline{f}^*, \underline{f}'] \\ p^*(x) & \textit{if } x \in [\underline{f}', \overline{f}^*] \\ p^*(x) + 2\varepsilon(t+\alpha) - t(x - \underline{f}^*) & \textit{if } x \in [\overline{f}^*, \overline{f}'] \\ p^*(x) + 2\varepsilon(t+\alpha) & \textit{if } x \in [\overline{f}', \overline{m}'], \end{cases}$$

where $(\underline{m}', \underline{f}', \overline{f}', \overline{m}') = (\underline{m}^* + \varepsilon, \underline{f}^* + 2\frac{t+\alpha}{t}\varepsilon, \overline{f}^* + 2\frac{t+\alpha}{t}\varepsilon, \overline{m}^* + \varepsilon)$

Notice that the firm moves the entire product portfolio to the right in order to alleviate the asymmetry. While doing this, it has to reduce the price of the product on the left differential pricing region and increase the prices on the right side. The changes in the price matches each other but the price increases apply to a larger area because $p^*(\underline{m}^*) > p^*(\overline{m}^*)$.

We illustrate the revenues of the firm under X^* and X' in Figure 13. The revenue that the firm loses under the alternative portfolio is the sum of areas L_1, L_2, L_3, L_4 whereas the firm's revenue gain by following X' is the sum of G_1, G_2, G_3, G_4. Notice that $L_2 = G_2$ and $L_4 = G_4$. The firms net gain from the alternative portfolio is

$$\begin{aligned} G_1 + G_3 - (L_1 + L_3) &= \varepsilon(t+2\alpha)\left[(\overline{b}^* - \overline{m}^*) - (\underline{m}^* - \underline{b}^*) - \varepsilon\right] \\ &\quad + 2\varepsilon(t+\alpha)\left[(\overline{m}^* - \overline{f}^*) - (\underline{f}^* - \underline{m}^*) - \frac{t+2\alpha}{t}\varepsilon\right] \\ &= \varepsilon(t+2\alpha)\left(1 + 2\frac{t+\alpha}{t}\right)\left[(\overline{b}^* - \overline{m}^*) - (\underline{m}^* - \underline{b}^*) - \varepsilon\right], \end{aligned}$$

where the second equality holds since $\frac{\overline{m}^* - \overline{b}^*}{\overline{f}^* - \overline{m}^*} = \frac{\underline{m}^* - \underline{b}^*}{\underline{f}^* - \underline{m}^*} = \frac{t}{t+2\alpha}$. The above equality shows that the firm has strictly positive net gain from X' for any $\varepsilon < (\overline{b}^* - \overline{m}^*) - (\underline{m}^* - \underline{b}^*)$, which exists because $\underline{m}^* - \underline{b}^* < \overline{b}^* - \overline{m}^*$. Therefore, the alternative portfolio improves the revenue of the firm, which contradicts with the optimality. Thus, we have that $p^*(\underline{m}^*) = p^*(\overline{m}^*)$.

A.5. Proof of Theorem 2

Denoting the size of the total market coverage $\overline{b} - \underline{b}$ as τ and the ratio between the fixed price region and the total coverage, $\frac{\overline{f} - \underline{f}}{\overline{b} - \underline{b}}$, as ρ, we can rewrite the firm's problem as:

$$\max_{\rho, \tau \le 1} \pi(\tau, \rho) \equiv (1-\rho)\tau p_g(\tau, \rho) + \rho\tau p_f(\tau, \rho) + 2\int_0^{\ell_d(\tau,\rho)} tz dz, \tag{7}$$

$$= \tau V - \frac{\tau^2(16\alpha^3 + 4\alpha^2(7+\rho^2) + 8\alpha(2-(1-\rho)\rho)t^2 + 3(1-\rho)^2 t^3)}{16(t+\alpha)^2}, \tag{8}$$

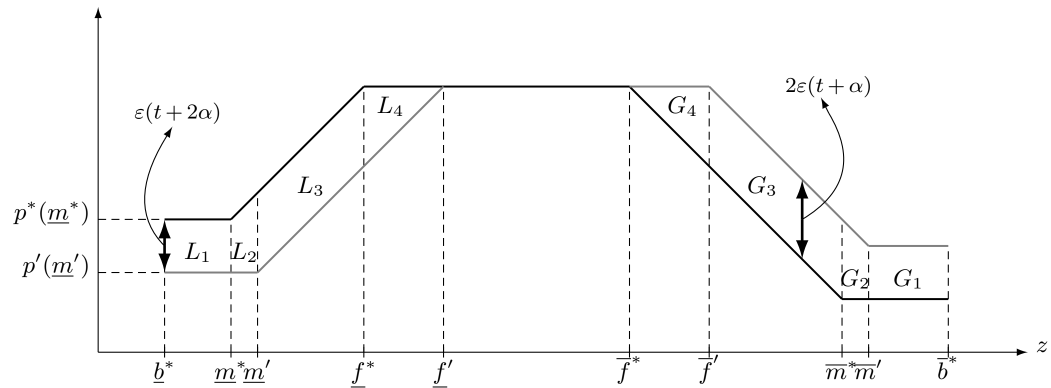


Figure 13 **The price that customers pay under the optimal price** $p^*(x)$ **(black line) and the alternative price** $p'(x)$ **(gray line).**

where $p_f(\tau,\rho) = V - \alpha\tau\frac{2\alpha+(1+\rho)t}{2(\alpha+t)}$, $p_g(\tau,\rho) = V - \tau\frac{4\alpha^2+4\alpha t+(1-\rho)t^2}{4(\alpha+t)}$, and $\ell_d(\tau,\rho) = \frac{t+2\alpha}{4(t+\alpha)}(1-\rho)\tau$. Note that p_f is the fixed price, p_g is the price of the guardrails, and ℓ_d is the size of the differential pricing region on each side of the fixed price region. We obtain these function by rewriting the optimal decisions in Proposition 2 as functions of τ and ρ.

