Monetary Policy and Adaptive Learning: An Overview of Recent Research

George W. Evans (University of Oregon)

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J. C. Trichet: “Understanding expectations formation as a process underscores the strategic interdependence that exists between expectations formation and economics.” (Zolotas lecture, 2005)

Ben S. Bernanke: “In sum, many of the most interesting issues in contemporary monetary theory require an analytical framework that involves learning by private agents and possibly the central bank as well.” (NBER, July 2007).
Outline

Introduction: Bounded rationality, multiplicity and stability under learning

The New Keynesian Model
— Structure & monetary policy rules
— Determinacy
— LS learning and E-stability
— Results for different interest rate rules

Further Developments: Extensions, related theoretical and empirical topics

Selected Topics in More Detail
— Monetary Policy under Perpetual Learning
— Learning and Inflation Persistence
— Explaining Hyperinflations
— Liquidity Traps
Introduction

• Since Lucas (1972, 1976) and Sargent (1973) the standard assumption in the theory of economic policy is rational expectations (RE). This assumes, for both private agents and policymakers,

  – knowledge of the correct form of the model

  – knowledge of all parameters, and

  – knowledge that other agents are rational & know that others know . . . .

• RE assumes too much and is therefore implausible. We need an appropriate model of *bounded rationality*. What form should this take?
• My general answer is given by the **Cognitive Consistency Principle**: economic agents should be about as smart as (good) economists. Economists forecast economic variables using econometric techniques, so a good starting point: model agents as “econometricians.”

• Neither private agents nor economists at central banks do know the true model. Instead economists formulate and estimate models. These models are re-estimated and possibly reformulated as new data becomes available. Economists engage in processes of learning about the economy.

• Do such processes of learning create **new tasks for monetary policy**? This presentation gives an overview of the rapidly growing literature on learning and monetary policy.
Starting Points:

- Forecasts (including private forecasts) of future inflation and output have a key role in monetary policy:
  1. Empirical evidence e.g. by (Clarida, Gali and Gertler 1998).
  2. Bank of England and ECB discuss private forecasts in addition to internal macro projections.

- The private sector is forward-looking (e.g. investment, savings decisions).

- Private agents and/or policy-makers are learning.
Fundamental Problems:

- There may be multiple equilibria, depending on interest rate policy.
- Policy may lead to expectational instability if expectations are not always rational.

These problems necessitate careful design of interest rate rule: Bullard and Mitra (2002), Evans and Honkapohja (2003a, 2006).


- Central message: Policy should facilitate learning by private agents.
The New Keynesian Model

- Log-linearized New Keynesian model (Clarida, Gali and Gertler 1999 and Woodford 2003 etc.).

1. “IS” curve

\[ x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t \]

2. the “New Phillips” curve

\[ \pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \]

where \( x_t = \) output gap, \( \pi_t = \) inflation, \( i_t = \) nominal interest rate. \( E_t^* x_{t+1} \), \( E_t^* \pi_{t+1} \) are expectations. Parameters \( \varphi, \lambda > 0 \) and \( 0 < \beta < 1 \).
- Observable shocks follow

\[
\begin{pmatrix}
  g_t \\
  u_t 
\end{pmatrix} = F \begin{pmatrix}
  g_{t-1} \\
  u_{t-1} 
\end{pmatrix} + \begin{pmatrix}
  \tilde{g}_t \\
  \tilde{u}_t 
\end{pmatrix}, 
F = \begin{pmatrix}
  \mu & 0 \\
  0 & \rho 
\end{pmatrix},
\]

where \( 0 < |\mu|, |\rho| < 1 \), and \( \tilde{g}_t \sim iid(0, \sigma^2_g) \), \( \tilde{u}_t \sim iid(0, \sigma^2_u) \).

- Some versions of the NK model incorporate inertia, i.e. \( \pi_{t-1} \) or \( x_{t-1} \).

