Interest Rate Pegs in New Keynesian Models[*]

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Abstract

John Cochrane asks: "Do higher interest rates raise or lower inflation?" We find that pegging the interest rate at a higher level will induce instability and most likely lead to falling inflation and output over time. Eventually, this will precipitate a change of policy.

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1 Introduction

Following the Financial Crisis of 2008-9 and the subsequent Great Recession, policy interest rates have been at or close to zero for extended periods. For example, in the US the federal funds rate was in the zero to 0.25% range from late 2008 through 2015. Although the federal funds rate was lifted by 0.25% in early 2016, in Europe there are now several countries that have near zero (or even negative) interest rates paid by central banks on bank reserves. Despite these low interest rates, the rate of inflation, while positive, has tended to remain below target, and there has been concern by policymakers that the economy may be evolving toward an unintended low-inflation equilibrium consistent with the neo-Fisherian view that low interest rates lead to low inflation. This view was discussed by Bullard (2010, 2015), and has been forcefully

argued by Cochrane (2015) who states, in his abstract, "Perhaps both theory and data are trying to tell us that, when conditions including adequate fiscal-monetary coordination operate, pegs can be stable and inflation responds positively to nominal interest rate increases." More recently, Bullard (2016) has noted that "market-based measures of inflation expectations have been declining in the US since the summer of 2014."[1] However, instead of advocating higher interest rates as does Cochrane (2015), Bullard (2016) argues that the deteriorating inflation expectations make a case for a slower pace of normalization to higher rates.

This range of views in part reflects the multiplicity of rational expectations equilibria (REE) noted, for example, by Benhabib, Schmidt-Grohe and Uribe (2001), hereafter BSU, and to the stability properties of the REE under adaptive learning. The unintended low inflation (or deflation) equilibrium corresponds to an interest-rate peg at zero or a near-zero level. Paths converging to that steady state were a major concern of BSU. However, the adaptive learning viewpoint appears to cast doubt on the relevance of this steady state. Instability under an interest-rate peg was first demonstrated in a monetary model by Howitt (1992). In New Keynesian (NK) models an analogous result appears in Evans and Honkapohja (2003), and for a discrete-time time version of the BSU model the instability result is provided in Evans, Guse and Honkapohja (2008) and Benhabib, Evans and Honkapohja (2014).[2]

In this paper we examine the relevance of the instability results to the policy debate, noted above, on interest rate pegs. We find that an NK framework augmented by learning delivers precisely the possibility of inflation expectations that are below target and declining. This arises because the theoretical instability results are asymptotic, may take time to emerge, and can manifest themselves by showing a gradual deterioration in inflation expectations. We show how this pattern can result from neo-Fisherian policies: under an interest rate peg a decline in inflation expectations leads to higher ex-ante real interest rates and to declining output. This outcome is avoided by a policy in which temporarily low interest rates are followed by an explicit, subsequent return to normalcy that is implemented by a standard Taylor rule.

2   The Model

We use the NK model based on a Rotemberg price friction. There is a continuum of identical, infinitely-lived households. The representative household chooses its consumption bundle and labor supply based on current and expected values of income, inflation and interest rates. There is a continuum of monopolistically competitive

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[1]A similar phenomenon has been observed recurrently in Europe and Japan, in which actual and expected inflation move close to the target inflation rate, only to then begin to decline.

[2]Concerns about REE of the type studied by Cochrane (2015) have been raised also by Garcia-Schmidt and Woodford (2015) and Evans and McGough (2015).

firms that hire labor and set prices for their differentiated goods based on current and expected values of output, inflation and interest rates. A quadratic adjustment cost is imposed to impart the nominal pricing friction. When closed with a Taylor rule, the symmetric REE of this entirely standard model satisfies the usual 3-equation system: see, e.g., Evans, Guse and Honkapohja (2008) for details.

We now modify the model to allow for boundedly-rational households and firms. Our modification is based on Evans, Honkapohja and Mitra (2016) (EHM) who employ the framework developed in Eusepi and Preston (2010). We refrain from elaborating on the details: see EHM for a full development.