Taking the derivative of the above profit function with respect to ρ and τ yields

$$\begin{aligned}\frac{\partial\pi(\tau,\rho)}{\partial\rho} &= \frac{t\tau^2\left[t(3t+4\alpha)-(4\alpha^2+8t\alpha+3t^2)\rho\right]}{8(t+\alpha)^2}\\ \frac{\partial\pi(\tau,\rho)}{\partial\tau} &= V - \frac{\tau(16\alpha^3+4\alpha^2(7+\rho^2)+8\alpha(2-(1-\rho)\rho)t^2+3(1-\rho)^2t^3)}{8(t+\alpha)^2}\end{aligned}$$

Note that $\frac{\partial\pi(\tau,\rho)}{\partial\rho} > 0$ for any $\rho < \frac{t(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)}$ and $\frac{\partial\pi(\tau,\rho)}{\partial\rho} \le 0$ otherwise. Therefore, the optimal ρ is $\rho^* \equiv \frac{t(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)}$. Furthermore, we have that

$$\frac{\partial\pi(\tau,\rho)}{\partial\tau}\Big|_{\rho=\rho^*} = V - \tau\frac{2\alpha(t+\alpha)(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)}.$$

Using the above equation, we have $\frac{\partial\pi(\tau,\rho)}{\partial\tau}\Big|_{\rho=\rho^*} > 0$ for any $\tau < \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)}$ and $\frac{\partial\pi(\tau,\rho)}{\partial\tau}\Big|_{\rho=\rho^*} \le 0$ otherwise. Therefore, the optimal size of the market coverage is

$$\tau^* = \begin{cases} 1 & \textit{if } \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} > 1 \\ \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} & \textit{if } \frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)} \le 1, \end{cases}$$

which leads to the optimal solution stated in the theorem because $\frac{V(t+2\alpha)(3t+2\alpha)}{2\alpha(t+\alpha)(3t+4\alpha)}$ can be rewritten as $\frac{V}{2\alpha}\left[1+\frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right]$. Furthermore, $\frac{V}{2\alpha}\left[1+\frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] > 1$ if and only if $\alpha < \alpha^*_{fc}(t)$ because $\frac{V}{2\alpha}\left[1+\frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right]$ is a decreasing function of α.

Using τ^* and ρ^*, the size of the product portfolio relative to the market coverage is

$$\begin{aligned}\frac{\overline{m}^*-\underline{m}^*}{\ell(B^*)} &= 2\frac{\underline{f}^*-\underline{m}^*}{\ell(B^*)} + \frac{\overline{f}^*-\underline{f}^*}{\ell(B^*)}\\ &= \frac{t+2\alpha}{t+\alpha}\left(\frac{\underline{f}^*-\underline{b}^*}{\ell(B^*)}\right) + \frac{\overline{f}^*-\underline{f}^*}{\ell(B^*)} = \frac{t+2\alpha}{2(t+\alpha)}(1-\rho^*)+\rho^*\\ &= \frac{t+2\alpha}{2(t+\alpha)} + \frac{t}{2(t+\alpha)}\rho^* = 1 - \frac{2t\alpha}{(t+2\alpha)(3t+2\alpha)}\end{aligned}$$

Finally, plugging in τ^* and ρ^* to $\pi(\tau,\rho)$, we obtain the optimal profit function stated in the theorem.

A.6. Proof of Corollary 1

We prove our claims by considering three cases: i) $\alpha \le V/2$, ii) $V/2 < \alpha \le \alpha^*_{fc}(t)$, and iii) $\alpha > \alpha^*_{fc}(t)$.

i) $\alpha \le \mathbf{V/2}$ **:** In this case, we have that $\ell(X^\circ) = 1$ and $\ell(F^*) = \frac{t(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)}$. Using these observations, we have that

$$1 - \frac{\ell(F^*)}{\ell(X^\circ)} = \frac{4\theta(\theta+1)}{3+4\theta(\theta+1)} = \frac{\theta}{\frac{3}{4(\theta+1)}+\theta} \ge \frac{\theta}{\theta+1}$$

Furthermore, our claim that $\ell(B^*) \le \ell(X^\circ) \le \ell(X^*)$ holds trivially because $\ell(B^*) = 1$.

ii) $\mathbf{V/2} < \alpha \le \alpha^*_{\mathbf{fc}}(\mathbf{t})$ **:** In this case, we have that $\ell(X^\circ) = V/(2\alpha)$ and $\ell(F^*) = \frac{t(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)}$. We also have that $\frac{V}{2\alpha}\left[1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] \ge 1$ since $\alpha \le \alpha^*_{fc}(t)$. Using these observations, we have that

$$1 - \frac{\ell(F^*)}{\ell(X^\circ)} \ge 1 - \left(\frac{\ell(F^*)}{\ell(X^\circ)}\right)\frac{V}{2\alpha}\left[1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] = \frac{t}{t+\alpha} = \frac{\theta}{\theta+1}.$$