**Policy Rules**

- Interest rate setting by a standard **Taylor rule**

\[
i_t = \chi_{\pi} \pi_t + \chi_x x_t \text{ where } \chi_{\pi}, \chi_x > 0.
\]

Also variations of the rule have been studied.
• **Optimal monetary policy**: Under full commitment minimize loss

\[ E_t \sum_{s=0}^{\infty} \beta^s \left[ \pi_{t+s}^2 + \alpha x_{t+s}^2 \right]. \]

We get the (timeless perspective) **optimal targeting rule**

\[ \lambda \pi_t + \alpha (x_t - x_{t-1}) = 0. \]

• How should one implement optimal policy?
1. “Fundamentals-based” reaction function

\[ i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t. \]

2. Expectations-based reaction function

\[ i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t \]

where

\[
\begin{align*}
\delta_L & = \frac{-\alpha}{\varphi(\alpha + \lambda^2)}, \\
\delta_\pi & = 1 + \frac{\lambda \beta}{\varphi(\alpha + \lambda^2)}, \\
\delta_x & = \varphi^{-1}, \quad \delta_g = \varphi^{-1}, \quad \delta_u = \frac{\lambda}{\varphi(\alpha + \lambda^2)}. 
\end{align*}
\]

Digression: Determinacy and Stability under Learning

DETERMINACY

Combining IS, PC and the \( i_t \) rule leads to a bivariate reduced form in \( x_t \) and \( \pi_t \). Letting \( y'_t = (x_t, \pi_t)' \) and \( v'_t = (g_t, u_t)' \) the model can be written

\[
\begin{pmatrix}
    x_t \\
    \pi_t 
\end{pmatrix}
= M \begin{pmatrix}
    E^*_t x_{t+1} \\
    E^*_t \pi_{t+1}
\end{pmatrix}
+ N \begin{pmatrix}
    x_{t-1} \\
    \pi_{t-1}
\end{pmatrix}
+ P \begin{pmatrix}
    g_t \\
    u_t
\end{pmatrix},
\]

\[y_t = ME^*_t y_{t+1} + Ny_{t-1} + Pv_t.\]

If the model is “determinate” there exists a unique stationary REE of the form

\[y_t = \bar{b}y_{t-1} + \bar{c}v_t.\]

Determinacy condition: compare \# of stable eigenvalues of matrix of stacked first-order system to \# of predetermined variables. If “indeterminate” there are multiple solutions, which include stationary sunspot solutions.
LEARNING

Under learning, agents have beliefs or a perceived law of motion (PLM)

\[ y_t = a + b y_{t-1} + c v_t, \]

where we now allow for an intercept, and estimate \((a_t, b_t, c_t)\) in period \(t\) based on past data.

- Forecasts are computed from the estimated PLM.
- New data is generated according to the model with the given forecasts.
- Estimates are updated to \((a_{t+1}, b_{t+1}, c_{t+1})\) using least squares.

Question: when is it the case that

\[ (a_t, b_t, c_t) \rightarrow (0, \bar{b}, \bar{c})? \]
Background on Learning in Macroeconomics

- Large literature on LS learning in macro models.


- Convergence of LS learning to RE can be shown in many standard models (but not always).

- Convergence to an REE depends on “expectational stability” (or “E-stability”) conditions.
LS LEARNING
E-STABILITY METHODOLOGY

Reduced form

\[ y_t = M E_t^* y_{t+1} + N y_{t-1} + P v_t. \]

Stability under learning is analyzed using E-stability:

Under the PLM (Perceived Law of Motion)

\[ y_t = a + b y_{t-1} + c v_t. \]

\[ E_t^* y_{t+1} = (I + b)a + b^2 y_{t-1} + (bc + cF) v_t. \]

This \(\rightarrow\) ALM (Actual Law of Motion)

\[ y_t = M(I + b)a + (Mb^2 + N)y_{t-1} + (Mbc + NcF + P)v_t. \]
This gives a mapping from PLM to ALM:

\[ T(a, b, c) = (M(I + b)a, Mb^2 + N, Mbc + NcF + P). \]

The optimal REE is a fixed point of \( T(a, b, c) \). If

\[ \frac{d}{d\tau}(a, b, c) = T(a, b, c) - (a, b, c) \]

is locally asymptotically stable at the REE it is said to be **E-stable**. See EH, Chapter 10, for details. The **E-stability conditions** can be stated in terms of the derivative matrices

\[
\begin{align*}
DT_a &= M(I + \bar{b}) \\
DT_b &= \bar{b}' \otimes M + I \otimes M\bar{b} \\
DT_c &= F' \otimes M + I \otimes M\bar{b},
\end{align*}
\]

where \( \otimes \) denotes the Kronecker product and \( \bar{b} \) denotes the REE value of \( b \).

**E-stability governs stability under LS learning.**
Results for Interest Rate Rules

- **Taylor rule** \( i_t = \chi_{\pi}\pi_t + \chi_x x_t \) yields determinacy and stability under LS learning if

\[
\lambda(\chi_\pi - 1) + (1 - \beta)\chi_x > 0.
\]

The "Taylor principle" \( \chi_\pi > 1 \) is sufficient. For forward-looking rules the conditions are more complex.