We start with behavioral rules for consumption and price-setting. The consumption function for household i is

$$\tilde{c}_t^i = (1 - \beta)\hat{E}_t \sum_{s \geq 0} \beta^s \tilde{y}_{t+s}^i - \frac{\beta^2 \bar{y}}{\pi^*}\hat{E}_t \sum_{s \geq 0} \beta^s \tilde{R}_{t+s} + \frac{\bar{y}}{\pi^*}\hat{E}_t \sum_{s \geq 1} \beta^s \tilde{\pi}_{t+s},$$

where $\tilde{y}_t^i$ is the income of household i, $\tilde{R}_t$ is the nominal interest rate factor and $\tilde{\pi}_t$ is the inflation factor, where all variables with tildes are expressed in deviation from steady state form. We use $\hat{E}_t$ to denote the subjective expectations of agents, where for simplicity we assume expectations are the same for all agents. We are studying behavior under learning so these expectations need not coincide with rational expectations. We assume that $\hat{E}_t x_t = x_t$ for generic variable x_t. Finally, $0 < \beta < 1$ is the discount factor, $\bar{y}$ is the steady state level of output, and π^* is the inflation target.

The firm j chooses its inflation rate to satisfy

$$\tilde{\pi}_t^j = (1 - \gamma_1)\hat{E}_t \sum_{s \geq 0} (\beta\gamma_1)^s \tilde{\pi}_{t+s} + \frac{a_2 \pi^*}{\bar{y}}\hat{E}_t \sum_{s \geq 0} (\beta\gamma_1)^s \tilde{y}_{t+s},$$

where $\tilde{y}_t$ denotes aggregate output. Here $0 < \gamma_1 < 1$ and $a_2 > 0$ depend on deep parameters and are given in EHM. For simplicity we omit exogenous shocks, which would be straightforward to include.

Assuming identical agents, $\tilde{y}_t^i = \tilde{y}_t$, $\tilde{c}_t^i = \tilde{c}_t$, and also that firms set the same price so that $\tilde{\pi}_t^j = \tilde{\pi}_t$, we can impose market clearing, i.e. $\tilde{y}_t = \tilde{c}_t$, and solve for the temporary equilibrium values of $\tilde{y}_t$ and $\tilde{\pi}_t$ in terms of the current interest rate and expectations:

$$\tilde{y}_t = -\frac{\beta\bar{y}}{\pi^*}\tilde{R}_t - \frac{\beta\bar{y}}{\pi^*}\hat{E}_t \sum_{s \geq 1} \beta^s \tilde{R}_{t+s} + \frac{1 - \beta}{\beta}\hat{E}_t \sum_{s \geq 1} \beta^s \tilde{y}_{t+s} + \frac{\bar{y}}{\beta\pi^*}\hat{E}_t \sum_{s \geq 1} \beta^s \tilde{\pi}_{t+s},$$

$$\tilde{\pi}_t = -\frac{\beta a_2}{\gamma_1}\tilde{R}_t - \frac{\beta a_2}{\gamma_1}\hat{E}_t \sum_{s \geq 1} \beta^s \tilde{R}_{t+s} + \frac{1 - \gamma_1}{\gamma_1}\hat{E}_t \sum_{s \geq 1} (\beta\gamma_1)^s \tilde{\pi}_{t+s}$$

$$+ \frac{a_2 \pi^*}{\gamma_1 \bar{y}}\left(\frac{1 - \beta}{\beta}\hat{E}_t \sum_{s \geq 1} \beta^s \tilde{y}_{t+s} + \frac{\bar{y}}{\beta\pi^*}\hat{E}_t \sum_{s \geq 1} \beta^s \tilde{\pi}_{t+s} + \hat{E}_t \sum_{s \geq 1} (\beta\gamma_1)^s \tilde{y}_{t+s}\right).$$