Furthermore, we have that

$$\ell(X^\circ) - \ell(X^*) = \frac{V}{2\alpha} - \left[1 - \frac{2t\alpha}{(t+2\alpha)(3t+2\alpha)}\right] \ge 0,$$

because $\frac{V}{2\alpha}\left[1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] \ge 1$ implies that $V/(2\alpha) \ge \left[1 - \frac{t\alpha}{(t+2\alpha)(3t+2\alpha)}\right]$. Finally, we have that $\ell(B^*) \le \ell(X^\circ)$ because $\ell(B^*) = 1$.

iii) $\alpha > \alpha^*_{\mathbf{fc}}(\mathbf{t})$ **:** In this case, we have that $\ell(X^\circ) = V/(2\alpha)$ and $\ell(F^*) = \frac{Vt}{2\alpha(t+\alpha)}$. Using these observations, we have that

$$1 - \frac{\ell(F^*)}{\ell(X^\circ)} = \frac{t}{t+\alpha} = \frac{\theta}{\theta+1}.$$

Furthermore, we have that

$$\ell(X^\circ) - \ell(X^*) = \frac{V}{2\alpha} - \frac{V}{2\alpha}\left[1 - \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] \ge 0.$$

Finally, we have that $\ell(B^*) \le \ell(X^\circ)$ because $\ell(B^*) = \ell(X^\circ)\left[1 + \frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right]$.

Appendix B: Proofs in Section 3

B.1. Proof of Proposition 3

For notational convenience, we let $\delta_i = \tilde{x}_{i+1} - \tilde{x}_i$ for $1 \le i < N$ and $\delta_N = \underline{f}^* - \tilde{x}_N$. Then, we have that $p^*(\tilde{x}_i) = p^*(\underline{m}^*) - t\sum_{j=1}^{i-1}\delta_j$ and $\sum_{i=1}^{N}\delta_i = \underline{f}^* - \underline{m}^*$.

Note that firm's revenue from customers outside the product portfolio (i.e., those in the $[\underline{b}^*, \underline{m}^*]$ interval) does not depend on the locations of the fixed products because the location of the guardrail products are fixed. Similarly, firm's revenue from customers in the uniform pricing region (i.e., those in the $[\underline{f}^*, \overline{f}^*]$ interval) also does not depend on the locations of the fixed products because the uniform pricing region is fixed. Therefore, we can focus our attention on the revenue from customers in the $[\underline{m}^*, \underline{f}^*]$ region.

Under any given portfolio of in-between products, customers whose ideal products are between $\tilde{x}_i$ and $\tilde{x}_{i+1}$ purchase the product at $\tilde{x}_i$ because

$$\begin{aligned} U(z, \tilde{x}_i, p^*) &= V - p(\tilde{x}_i) - t(z - \tilde{x}_i) - \alpha(\overline{m}^* - \underline{m}^*) = t(\underline{f}^* - z) \\ &> t(\underline{f}^* - \tilde{x}_{i+1}) > V - p(\tilde{x}_{i+1}) - t(\tilde{x}_{i+1} - z) - \alpha(\overline{m}^* - \underline{m}^*) = U(z, \tilde{x}_{i+1}, p^*), \end{aligned}$$

for any $\tilde{x}_i < z < \tilde{x}_{i+1}$. Therefore, we can write the firm's revenue in the $[\underline{m}^*, \underline{f}^*]$ region as

$$\begin{aligned}\pi_{ib}(\delta) &= \delta_1 p^*(\underline{m}^*) + \sum_{i=2}^{N} p^*(\tilde{x}_i)\delta_i = p^*(\underline{m}^*)\sum_{i=1}^{N}\delta_i - t\sum_{i=2}^{N}\delta_i\sum_{j=1}^{i-1}\delta_j \\ &= p^*(\underline{m}^*)[\underline{f}^* - \underline{m}^*] - t\sum_{i=2}^{N}\delta_i\sum_{j=1}^{i-1}\delta_j,\end{aligned}$$

where $\delta = \{\delta_1, \ldots, \delta_N\}$. Then, the firm solves

$$\begin{aligned}\max_{\delta} \quad & \pi_{ib}(\delta) \\ s.t. & \sum_{i=1}^{N}\delta_i = \underline{f}^* - \underline{m}^*.\end{aligned}$$

First order conditions for the above problem are

$$\begin{aligned}&\frac{\partial \pi_{ib}(\delta)}{\partial \delta_i} + \gamma = -\sum_{j=1}^{i-1}\delta_j - \sum_{j=i+1}^{N}\delta_j + \gamma = 0 \ \textit{for all}\ 0 \le i \le N, \\ &\sum_{i=1}^{N}\delta_i = \underline{f}^* - \underline{m}^*,\end{aligned}$$

where γ is the Lagrangian multiplier for the equality constraint. Using the first set of equations, we have that $\delta_l = \delta_m$ for any $l, m \in \{1, \ldots, N\}$. Then, we have that $\delta_i = (\underline{f}^* - \underline{m}^*)/N$ by the last equation.