- **Optimal monetary policy under commitment:**

  1. Fundamentals based reaction function: instability under learning and also indeterminacy can arise.

  2. Expectations-based rule: learnability and determinacy. (figures)
Instability under fundamentals-based rule
Figure 2

Stability under expectations-based rule
• **Hybrid rules**: determinacy and learnability only under certain parameter restrictions.

**Practical Concerns**

• Measurement errors in private expectations: CB can use internal VAR forecasts as a proxy (Honkapohja and Mitra 2005a, 2006).

• Non-observability of current variables: use $E^*_t y_t$ (Bullard and Mitra 2002).

Estimation of structural parameters: CB estimates structural parameters, convergence of simultaneous learning under expectations-based rule (Evans and Honkapohja 2003a,b).
Further Developments

I. Extensions of Monetary Policy Models

- Optimal policy when agents are learning (Orphanides and Williams 2005a, Molnar and Santoro 2005, Gaspar, Smets and Vestin 2005, 2006),

- Learning and macroeconomic policy against liquidity traps (Evans and Honkapohja 2005b, Evans, Guse and Honkapohja 2007),

- Learning and supply-side effects of monetary policy (Kurozumi 2004, Llosa and Tuesta 2007),

- Learning in NK models with capital (Kurozumi 2007, Duffy and Xiao 2007),

- Communication by central bank: (i) constant-interest-rate projections (Honkapohja and Mitra 2005b), (ii) communicating targets and rules (Eu-sepi and Preston 2007).

- Allowing for uncertainty in structural parameters (Evans and McGough 2007)

- Learning, endogenous inattention and the great moderation (Branch, Carlson, Evans and McGough 2006, 2007)
II. Theoretical Topics

- Learning and the Lucas critique (Evans and Ramey 2006),

- Monetary policy with near-rational expectations (Bullard, Evans and Honkapohja 2007, Woodford 2006),


- Heterogeneous expectations, learning and policy (Evans, Honkapohja and Marimon 2001, Giannitsarou 2003, Honkapohja and Mitra 2006),
- Dynamic predictor selection and inflation (Brock and Hommes 1997, Branch and Evans 2006a,b, Brazier, Harrison, King and Yates 2006).

III. Empirical Topics

- 1970’s Great Inflation (Bullard & Eusepi 2005, Orphanides & Williams 2005b),

- Explaining hyperinflations (Marcet and Nicolini 2003),

- Policymaker’s model uncertainty and CB learning (Cogley and Sargent 2005, Sargent, Williams and Zha 2006),

- Learning and sources of macroeconomic persistence (Milani 2005, 2007).
Selected Topics in More Detail

(i) Monetary policy under perpetual learning.

Orphanides and Williams (2005a)

- Lucas-type aggregate supply curve for inflation $\pi_t$:
  \[
  \pi_{t+1} = \phi \pi^e_{t+1} + (1 - \phi)\pi_t + \alpha y_{t+1} + e_{t+1},
  \]

- Output gap $y_{t+1}$ is set by monetary policy up to white noise control error
  \[
  y_{t+1} = x_t + u_{t+1}.
  \]

- Policy objective function $\mathcal{L} = (1 - \omega)Var(y) + \omega Var(\pi - \pi^*)$ gives rule
  \[
  x_t = -\theta(\pi_t - \pi^*).
  \]

where under RE $\theta = \theta^P(\omega, \phi, \alpha)$. 
Learning: Under RE inflation satisfies

$$\pi_t = \bar{c}_0 + \bar{c}_1 \pi_{t-1} + \nu_t.$$  

Under learning private agents estimate coefficients by constant gain (or discounted) least squares. Older data dated discounted at rate $$(1 - \kappa)$$.

- Discounting of data natural if agents are concerned to track structural shifts.

- There is empirical support for constant gain learning.

- With constant gain, LS estimates fluctuate randomly around $$(\bar{c}_0, \bar{c}_1)$$: there is “perpetual learning” and

$$\pi_{t+1}^e = c_{0,t} + c_{1,t} \pi_t.$$
Results:

- Perpetual learning increases inflation persistence.

- Naive application of RE policy leads to inefficient policy. Incorporating learning into policy response can lead to major improvement.
- Efficient policy is more hawkish, i.e. under learning policy should increase $\theta$ to reduce persistence. This helps guide expectations.

- Following a sequence of unanticipated inflation shocks, inflation doves (i.e. policy-makers with low $\theta$) can do very poorly, as expectations become detached from RE.