We assume the policy rule takes the form $\tilde{R}_t = \frac{R^*\phi}{\pi^*}\hat{E}_t\tilde{\pi}_{t+1}$, where $\phi \geq 0$ and R^* is the target nominal interest-rate factor consistent with the inflation target and the steady state real interest-rate factor: $R^* = \pi^*/\beta$. It follows that

$$\tilde{R}_t = \frac{\phi}{\beta}\hat{E}_t\tilde{\pi}_{t+1}.$$

Here ϕ is a policy parameter, which is usually set to a value $\phi > 1$ in accordance with the Taylor principle since this is needed for determinacy of the REE.[3] Setting $\phi = 0$ corresponds to an interest-rate peg.[4]

In this nonstochastic setting there is a perfect-foresight REE given by $\tilde{y}_t = \tilde{\pi}_t = \tilde{R}_t = 0$, and if $\phi > 1$ and not too large it can be shown that this is the unique non-explosive perfect foresight solution: see Bullard and Mitra (2002).[5] If $0 \leq \phi < 1$ the steady state is indeterminate and the set of REE includes a continuum of perfect-foresight solutions that converge to it.

We introduce adaptive learning into the NK model following Evans and Honkapohja (2001), Eusepi and Preston (2010) and EHM. Given our simple set-up it is sufficient to assume that agents use a Perceived Law of Motion (PLM) for $(\tilde{y}_t, \tilde{\pi}_t)$ of the form

$$\tilde{y}_t = \delta^y + noise_t \text{ and } \tilde{\pi}_t = \delta^\pi + noise_t,$$

where $noise_t$ indicate unforecastable perceived white-noise disturbances, and thus $\hat{E}_t\tilde{y}_{t+s} = \delta^y$ and $\hat{E}_t\tilde{\pi}_{t+s} = \delta^\pi$. We discuss below how their beliefs δ^y, δ^π evolve over time under learning. For given beliefs output and inflation are determined by combining the temporary equilibrium equations and the policy rule. This yields

$$\begin{pmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{pmatrix} = \begin{pmatrix} 1 & -\frac{\bar{y}}{\pi^*}\frac{\phi-1}{1-\beta} \\ \frac{a_2\pi^*}{\bar{y}\gamma_1(1-\beta\gamma_1)} & \frac{\beta(1-\gamma_1)}{1-\beta\gamma_1} - \frac{a_2(\phi-1)}{\gamma_1(1-\beta)} \end{pmatrix} \begin{pmatrix} \delta^y \\ \delta^\pi \end{pmatrix} \equiv T(\delta),$$

where $\delta = (\delta^y, \delta^\pi)'$. We note that generically the unique fixed point of this equation is the REE $\delta^y = \delta^\pi = 0$.

Under adaptive learning the parameters δ^y, δ^π are updated over time using observed data. Specifically we assume

$$\delta^y_t = \delta^y_{t-1} + \kappa(\tilde{y}_{t-1} - \delta^y_{t-1}) \text{ and } \delta^\pi_t = \delta^\pi_{t-1} + \kappa(\tilde{\pi}_{t-1} - \delta^\pi_{t-1}).$$

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[3] For convenience we assume $\tilde{R}_t$ depends on expected inflation rather than contemporaneous inflation and we do not include a dependence on contemporaneous or expected output.

[4] Throughout this paper we have assumed for simplicity that policymakers know the steady state real interest rate and that the interest-rate target is consistent with the inflation target: $R^* = \pi^*/\beta$. However, none of our results depend on this.

[5] If stochastic productivity, preferences, mark-ups or government spending shocks are included then the corresponding REE also depends on these shocks.

It follows that

$$\delta_t = \delta_{t-1} + \kappa \left(T(\delta_{t-1}) - \delta_{t-1} \right),$$

where $0 < \kappa < 1$ is the "gain" parameter. Note that given initial beliefs δ_0 the above difference equation generates the time-path of both beliefs (δ^y, δ^π) and realized aggregates $(\tilde{y}, \tilde{\pi})$.

The asymptotic behavior of the economy is characterized by the following Proposition, which is straightforward to show:

Proposition 1 *The perfect foresight steady state* $(\tilde{y}, \tilde{\pi}) = (\delta^y, \delta^\pi) = (0, 0)$ *is stable under adaptive learning for* $\kappa > 0$ *sufficiently small if and only if* $\phi > 1$.