B.2. Proof of Lemma 1

We first consider the case where the firm serves the entire market, i.e., $\alpha < \alpha^*_{fc}(t)$. Using the fact that the fixed products are uniformly distributed, the firm's revenue under the heuristic model is

$$\begin{aligned}\tilde{\Pi}(N) &= \lambda\left[V - \frac{\alpha\left(8\alpha^3 N + 2\alpha^2(13Nt + t) + \alpha(27N+1)t^2 + 9Nt^3\right)}{N(t+2\alpha)(3t+2\alpha)^2}\right] \\ &= \lambda\left[V - \alpha\left(1 - \frac{\alpha t((3N-1)t + 2\alpha(N-1))}{N(t+2\alpha)(3t+2\alpha)^2}\right)\right] \\ &= \lambda\left[V - \alpha\left(1 - \frac{\alpha t}{(t+2\alpha)(3t+2\alpha)} + \frac{t\alpha}{N(3t+2\alpha)^2}\right)\right] = \Pi^* - \lambda\frac{t\alpha^2}{N(3t+2\alpha)^2}.\end{aligned}$$

Thus, $\lim_{N\to\infty}\tilde{\Pi}(N) = \Pi^*$ since $\lim_{N\to\infty}\frac{t\alpha^2}{N(3t+2\alpha)^2} = 0$.

When $\alpha \ge \alpha^*_{fc}(t)$, we can, similarly, write the firm's revenue under the heuristic model as

$$\tilde{\Pi}(N) = \Pi^* - \lambda V^2 \frac{t(t+2\alpha)^2}{4N(t+\alpha)^2(3t+4\alpha)^2},$$

which also implies that $\lim_{N\to\infty}\tilde{\Pi}(N) = \Pi^*$ since $\lim_{N\to\infty}\frac{t(t+2\alpha)^2}{4N(t+\alpha)^2(3t+4\alpha)^2} = 0$.

B.3. Proof of Lemma 2

We first note that $\Delta(\alpha, t, N)$ is increasing in N by Lemma 1. Thus, it is sufficient to show that $\Delta(\alpha, t, 1) > 0$. We prove this claim by considering three cases: i) $\alpha \le V/2$, ii) $V/2 < \alpha \le \alpha^*_{fc}(t)$, and iii) $\alpha > \alpha^*_{fc}(t)$.

i) $\alpha \le \mathbf{V/2}$: In this case, we have that

$$\Delta(\alpha, t, 1) = \frac{2(t\alpha)^2}{(V-\alpha)(t+2\alpha)(3t+2\alpha)^2} > 0,$$

where the inequality holds since $\alpha \le V/2$.

ii) $\mathbf{V/2} < \alpha \leq \alpha^*_{\mathbf{fc}}(\mathbf{t})$: In this case, we have that

$$\Delta(\alpha,t,1) = \frac{4\alpha\left(V - \frac{\alpha\left(8\alpha^3+28\alpha^2 t+28\alpha t^2+9t^3\right)}{(t+2\alpha)(3t+2\alpha)^2}\right)}{V^2} - 1 \geq \frac{2\alpha}{V} - 1 > 0$$

where the first inequality holds because we have that $(t+2\alpha)(3t+2\alpha)^2 \geq \frac{2\alpha(8\alpha^3+26\alpha^2 t+26\alpha t^2+9t^3)}{V}$ due to the fact that $\frac{V}{2\alpha}\left[1+\frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right] \geq 1$ when $\alpha \leq \alpha^*_{fc}(t)$. Furthermore, the second inequality holds since $\alpha > V/2$.

iii) $\alpha > \alpha^*_{\mathbf{fc}}(\mathbf{t})$: In this case, we have that

$$\Delta(\alpha,t,1) = \frac{\alpha t^2(2t+3\alpha)}{(t+\alpha)^2(3t+4\alpha)^2} > 0.$$

B.4. Proof of Proposition 4

As we have that $\alpha < V/2$, the firm covers the entire market in both *Uniform Pricing* and the *Preset Products* models. Thus, for any $\alpha < V/2$, we have that

$$\Delta(\alpha,t,N) = \frac{t\alpha^2}{(t+2\alpha)(3t+2\alpha)(V-\alpha)} - \lambda\frac{t\alpha^2}{N(V-\alpha)(3t+2\alpha)^2}$$

By taking the derivative of $\Delta(\alpha,t,N)$ with respect to α and N, we have that $\frac{d^2\Delta(\alpha,t,N)}{d\alpha dN} > 0$ for any $\alpha < V/2$. This implies that $\frac{d\Delta(\alpha,t,N)}{d\alpha}$ is increasing in N. Then, we have that $\Delta(\alpha,t,N)$ is increasing in α for any $\alpha < V/2$ and $N \geq 2$ because

$$\frac{d\Delta(\alpha,t,2)}{d\alpha} = \alpha t\left(\frac{8\alpha^4+28\alpha^3 t+8\alpha^2 tV+2\alpha t^2(24V-\alpha)+15t^3(2V-\alpha)}{2(V-\alpha)^2(t+2\alpha)^2(3t+2\alpha)^3}\right) > 0,$$

when $\alpha < V/2$.

B.5. Proof of Proposition 5

1. For any $\alpha < V/2$, we have that $\Delta(\alpha,t) = \frac{t\alpha^2}{(t+2\alpha)(3t+2\alpha)(V-\alpha)}$ and $\frac{\partial\Delta(\alpha,t)}{\partial\alpha} = \frac{\alpha t\left(4\alpha^3+8Vt\alpha+3t^2(2V-\alpha)\right)}{(t+2\alpha)^2(3t+2\alpha)^2(V-\alpha)^2}$. Then, $\Delta(\alpha,t)$ is increasing in α for any $\alpha < V/2$ because $\frac{\partial\Delta(\alpha,t)}{\partial\alpha} > 0$ when $\alpha < V/2$.