- If agents know $\pi^*$ and only estimate the AR(1) parameter the policy trade-off is more favorable.
(ii) Learning and inflation persistence

Preceding analysis was an example of empirics based on calibration. Now we consider work that estimates learning dynamics (Milani 2005, 2007).

- The source of inflation persistence is subject to dispute. For good empirical fit, a backward-looking component is needed in the NK Phillips curve.

- The bulk of the literature assumes RE, but learning could be a reason for inflation persistence.

- Incorporate indexation to Calvo price setting: non-optimized prices indexed to past inflation. This yields

\[ \pi_t - \gamma \pi_{t-1} = \delta x_t + \beta E_t(\pi_{t+1} - \gamma \pi_t) + u_t, \]

where \( x_t \) is the output gap. Earlier work under RE finds \( \gamma \approx 1 \).
Inflation under learning

- The preceding can be written as

\[
\pi_t = \frac{\gamma}{1 + \beta \gamma} \pi_{t-1} + \frac{\beta}{1 + \beta \gamma} E_t^* \pi_{t+1} + \frac{\delta}{1 + \beta \gamma} x_t + u_t.
\]

For expectations assume a PLM

\[
\pi_t = \phi_{0,t} + \phi_{1,t} \pi_{t-1} + \varepsilon_t
\]

Agents use data \(\{1, \pi_i\}_{0}^{t-1}\) to estimate \(\phi_0, \phi_1\) using constant gain LS.

- The implied ALM is

\[
\pi_t = \frac{\beta \phi_{0,t}(1 + \phi_{1,t})}{1 + \beta \gamma} + \frac{\gamma + \beta \phi_{1,t}^2}{1 + \beta \gamma} \pi_{t-1} + \frac{\delta}{1 + \beta \gamma} x_t + u_t.
\]
• Alternatively, could use with real marginal cost as the driving variable.

Empirical results

• **Data:** GDP deflator, output gap is detrended GDP, real marginal cost is proxied by deviation of labor income share from 1960:01 to 2003:04.

• **Initialization:** agents’ initial parameter estimates obtained by using presample data 1951-1959.
• **Methodology:** PLM estimated from constant-gain learning using

\[ \kappa = 0.015. \]

Then estimate ALM using nonlinear LS, which separates learning effects from structural effects.

**Note:** Simultaneous estimation of learning rule and the model would be much more ambitious.

• PLM parameters:

(i) \( \phi_{1,t} \) initially low in 1950s and 60s, then higher (up to 0.958), then some decline to values above 0.8.
(ii) \( \phi_{0,t} \) initially low, then became much higher and then gradual decline after 1980.
• ALM structural estimates: Degree of indexation $\gamma = 0.139$ (with output gap & $\kappa = 0.015$) and declining for higher $\kappa$.

• Model fit criterion (Schwartz’ BIC) suggest values $\kappa \in (0.015, 0.03)$, with best fit at $\kappa = 0.02$.

• Conclusion: Estimates for $\gamma$ not significantly different from zero. Results are very different from those obtained under RE, which finds $\gamma \approx 1$.

• Milani has also estimated full NK models under learning. He finds that also the degree of habit persistence is low in IS curve.
(iii) Explaining Hyperinflations

The seigniorage model of inflation extended to open economies and occasional exchange rate stabilizations explain hyperinflation episodes during the 1980s (Marcet and Nicolini 2003).

Basic hyperinflation model (seigniorage model of inflation)

\[
\frac{M^d_t}{P_t} = \phi - \phi \gamma (P^e_{t+1}/P_t) \text{ if } 1 - \gamma (P^e_{t+1}/P_t) > 0 \text{ and } 0 \text{ otherwise,}
\]
gives money demand. This is combined with exogenous government purchases \(d_t = d > 0\) financed entirely by seigniorage:

\[
M_t = M_{t-1} + d_t P_t
\]

\[
\frac{P_t}{P_{t-1}} = \frac{1 - \gamma (P^e_t/P_{t-1})}{1 - \gamma (P^e_{t+1}/P_t) - d/\phi}.
\]
Under RE/perfect foresight, for \( d > 0 \) not too large, there are two steady states \( \beta = \frac{P_t}{P_{t-1}}, \beta_L < \beta_H \), with a continuum of paths converging to \( \beta_H \).

