Expectations, Learning and Macroeconomic Policy

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Lecture 4

Liquidity traps, learning and stagnation

Evans, Guse & Honkapohja (EER, 2008), Evans & Honkapohja (2009)
Background

- A standard characterization of **monetary policy** is that the CB (Central Bank) follows a Taylor-type rule in which the **interest rate** \( R_t \) **responds more than one-for-one to inflation** \( \pi_t \) near the inflation target \( \pi^* \).

- The ZLB (**zero lower bound**) for \( R_t - 1 \) implies a **second unintended steady-state** \( \pi_L \) of any (continuous) “global” Taylor rule. See Figure.

- Under perfect foresight (& rational expectations) the low steady state is “indeterminate,” i.e. has multiple perfect foresight paths that converge to it (a low \( R \) **liquidity trap**). See Benhabib, Schmitt-Grohe and Uribe (2001, 2002).
Multiple steady states with global Taylor rule.
Here $R =$ nominal interest rate factor (e.g. 1.06), $\pi =$ inflation factor (e.g. 1.02), and $\beta^{-1} =$ steady state real interest rate factor (e.g. $\beta = 0.96$).
Outline

• We consider a fairly standard NK (New Keynesian) model with a global Taylor rule and a standard fiscal policy setting.

• We consider the solutions both under RE (rational expectations) and when private agents form expectations of future consumption and inflation using adaptive learning.

• We find: under learning the $\pi^*$ solution is locally stable, but if expectations are too pessimistic they follow unstable paths – deflationary spirals.
• To prevent deflationary spirals we consider procedures that suspend normal policies and replace them with aggressive policies when $\pi$ falls to some threshold $\tilde{\pi} < \pi^*$. 

• Fiscal as well as aggressive monetary policy may be needed. 

• The aggressive policies must be based on an inflation threshold. Using an output threshold is not sufficient.
The Model

We use a standard discrete-time, stochastic NK ("New Keynesian") model.

A continuum of households produce a differentiated consumption good under conditions of
(i) monopolistic competition and
(ii) price-adjustment costs.
Private Sector

Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_{t,j} \left( c_{t,j}, \frac{M_{t-1,j}}{P_t}, h_{t,j}, \frac{P_{t,j}}{P_{t-1,j}} - 1 \right)$$

subject to

$$c_{t,j} + m_{t,j} + b_{t,j} + \gamma_{t,j} = m_{t-1,j} \pi_t^{-1} + R_{t-1} \pi_t^{-1} b_{t-1,j} + \frac{P_{t,j}}{P_t} y_{t,j},$$

where $c_{t,j}$ is the consumption aggregator of agent $j$, where

$$y_{t,j} = h_{t,j}^{\alpha}, \quad P_{t,j} = \left( \frac{y_{t,j}}{Y_t} \right)^{-1/\nu} P_t$$

and

$$U_{t,j} = \frac{c_{t,j}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left( \frac{M_{t-1,j}}{P_t} \right)^{1-\sigma_2} - \frac{h_{t,j}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left( \frac{P_{t,j}}{P_{t-1,j}} - 1 \right)^2.$$
Monetary and Fiscal Policy

Government purchases $g_t$ are given by

$$g_t = \bar{g} + u_t$$

where $u_t$ is exogenous AR(1), stationary. Lump-sum taxes $\Upsilon_t$ are given by

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1} + \eta_t,$$

$\eta_t$ white noise, and $\beta^{-1} - 1 < \kappa < 1$. Real 1-period debt $b_t$ evolves as:

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}.$$

Monetary Policy:

$$R_t - 1 = \theta_t f(\pi_t),$$

where $\theta_t$ is AR(1) with $E\theta_t = 1$ and $f(\pi)$ as shown earlier → two steady states $0 < \pi_L < \pi^*$. 
Key Equations

The equilibrium of the model is given by the monetary and fiscal policy, market clearing, and the private-sector optimization equations:

\[
\frac{\alpha \gamma}{\nu} (\pi_t - 1) \pi_t = h_t \left( h_t^\varepsilon - \alpha \left( 1 - \frac{1}{\nu} \right) h_t^{\alpha - 1} c_t^{-\sigma_1} \right) \\
+ \beta \frac{\alpha \gamma}{\nu} E_t \left( (\pi_{t+1} - 1) \pi_{t+1} \right),
\]

\[
c_t^{-\sigma_1} = \beta R_t E_t \left( \pi_{t+1}^{-1} c_{t+1}^{-\sigma_1} \right),
\]

and a money demand equation.