2. For any $V/2 \leq \alpha < \alpha^*_{fc}(t)$, we have that $\Delta(\alpha,t) = \frac{4\alpha\left(V-\frac{\alpha(t+\alpha)(3t+4\alpha)}{(t+2\alpha)(3t+2\alpha)}\right)}{V^2} - 1$. Furthermore, we have that

$$\frac{\partial^2\Delta(\alpha,t)}{\partial\alpha^2} = -\frac{8\left(64\alpha^6+384\alpha^5 t+912\alpha^4 t^2+1036\alpha^3 t^3+612\alpha^2 t^4+189\alpha t^5+27t^6\right)}{V^2(t+2\alpha)^3(3t+2\alpha)^3} < 0 \tag{9}$$

$$\frac{\partial\Delta(\alpha,t)}{\partial\alpha}\Big|_{\alpha=V/2} = \frac{t\left(9t^2+8tV+V^2\right)}{(t+V)^2(3t+V)^2} > 0 \tag{10}$$

$$\frac{\partial\Delta(\alpha,t)}{\partial\alpha}\Big|_{\alpha=\alpha^*_{fc}(t)} = \frac{t(3t^2-4\alpha^*_{fc}(t)^2)}{(t+\alpha^*_{fc}(t))^2(3t+4\alpha^*_{fc}(t))^2}. \tag{11}$$

For any $\alpha \geq \alpha^*_{fc}(t)$, we have that $\Delta(\alpha,t) = \frac{\alpha t}{(\alpha+t)(4\alpha+3t)}$ and thus

$$\frac{\partial\Delta(\alpha,t)}{\partial\alpha} = \frac{t\left(3t^2-4\alpha^2\right)}{(t+\alpha)^2(3t+4\alpha)^2}. \tag{12}$$

Using the above observations, we prove our claim by considering two cases: i) $t < \frac{4V}{3+2\sqrt{3}}$ and ii) $t \geq \frac{4V}{3+2\sqrt{3}}$. We also want to note that $t > \frac{4V}{3+2\sqrt{3}} \Leftrightarrow \alpha^*_{fc}(t) < t\sqrt{3}/2$ because letting $h(\alpha) = \frac{V}{2\alpha}\left[1+\frac{t\alpha}{(t+\alpha)(3t+4\alpha)}\right]$, we have that $h(\alpha)$ is decreasing in α and by the definition of $h(\alpha^*_{fc}(t)) = 1$. Then, we have that

$$\alpha^*_{fc}(t) > t\sqrt{3}/2 \Leftrightarrow h(t\sqrt{3}/2) > 1 \Leftrightarrow \left(\frac{2}{\sqrt{3}}-1\right)4V/t > 1 \Leftrightarrow t < \frac{4V}{3+2\sqrt{3}}.$$

i) $\mathbf{t < \frac{4V}{3+2\sqrt{3}}}$: First note that $\alpha^*_{fc}(t) > t\sqrt{3}/2$ in this case. (9) above shows that $\Delta(\alpha,t)$ is concave in α when $V/2 \le \alpha < \alpha^*_{fc}(t)$. In other words, $\frac{\partial\Delta(\alpha,t)}{\partial\alpha}$ is decreasing in α for any $V/2 \le \alpha < \alpha^*_{fc}(t)$. Furthermore, (10) shows that $\Delta(\alpha,t)$ is increasing at $\alpha = V/2$, and (11) shows that $\frac{\partial\Delta(\alpha,t)}{\partial\alpha} < 0$ at $\alpha = \alpha^*_{fc}(t)$ since $\alpha^*_{fc}(t) > t\sqrt{3}/2$. Combining these three observations, we have that there must be an $\hat{\alpha} \in (V/2, \alpha^*_{fc}(t))$ such that $\Delta(\alpha,t)$ is increasing in α up to $\hat{\alpha}$ and decreasing afterwards until $\alpha^*_{fc}(t)$. Finally, (12) shows that $\frac{\partial\Delta(\alpha,t)}{\partial\alpha} < 0$ for any $\alpha \ge \alpha^*_{fc}(t)$ since $\alpha^*_{fc}(t) > t\sqrt{3}/2$.

ii) $\mathbf{t \ge \frac{4V}{3+2\sqrt{3}}}$: In this case, we have that $\alpha^*_{fc}(t) \le t\sqrt{3}/2$. As $\alpha^*_{fc}(t) \le t\sqrt{3}/2$, (9)-(11) now imply that $\frac{\partial\Delta(\alpha,t)}{\partial\alpha} > 0$ for any $V/2 \le \alpha < \alpha^*_{fc}(t)$. Furthermore, for the range of $\alpha \ge \alpha^*_{fc}(t)$, we have that $\Delta(\alpha,t)$ is maximized at $\alpha = t\sqrt{3}/2$ due to (12) since $\alpha^*_{fc}(t) \le t\sqrt{3}/2$.