This result is well-known and fully in accordance with, e.g., Bullard and Mitra (2002) and EHM.[6] It is typically obtained via the following continuous-time approximation, which can be shown to hold even in the stochastic case:

$$\dot{\delta} = T(\delta) - \delta,$$

where $\dot{\delta} = d\delta/d\tau$ with $\tau = \kappa t$. This approximation corresponds to the "E-stability" differential equation in the adaptive learning literature. More generally an REE is said to be E-stable if and only if it is corresponds to a Lyapunov stable fixed point of the differential equation associated with the learning algorithm.

Note that in particular, an interest-rate peg, i.e. $\phi = 0$, is unstable under adaptive learning. We now use this framework to assess the consequences of neo-Fisherian policies.

3   Policy experiments

Suppose the Central Bank (CB) wants to increase inflation and relies on the neo-Fisherian logic that an increase in the interest rate will necessarily lead, at least eventually, to a higher inflation rate. To study this under learning, suppose that initially the economy is in a steady state with zero inflation, a zero output gap and nominal interest factor $R = \beta^{-1}$. (Note that we now express variables in levels instead of deviation from mean form.) We set $\beta = 0.99$ and data is quarterly.[7] Suppose that the CB at $t = 0$ announces an increase in the inflation target to $\pi^* = 1.005$ quarterly, i.e. 2% per year, and raises the corresponding interest rate peg from $R^* = 1.0101$ to $R^* = 1.01515$. There is an REE in which inflation immediately jumps to its new

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[6] Arifovic et al. (2013) find that under certain social learning mechansims implemented by genetic algorithms the REE can be stable even when the Taylor principle is not satisfied.

[7] We use the same model parameters as in EHM. We set government spending to zero for simplicity. For $\pi^* = 1.005$ this leads to values $\bar{y} = 0.955$, $\gamma_1 = 0.805$, $a_2 = 0.118$.

target and the output gap remains zero. Under learning we suppose agents place a high weight on this outcome, but retain a small doubt that inflation will increase by the full amount. To capture this, suppose that agents at $t = 0$ adjust their expectations to $\pi^e = \delta_0^\pi = 1.004992$, i.e. very slightly below the new inflation target of $\pi^* = 1.005$. They then revise their expectations over time in response to data using the adaptive learning rule specified above. In all Figures we use a gain parameter of $\kappa = 0.01$.
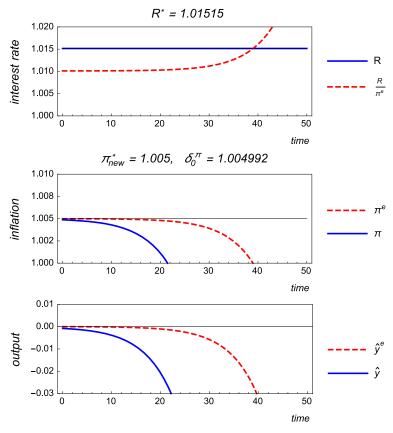

Figure 1: Increase in interest rate peg. Here π^e and $\hat{y}^e$ denote, respectively, the paths of expected inflation and expected output.

Figure 1 gives the results for inflation, output and the interest rate. In the figures we use $\hat{y}$ to denote output in proportional deviation from mean form. Because δ_0^π is very close to RE, inflation and output remain little changed for several periods. However, since π_t is (slightly) below $\pi^* = 1.005$, inflation expectations gradually decline and by period $t = 10$ inflation and output are measurably below the target values. Expectations of inflation and output then start to rapidly decline, eventually pushing the economy into a serious recession accompanied by falling inflation that turns to deflation. The central mechanism for instability is that, with a nominal

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interest rate peg, lower expected inflation leads to a higher ex-ante real interest rate that lowers consumption demand and equilibrium output.