The first equation is a NK Phillips curve. The second is the Euler equation for \( c_t \) (the NK IS curve).
Rational Expectations

Consider the stochastic system for “small shocks,” i.e. if the random exogenous shocks have small support, and RE.

For the \((c_t, \pi_t)\) block there is a **stochastic steady-state** solution

\[
\begin{pmatrix} c_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} c \\ \pi \end{pmatrix} + \begin{pmatrix} G_{cu} & G_{c\theta} \\ G_{\pi u} & G_{\pi\theta} \end{pmatrix} \begin{pmatrix} u_t \\ \tilde{\theta}_t \end{pmatrix},
\]

near each of the two steady states.

**Proposition 1.** *In the linearized model there are two steady states \(\pi^* > \pi_L\). For \(\gamma > 0\) sufficiently small, the steady state \(\pi = \pi^*\) is locally determinate (i.e. locally unique) and the steady state \(\pi = \pi_L\) is locally indeterminate.*

We now consider the situation under adaptive learning.
Learning and Expectational Stability

We now replace RE by private agent learning.

We return to the nonlinear system of equations so that we can study the global properties of the system under learning.

We assume that agents estimate the linear projection

\[
    c_{t+1} = a_c + d u_t + e \theta_t + \epsilon_{c,t+1}
\]
\[
    \pi_{t+1} = a_\pi + f u_t + g \theta_t + \epsilon_{\pi,t+1},
\]

and use it to make forecasts.
**Timing:** At end of period $t-1$, agents use LS to update estimated coefficients $a_{c,t-1}, d_{t-1}, e_{t-1}, a_{\pi,t-1}, f_{t-1}, g_{t-1}$. Then, at the start of $t$ agents form forecasts

$$c^e_{t+1} = a_{c,t-1} + d_{t-1}u_t + e_{t-1}\theta_t$$
$$\pi^e_{t+1} = a_{\pi,t-1} + f_{t-1}u_t + g_{t-1}\theta_t.$$ 

This determines actual $c_t, \pi_t$. Then at the end of $t$ the coefficients are updated using the new data point.

**Do estimates and forecasts converge (approximately) to RE corresponding to the $\pi^*$ or $\pi_L$ equilibrium?** This can be analyzed using E-stability.
We make a simplification that does not affect any of our key results. It turns out that stability is governed by the stability of the intercepts, not the coefficients for exogenous shocks.

Thus for simplicity we now assume that \( u_t \) and \( \theta_t \) are iid and drop them from the regression.

In effect, private agents simply estimate the unknown means of \( \pi_t \) and \( c_t \). Coefficient updating is then given by

\[
\begin{align*}
\pi_{t+1}^e &= \pi_t^e + \phi_t (\pi_{t-1}^e - \pi_t^e) \\
c_{t+1}^e &= c_t^e + \phi_t (c_{t-1}^e - c_t^e),
\end{align*}
\]

where \( \phi_t \) is the "gain sequence" (e.g. \( \phi_t = t^{-1} \), i.e. decreasing gain, or \( \phi_t = \phi \) for \( 0 < \phi < 1 \), i.e. constant gain).

The system under learning consists of the original system but with RE replaced by adaptive learning.
The $\pi_t, c_t$ block under learning is given by

$$
\frac{\alpha \gamma}{\nu} (\pi_t - 1) \pi_t = \beta \frac{\alpha \gamma}{\nu} (\pi_{t+1}^e - 1) \pi_{t+1}^e + (c_t + g_t)^{(1+\varepsilon)/\alpha} \\
-\alpha \left(1 - \frac{1}{\nu}\right) (c_t + g_t)c_t^{-\sigma_1}
$$

$$
c_t = c_{t+1}^e (\pi_{t+1}^e / \beta R_t)^{\sigma_1},
$$

together with the interest-rate rule for $R_t$. $c_{t+1}^e, \pi_{t+1}^e$ are given by adaptive learning, as above. Note: we are assuming Euler-equation learning.