Appendix C: Proofs in Section 4

C.1. Proof of Proposition 6

For any given product portfolio $X = [\underline{m}, \overline{m}]$ and the market coverage $B = [\underline{b}, \overline{b}]$, consider the price function

$$\hat{p}_L(x,X,B) = \min\{V - C(x,X), \hat{p}_\ell(\underline{m}) + t(x-\underline{m}), \hat{p}_r(\overline{m}) + t(\overline{m} - x)\},$$

where $\hat{p}_\ell(\underline{m})$ and $\hat{p}_\ell(\overline{m})$ are chosen such that the customers at the market coverage boundaries obtain zero utility from the guardrail products, i.e., $\hat{p}_\ell(\underline{m}) = V - C(\underline{b}, X) - t(\underline{m} - \underline{b})$ and $\hat{p}_\ell(\overline{m}) = V - C(\overline{b}, X) - t(\overline{b} - \overline{m})$.

Similar to the proof of Proposition 2, we first prove that $p^\star(x)$ must be equal to $\hat{p}_L(x, X^\star, B^\star)$. Note that market coverages under both price functions are the same by the construction of $\hat{p}_L(x, X^*, B^*)$. Under the optimal price function $p^*(x)$, a customer located at z should pay less than $V - C(\underline{z}, X)$ because otherwise her utility would be negative. Moreover, customers located in $X^\star$ should not pay more than $p^*(\underline{m}) + t(z - \underline{m}^*)$ or $p^*(\overline{m}) + t(\overline{m}^* - z)$ because otherwise they can improve their utility by purchasing the guardrail products. Therefore, for any $z \in X^\star$, we should have that

$$p^\star(b^\star(z, p^\star)) \le \hat{p}_L(z, X^\star, B^\star).$$

We can also show that all customers located in $X^\star$ will purchase their ideal products under $\hat{p}_L(x)$, similar to Lemma 3 using the convexity of $C(z,X)$. Specifically, we use the convexity of $C(z,X)$ to show that customers located at $z \in X^o \equiv \{x : \hat{p}_L(x,X,B) = V - C(x,X)\}$ purchase their ideal products. To prove this result, it is sufficient to show that the utility of customers $z \in X^o$ from a product located at $x \in X^o$ with $x < z$ is less than zero under $\hat{p}_L(x,X,B)$. For any $x \in X^o$ with $x < z$, we have that

$$\begin{aligned} U(z,x,\hat{p}_L) &= V - C(z,X) - [V - C(x,X)] - t(z-x) = C(x,X) - C(z,X) - t(z-x) \\ &= -\int_x^z C'(s,X)dS - t(z-x) = -\int_x^z [C'(s,X) + t]dS < 0, \end{aligned}$$

where the third equality holds by the *Fundamental Theorem of Calculus* and the last inequality holds because $C'(x^o, X) > -t$ at $x^o = \min\{x : x \in X^o\}$ and thus $C'(x^o, X) > -t$ for any $x \in X^o$ because $C(x,X)$ is convex.

We can also show that customers outside the product portfolio buy the guardrails under the optimal price $p^\star(z)$, similar to Lemma 4. Note also that customers outside the product portfolio buy the guardrails under $\hat{p}_L(z, X^\star, B^\star)$ by construction. Then, as the market coverages are the same under both $\hat{p}_L(z, X^\star, B^\star)$ and $p^\star(x)$, the above inequality implies that the firm's revenue under $\hat{p}_L(z, X^\star, B^\star)$ is an upper bound for its optimal revenue, which proves that $p^\star(x)$ must be equal to $\hat{p}_L(z, X^\star, B^\star)$.

Finally, we obtained the customers' most preferred products by the construction of $\hat{p}_L(z, X^\star, B^\star)$.

C.2. Proof of Proposition 7

Using our result from Proposition 6, the profit becomes a function of the guardrail products and the endpoints of the market coverage. We can also assume (without loss of generality) that the market coverage is symmetric around the half-point, i.e. $\overline{b} = 1 - \underline{b}$. Then, we can rewrite the firm's revenue as a function of $\underline{b}$, $\underline{m}$, and $\overline{m}$ as follows:

$$\begin{aligned}\pi_C(\underline{b}, \underline{m}, \overline{m}) = &(\underline{m} - \underline{b})p^\star(\underline{m}) + \int_{\underline{m}}^{\underline{f}^\star(\underline{b},\underline{m},\overline{m})} p^\star(\underline{m}) + t(z - \underline{m})dz \\ &+ \int_{\underline{f}^\star(\underline{b},\underline{m},\overline{m})}^{\overline{f}^\star(\underline{b},\underline{m},\overline{m})} V - \hat{C}(z, X)dz + \int_{\overline{f}^\star(\underline{b},\underline{m},\overline{m})}^{\overline{m}} p^\star(\overline{m}) + t(\overline{m} - z)dz + (1 - \underline{b} - \overline{m})p^\star(\overline{m}),\end{aligned}$$

where $\underline{f}^\star(\underline{b}, \underline{m}, \overline{m}) \equiv \max\{z : V - \hat{C}(z, X) = p^\star(\underline{m}) + t(z - \underline{m})\}$ and $\overline{f}^\star(\underline{b}, \underline{m}, \overline{m}) \equiv \max\{z : V - \hat{C}(z, X) = p^\star(\overline{m}) + t(\underline{m} - z)\}$. Note that it is possible to have $\underline{f}^\star > \overline{f}^\star$, but in this proof, we focus on the case where $\underline{f}^\star \le \overline{f}^\star$. The proof is almost the same when $\underline{f}^\star > \overline{f}^\star$, in fact with less complicated expressions.