Under learning the PLM is

\[
\frac{P_{t+1}}{P_t} = \beta,
\]

and the implied ALM is

\[
\frac{P_t}{P_{t-1}} = \frac{1 - \gamma\beta}{1 - \gamma\beta - d/\phi} \equiv T(\beta).
\]

Steady state learning: agents estimate \( \beta \) based on past inflation:

\[
\left(\frac{P_{t+1}}{P_t}\right)^e = \beta_t
\]

\[\beta_t = \beta_{t-1} + t^{-1}(P_{t-1}/P_{t-2} - \beta_{t-1}).\]
Steady state learning in hyperinflation model

Since $0 < T'(\beta_L) < 1$ and $T'(\beta_H) > 1$, $\beta_L$ is E-stable, and therefore locally stable under learning, while $\beta_H$ is not.
Hyperinflation stylized facts

- Facts:
  - Recurrence of hyperinflation episodes.
  - ERR (exchange rate rules) stop hyperinflations, though new hyperinflations eventually occur.
  - During a hyperinflation, seigniorage and inflation are not highly correlated.
  - Hyperinflations only occur in countries where seigniorage is on average high.

- These facts are difficult to reconcile with RE.
Marcet-Nicolini’s Approach:

- The low inflation steady state is locally learnable.

- A sequence of adverse shocks can create explosive inflation. When inflation rises above $\beta^U$ inflation is stabilized by moving to an ERR.

- The learning dynamics lead to periods of stability alternating with occasional eruptions into hyperinflation.
• The learning approach can explain all the stylized facts.

Hyperinflations under learning
(iv) Liquidity Traps


Possibility of a “liquidity trap” under a global Taylor rule subject to zero lower bound. Benhabib, Schmitt-Grohe and Uribe (2001, 2002) analyze this for RE.

Multiple steady states with global Taylor rule.
What happens under learning?

- Evans and Honkapohja (2005b) analyze a flexible-price perfect competition model:
  – deflationary paths possible
  – switch to aggressive money supply rule at low $\pi$ avoids liquidity traps.

- Evans, Guse and Honkapohja (2007) consider a model with (i) monopolistic competition (ii) price-adjustment costs. Monetary policy follows a global Taylor-rule. Fiscal policy is standard: exogenous government purchases $g_t$ and Ricardian tax policy that depends on real debt level.
• The key equations are the PC and IS curves

\[
\frac{\alpha \gamma}{\nu} (\pi_t - 1) \pi_t = \beta \frac{\alpha \gamma}{\nu} (\pi_{t+1}^e - 1) \pi_{t+1}^e \\
+ (c_t + g_t)^{(1+\varepsilon)/\alpha} - \alpha \left(1 - \frac{1}{\nu}\right) (c_t + g_t)c_t^{-\sigma_1}
\]

\[
c_t = c_{t+1}^e (\pi_{t+1}^e / \beta R_t)^{\sigma_1},
\]

There are also money and debt equations.

• Two stochastic steady states at \(\pi_L\) and \(\pi_H\). Under “steady-state” learning, \(\pi^*\) is locally stable but \(\pi_L\) is not.

• Pessimistic expectations \(c^e, \pi^e\) can lead to a deflationary spiral and stagnation.
$\pi^e$ and $c^e$ dynamics under normal policy
• To avoid this we recommend adding aggressive policies at an inflation threshold \( \tilde{\pi} \), where \( \pi_L < \tilde{\pi} < \pi^* \).

• Setting the R-rule so that \( \pi_L \) is a deflationary rate, a natural choice is \( \tilde{\pi} = 1 \), i.e. zero net inflation.

• If \( \pi_t \) falls to \( \tilde{\pi} \) then

  - \( R_t \) should be reduced as needed to near the zero lower bound \( R = 1 \).
  - If necessary, then \( g_t \) should also be increased.

Thus both aggressive fiscal and monetary policy may be needed. Policy needs to focus on inflation, not expansionary spending per se.
Inflation threshold $\tilde{\pi}$, $\pi_L < \tilde{\pi} < \pi^*$, for aggressive monetary policy and, if needed, aggressive fiscal policy.
Conclusions

- Expectations play a large role in modern macroeconomics. People are smart, but boundedly rational. Cognitive consistency principle: economic agents should be about as smart as (good) economists, e.g. model agents as econometricians.

- Stability of RE under private agent learning is not automatic. Monetary policy must be designed to ensure both determinacy and stability under learning.

- Policymakers may need to use policy to guide expectations. Under learning there is the possibility of persistent deviations from RE, hyperinflation, and deflationary spirals with stagnation. Appropriate monetary and fiscal policy design can minimize these risks.