These “temporary equilibrium” PC and IS equations determine $\pi_t, c_t$ given expectations. The temporary equilibrium system is completed by the money equation and the bond evolution equation.

Under learning does the system evolve towards $\pi^*$ or towards $\pi_L$?
Stability under learning

Formally we can write the temporary equilibrium system as

\[
\begin{align*}
\pi_t &= F_\pi(\pi_{t+1}^e, c_{t+1}^e, u_t, \theta_t) \\
c_t &= F_c(\pi_{t+1}^e, c_{t+1}^e, u_t, \theta_t),
\end{align*}
\]

with

\[
\begin{align*}
\pi_{t+1}^e &= \pi_t^e + \phi_t(\pi_{t-1}^e - \pi_t^e) \\
c_{t+1}^e &= c_t^e + \phi_t(c_{t-1}^e - c_t^e).
\end{align*}
\]

This is a stochastic recursive algorithm, whose convergence properties can be analyzed, as usual, using E-stability.
Local stability under learning is determined by E-stability. A stochastic steady state is E-stable if the differential equation

\[
\begin{pmatrix}
\frac{d\pi^e}{d\tau} \\
\frac{dc^e}{d\tau}
\end{pmatrix} = \begin{pmatrix}
T\pi(\pi^e, c^e) \\
Tc(\pi^e, c^e)
\end{pmatrix} - \begin{pmatrix}
\pi^e \\
c^e
\end{pmatrix}
\]

is locally asymptotically stable at the steady state \((\pi, c)\), where

\[
T\pi(\pi^e, c^e) = EF\pi(\pi^e, c^e, u_t, \theta_t)
\]
\[
Tc(\pi^e, c^e) = EFc(\pi^e, c^e, u_t, \theta_t).
\]

\(T(\pi^e, c^e)\) maps the Perceived Law of Motion to the Actual Law of Motion.
Proposition 2. For $\gamma > 0$ sufficiently small, the (stochastic) steady state at $\pi = \pi^*$ is locally stable under learning and the steady state at $\pi = \pi_L$ is locally unstable under learning, taking the form of a saddle point.

See Figure. **Local stability of** $\pi^*$ **is reassuring, but** the instability under learning of $\pi_L$ **comes with a danger:** the possibility of **deflationary spirals leading to stagnation** (a deflation trap).

If expectations $\pi^e, c^e$ are initially low enough then actual $\pi, c$, and output $y$ are low and this is self-reinforcing under learning.
$\pi^e$ and $c^e$ dynamics under normal policy
Adding Aggressive Monetary Policy

Can the deflation trap be avoided if we modify monetary policy to be more aggressive when we approach the expectational danger zone? We consider the following change to monetary policy:

\[
R_t = \begin{cases} 
1 + \theta_t f(\pi_t) & \text{if } \pi_t > \tilde{\pi} \\
\hat{R} & \text{if } \pi_t < \tilde{\pi},
\end{cases}
\]

where \(\hat{R} > 1\) is close to the ZLB of 1, and

\[
\hat{R} \leq R_t \leq 1 + \theta_t f(\pi_t) \text{ if } \pi_t = \tilde{\pi}.
\]

Thus if \(\pi_t\) threatens to fall below some threshold \(\tilde{\pi}\), we suspend the global Taylor rule and reduce \(R_t\) as needed to try to maintain \(\pi_t = \tilde{\pi}\), if necessary reducing \(R_t\) all the way to \(\hat{R}\). See Figure.
Aggressive monetary policy for $\pi \leq \bar{\pi}$. 
It turns out that **aggressive monetary policy is not enough** to avoid deflationary spirals.

**Proposition 3.** *There is a steady state at \( \hat{\pi} = \beta \hat{R} \) and there is no steady state value for \( \pi_t \) below \( \hat{\pi} \). For all \( \gamma > 0 \) sufficiently small the steady state at \( \hat{\pi} = \beta \hat{R} \) is a saddle point under learning.*

While the region of stability may increase, the possibility of a deflationary trap remains.
Two steady states with standard fiscal policy and $\pi_L < \tilde{\pi} < \pi^*$. 
Combined Monetary and Fiscal Policy

Our recommended policy is to add an inflation floor or threshold $\tilde{\pi}$, with $\pi_L < \tilde{\pi} < \pi^*$, and to use both aggressive monetary and fiscal policy, if needed, to ensure this floor is achieved. If $\pi_L < 1$ then $\tilde{\pi} = 1$ can be used.