We prove the symmetry of $X^\star$ by contradiction, so that we suppose $X^\star$ is not symmetric inside $B^\star$, which implies that $\underline{m}^\star + \overline{m}^\star \neq 1$. Then, we define $h(\varepsilon) = \pi_C(\underline{b}^\star, \underline{m}^\star + \varepsilon, \overline{m}^\star + \varepsilon)$, for any $\varepsilon \in [\underline{b}^\star - \underline{m}^\star, 1 - \underline{b}^\star - \overline{m}^\star]$. Note that $h(\varepsilon)$ is the firm's profit when the both ends of the product portfolio is moved by ε. Thus, we can establish the symmetry of X^* by showing that $h(\varepsilon)$ is maximized at $\varepsilon^* = (1 - \underline{m}^\star - \overline{m}^\star)/2$.

By taking the derivative of $h(\varepsilon)$ with respect to ε and after some algebra, we have that

$$h'(\varepsilon) = p^\star(\underline{m} + \varepsilon) - p^\star(\overline{m} + \varepsilon) + \left[\underline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon) - \underline{b}\right]\frac{dp^\star(\underline{m} + \varepsilon)}{d\varepsilon} + \left[1 - \underline{b} - \overline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon)\right]\frac{dp^\star(\overline{m} + \varepsilon)}{d\varepsilon}.$$

Using the explicit functional form of $\hat{C}(z, X)$, we have that $p^\star(\underline{m} + \varepsilon^*) = p^\star(\overline{m} + \varepsilon^*)$, $\underline{f}^\star(\underline{b}, \underline{m} + \varepsilon^*, \overline{m} + \varepsilon^*) + \overline{f}^\star(\underline{b}, \underline{m} + \varepsilon^*, \overline{m} + \varepsilon^*) = 1$, and $\frac{dp^\star(\underline{m}+\varepsilon)}{d\varepsilon}\Big|_{\varepsilon=\varepsilon^*} = -\frac{dp^\star(\overline{m}+\varepsilon)}{d\varepsilon}\Big|_{\varepsilon=\varepsilon^*}$. Combining these findings, we obtain that $h(\varepsilon^*) = 0$. Furthermore, we have that

$$\begin{aligned}h''(\varepsilon) = &\left[1 + \frac{d\underline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon)}{d\varepsilon}\right]\frac{dp^\star(\underline{m} + \varepsilon)}{d\varepsilon} - \left[1 + \frac{d\overline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon)}{d\varepsilon}\right]\frac{dp^\star(\overline{m} + \varepsilon)}{d\varepsilon} \\ &+ \left[\underline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon) - \underline{b}\right]\frac{d^2p^\star(\underline{m} + \varepsilon)}{d\varepsilon^2} + \left[1 - \underline{b} - \overline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon)\right]\frac{d^2p^\star(\overline{m} + \varepsilon)}{d\varepsilon^2} \le 0.\end{aligned}$$

The above inequality holds true because $p^\star(\underline{m} + \varepsilon^*)$ is concave and decreasing, $p^\star(\overline{m} + \varepsilon^*)$ is concave and increasing, $\underline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon)$ and $\overline{f}^\star(\underline{b}, \underline{m} + \varepsilon, \overline{m} + \varepsilon)$ are increasing in ε. Thus, $h(\varepsilon)$ must be maximized at $\varepsilon^* = (1 - \underline{m}^\star - \overline{m}^\star)/2$.

C.3. Proof of Theorem 3

We first want to note that

$$\underline{f}^\star(\underline{b}, \underline{m}) = \frac{2\alpha - t + \sqrt{\left[2\alpha(1 - 2\underline{b}) - t\right]^2 + 16\alpha t(\underline{m} - \underline{b})}}{4\alpha},$$

which is an increasing function of $\underline{m}$. Furthermore, $\underline{f}^\star(\underline{b}, \underline{b}) = \max\{b, 1 - b - t/(2\alpha)\}$, and thus we have that $\underline{f}^\star(\underline{b}, \underline{b}) \ge 1/2 \Leftrightarrow \underline{b} \le 1/2 - t/(2\alpha)$. With the help of this property, we can prove our claim by considering three cases: i) $\underline{b}^\star > 1/2 - t/(2\alpha)$, ii) $\underline{b}^\star < 1/2 - t/(2\alpha)$, and iii) $\underline{b}^\star = 1/2 - t/(2\alpha)$.

Case 1: $\underline{\mathbf{b}}^\star > \mathbf{1/2 - t/(2\alpha)}$: In this case, we have that $\underline{f}^\star(\underline{b}^\star, \underline{b}^\star) < 1/2$. Due to the monotonicity of $\underline{f}^\star(\underline{b}, \underline{m})$, we should also have that $\underline{f}^\star(\underline{b}^\star, \underline{m}) < 1/2$ for all $\underline{m} < x^0$ for some $x^0 > \underline{b}^\star$. Then, for any $\underline{b}^\star \le \underline{m} < x^0$, we have that

$$\Pi_C(\underline{b}^\star, \underline{m}) = 2\left[(\underline{m} - \underline{b}^\star)p^\star(\underline{m}) + \int_{\underline{m}}^{\underline{f}^\star(\underline{b}^\star,\underline{m})} p^\star(\underline{m}) + t(z-\underline{m})dz + \int_{\underline{f}^\star(\underline{b}^\star,\underline{m})}^{1/2} V - \hat{C}(z,X)dz\right].$$

Taking the derivative of the above profit function with respect to $\underline{m}$ and after some algebra, we have that

$$\frac{d\Pi_C(\underline{b}^\star, \underline{m})}{d\underline{m}}\Big|_{\underline{m}=\underline{b}^\star} = \frac{\left[t - \alpha(1-2\underline{b}^\star)\right]^2}{\alpha} > 0,$$

where the inequality holds because $\underline{b}^\star > 1/2 - t/(2\alpha)$. However, this contradicts with the optimality of $\underline{m}^\star = \underline{b}^\star$. Hence, we must have that $\underline{m}^\star > \underline{b}^\star$ under this case.