It is important to note that we have assumed in Figure 1 that agents place a very high weight on the ability of the CB to achieve its higher inflation target, initially setting their expectations for quarterly inflation at 0.4992% instead of 0.5%. Bearing in mind that CB policy only indirectly affects the inflation rate, we regard our assumption on initial expectations as strongly favoring the neo-Fisherian position. Our demonstration of instability requires only that going forward in time agents revise their expectations in light of the actual evolution of inflation. Indeed, even if agents initially fully adjusted their initial expectations to the announced target, the slightest stochasticity will trigger a destabilizing path.
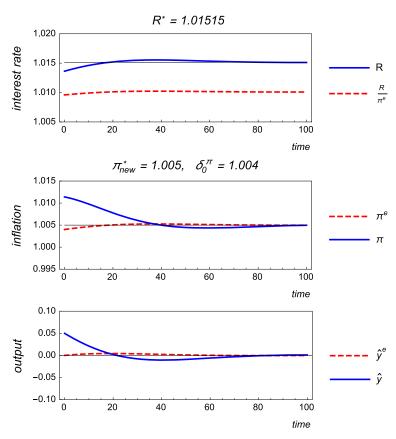

Figure 2: Increase in inflation target implemented by Taylor rule with $\phi = 1.5$, assuming partial initial adjustment of expectations.

The preceding does not, of course, imply that the inflation target is unattainable. Suppose that when the CB increases its inflation target it instead employs an interest-rate rule that satisfies the Taylor principle, e.g. $\phi = 1.5$. This policy experiment is shown in Figure 2, where we now set $\delta_0^\pi = 1.004$, i.e. inflation expectations

adjust part-way to the new target (similar qualitative results obtain if expectations initially adjust almost all the way, as before, or not at all). Policy dictates an initial increase in interest rates smaller than recommended by the peg. In line with the Taylor principle, the resulting ex ante real interest rate falls slightly, leading to initial increases in inflation and output. Over time the economy converges to the new REE as expectations adjust under the adaptive learning rule.
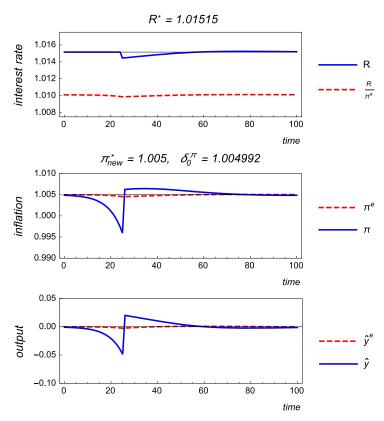

Figure 3: Increase in interest rate peg, with large initial adjustment of expectations, and with later implementation of Taylor rule at $t = 25$ with $\phi = 1.5$.

Finally, returning to the case of the interest rate peg that was illustrated in Figure 1, suppose that the CB, after observing declining inflation expectations and reductions in both inflation and output, decides after a number of periods to shift from a peg to an interest-rate rule that satisfies the Taylor principle. Figure 3 illustrates the results for initial $\delta_0^\pi = 1.004992$, the value used in Figure 1. At time $t = 25$, with output just over 5% below the steady state level and the economy now experiencing deflation, the CB implements an interest-rate rule with $\phi = 1.5$. The adoption of this new policy leads to an immediate reduction in interest rates and a corresponding reduction in real interest rates below steady state values. This stimulates demand, resulting in a contemporaneous increase in output and inflation, with eventual convergence to the new steady state.