Case 2: $\underline{\mathbf{b}}^\star < \mathbf{1/2 - t/(2\alpha)}$: In this case, we have that $\underline{f}^\star(\underline{b}^\star, \underline{b}^\star) \ge 1/2$. Due to the monotonicity of $\underline{f}^\star(\underline{b}, \underline{m})$, we should also have that $\underline{f}^\star(\underline{b}^\star, \underline{m}) \ge 1/2$ for all $\underline{m} > \underline{b}^\star$. Then, for any $\underline{b}^\star \le \underline{m}$, we have that

$$\Pi_C(\underline{b}^\star, \underline{m}) = 2\left[(\underline{m} - \underline{b}^\star)p^\star(\underline{m}) + \int_{\underline{m}}^{1/2} p^\star(\underline{m}) + t(z-\underline{m})dz\right].$$

Taking the derivative of the above profit function with respect to $\underline{m}$ and after some algebra, we have that

$$\frac{d\Pi_C(\underline{b}^\star, \underline{m})}{d\underline{m}}\Big|_{\underline{m}=\underline{b}^\star} = (1-2\underline{b}^\star)\left[\alpha(1-2\underline{b}^\star) - t\right] > 0,$$

where the inequality holds because $\underline{b}^\star < 1/2 - t/(2\alpha)$. However, this contradicts with the optimality of $\underline{m}^\star = \underline{b}^\star$. Hence, we must have that $\underline{m}^\star > \underline{b}^\star$ under this case.

Case 3: $\underline{\mathbf{b}}^\star = \mathbf{1/2 - t/(2\alpha)}$: In this case, we have that $\underline{f}^\star(\underline{b}^\star, \underline{b}^\star) = 1/2$. Then, similar to Case 2, the derivative of the firm's profit function with respect to $\underline{m}$ is

$$\frac{d\Pi_C(\underline{b}^\star, \underline{m})}{d\underline{m}}\Big|_{\underline{m}=\underline{b}^\star} = (1-2\underline{b}^\star)\left[\alpha(1-2\underline{b}^\star) - t\right] = 0,$$

where the last equality holds because $\underline{b}^\star = 1/2 - t/(2\alpha)$. Thus we must have $\underline{b}^\star = \underline{m}^\star$ when $\underline{b}^\star = 1/2 - t/(2\alpha)$. However, this is not sufficient to prove that $\underline{b}^\star = 1/2 - t/(2\alpha)$ is the optimal decision because the firm can improve its profits when $\alpha \ne \max\{t, 5t^2/(2V)\}$ as follows: Consider the firm's profit function when $\underline{m} = \underline{b}$, i.e., $\Pi_C(\underline{b}, \underline{b})$. We have that

$$\frac{d\Pi_C(\underline{b}, \underline{b})}{d\underline{b}}\Big|_{\underline{b}=1/2-t/(2\alpha)} = \frac{5t^2}{2\alpha} - V.$$

The above derivative is zero only when $\alpha = 5t^2/(2V)$. We also want to note that $\underline{b}^\star = 1/2 - t/(2\alpha) \ge 0$ implies that $\alpha \ge t$. Using these two properties, we will show that the firm can do better than $\underline{b}^* = \underline{m}^* = 1/2 - t/(2\alpha)$ considering $5t^2/(2V) > t$ and $5t^2/(2V) \le t$ separately.

- $\mathbf{5t^2/(2V) > t}$: The above derivative is negative for $\alpha > 5t^2/(2V)$, and thus the firm improves its profit by deviating from $\underline{b}^* = \underline{m}^* = 1/2 - t/(2\alpha)$ to $\underline{b}' = \underline{m}' = 1/2 - t/(2\alpha) - \epsilon$ for some small $\epsilon > 0$. Similarly, the above derivative is positive for $t \le \alpha < 5t^2/(2V)$, and thus the firm improves its profit by deviating from $\underline{b}^* = \underline{m}^* = 1/2 - t/(2\alpha)$ to $\underline{b}' = \underline{m}' = 1/2 - t/(2\alpha) + \epsilon$ for some small $\epsilon > 0$. Hence, $\underline{b}^* = \underline{m}^* = 1/2 - t/(2\alpha)$ cannot be optimal as long as $\alpha \ne 5t^2/(2V)$.

• $\mathbf{5t^2/(2V) \leq t}$: The above derivative is negative for $\alpha \geq t$, and thus the firm improves its profit by deviating from $\underline{b}^* = \underline{m}^* = 1/2 - t/(2\alpha)$ to $\underline{b}' = \underline{m}' = 1/2 - t/(2\alpha) - \epsilon$ for some small $\epsilon > 0$ when $1/2 - t/(2\alpha) \neq 0$. Hence, $\underline{b}^* = \underline{m}^* = 1/2 - t/(2\alpha)$ cannot be optimal as long as $\alpha \neq t$.

Combining these two observations proves that $\underline{b} = \underline{m} = 1/2 - t/(2\alpha)$ cannot be optimal when $\alpha \neq \max\{t, 5t^2/(2V)\}$.