4 Monetary policy normalization

Beginning in December 2015 the US Federal Reserve embarked on a normalization program in which the policy interest rate was increased above the 0 to 0.25% range for the first time since December 2008. This was in response to the recovery of the economy from the Great Recession, in particular to steady if unexciting growth of GDP and a substantial strengthening of labor markets in the 2014-2015 period. In December 2015 the Federal Reserve also indicated that it planned to follow a normalization policy in which interest rates were to be further increased over 2016. In early 2016 some concerns were raised regarding the pace of normalization. In particular, St Louis FRB President James Bullard, in his February 2016 talk to the CGA, noted that inflation expectations had been declining since July 2015 and in February 2016 were around 1.6% p.a., significantly below the 2% p.a. target. Bullard argued that this, together with other developments, suggested the need to slow the pace of normalization (as well as to make it more data dependent). This economic situation has therefore given increased urgency to the policy debate between the neo-Fisherian view that interest rates should be increased and the standard view that interest rates should be kept low when inflation expectations are below target.
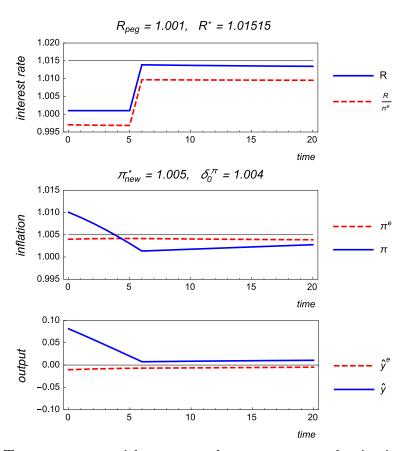

Figure 4: Temporary peg with announced return to normalcy in six quarters.

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As we have shown, the adaptive learning viewpoint argues forcefully against the neo-Fisherian view and in support of the standard view. We illustrate this using two final figures. Both contemplate an announced policy of holding the effective interest rate at $R_{peg} = 1.001$, i.e. 0.4% p.a., approximately its February 2016 value, for 5 quarters, before normalizing interest rates. In both figures we set initial inflation expectations at $\delta_0^\pi = 1.004$, corresponding to the February 2016 value of 1.6% p.a., and we set $\delta_0^y = -.01$, i.e. 1% below steady state output, reflecting some concern about the strength of the recovery.[8]
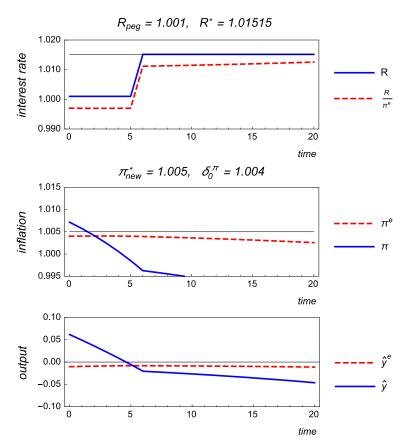

Figure 5: Neo-Fisherian peg-to-peg policy.

Figure 4 considers a return to normalcy implemented by the adoption of a standard Taylor rule beginning in period $t = 6$ and we set $\phi = 1.5$. In contrast, for Figure 5, in the spirit of a neo-Fisherian policy, we set a new interest-rate peg at $R^* = 1.01515$

[8]In these Figures we also make the assumption that in forming their expectations agents believe inflation and output will be at their steady state values beginning five years hence: $\delta_{t+20}^\pi = 1.005$ and $\delta_{t+20}^y = 0$ for all t. This seems a realistic modification for addressing the specific policy issues examined in these two Figures.

beginning in period $t = 6$. In both figures we are assuming that agents build the announced interest rate policy into their decision rules. In Figure 4 we see that the policy has the desired effect: the temporary peg at a low R maintains a low real interest rate, which stimulates aggregate demand leading to higher output and inflation; and the return the return to normalcy provides a smooth return to the steady state.

In contrast, under the neo-Fisherian approach of Figure 5, the temporary low peg has a similar initial impact, providing some initial economic stimulus, but the situation soon deteriorates: inflation expectations decline leading to a rise in the real interest rate, which depresses aggregate demand and leads an unstable path for the same reasons as in Figure 1.

5   Conclusion

Following the Great Recession, many countries have experienced repeated periods with realized and expected inflation below target levels set by policymakers. Should policy respond to this by keeping interest rates near zero for a longer period or, in line with neo-Fisherian reasoning, by increasing the interest rate to the steady-state level corresponding to the target inflation rate? We have shown that neo-Fisherian policies, in which interest rates are set according to a peg, impart unavoidable instability. In contrast, a temporary peg at low interest rates, followed by later imposition of the Taylor rule around the target inflation rate, provides a natural return to normalcy, restoring inflation to its target and the economy to its steady state.

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