Our policy is feasible because fiscal policy can guarantee an inflation floor:

**Lemma** Given expectations $c_{t+1}^e$ and $\pi_{t+1}^e$ and setting $R_t = \hat{R}$, any value of $\pi_t > 1/2$ can be achieved by setting $g_t$ sufficiently high.

This follows by implicitly differentiating the Phillips curve equation.
Our recommended policy: Follow normal monetary and fiscal policy provided \( \pi_t \geq \tilde{\pi} \). Reduce \( R_t \) as needed and increase \( g_t \) if necessary to ensure \( \pi_t \geq \tilde{\pi} \).

Policy needs to focus on inflation, not expansionary spending *per se*.

**Proposition 4.** If \( \pi_L < \tilde{\pi} < \pi^* \) then \( \pi^* \) is the unique steady state and it is stable under learning.

Thus for \( \pi_L < \tilde{\pi} < \pi^* \) our recommended policy eliminates the deflation trap. See Figure.
Inflation threshold $\tilde{\pi}$, $\pi_L < \tilde{\pi} < \pi^*$, for aggressive monetary policy and, if needed, aggressive fiscal policy.
Stochastic Simulations

We set $\pi^* = 1.02$ (with $\pi_L = 0.975$), $\tilde{\pi} = 1$ and $\phi = 1/30$. We start with a pessimistic expectations shock at $t = 0$ ($\pi^e$ falls to 1.01 and $c^e$ falls by about 8%), large enough to lead to deflationary spirals.

We examine the paths if initially normal policies are used, and then our recommended policy is introduced at $t_1 = 150$ vs. $t_1 = 80$. These are compared to the results if the policy is initially in place. See Figs.

Introducing our policy earlier, at $t_1 = 80$ avoids the worst part of the stagnation. Having the policy in place when the shock occurs is best.

Setting $\pi^*$ higher, e.g. $\pi^* = 1.05$ can avoid the need for fiscal policy for the larger shock. However there is an efficiency loss of a higher inflation target.
Extension: infinite horizon learning

– Our preceding analysis was under the assumption that agents’ decision rules had a short planning horizon, based on subjective Euler equations.

– Commitment to low interest rates cannot be studied in that setting.

– In Evans & Honkapohja (2009) we consider a modification of the set-up. We replace Euler-equation learning with infinite-horizon decision rules, as in Marcet and Sargent 1989, Preston 2005, 2006 or Evans, Honkapohja & Mitra (2009, 2010).
– In this setting agents solve forward their Euler equations & use their life-time budget constraint. Now under learning they must, at each time, forecast the whole future time path. The temporary equilibrium equations are

\[ Q_t = \frac{\nu}{\gamma} \sum_{j=0}^{\infty} \alpha^{-1} \beta^j \left( y_{t+j}^e \right)^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} \sum_{j=0}^{\infty} \beta^j \left( \frac{y_{t+j}^e}{x_{t+j}^e} \right), \]

where \( Q_t = (\pi_t - 1) \pi_t, \) and \( x_{t+j}^e = y_{t+j}^e - g_{t+j}^e, \) and

\[ c_t = (1 - \beta) \left( y_t - g_t + \sum_{j=1}^{\infty} (D_{t, t+j}^e)^{-1} x_{t+j}^e \right). \]

– Our central results extend to this setting. We continue to find that fiscal policy may be needed. Monetary policy alone, even if policy commits to zero net interest rates for ever, may be insufficient to avoid the deflation trap.
Conclusions

• We take seriously the multiple equilibrium problem emphasized by the RE literature on the ZLB.

• However, the adaptive learning approach provides another perspective and is in some ways more alarming: large pessimistic shocks can lead to unstable deflationary spirals.

• To avoid this normal policy must be replaced by aggressive monetary and fiscal policy triggered if inflation falls below a threshold $\tilde{\pi} > \pi_L$.

• Output thresholds are inadequate. The key is to stabilize inflation.
Conclusions to Lectures

• Expectations play a large role in modern macroeconomics. People are smart, but boundedly rational. Cognitive consistency principle: economic agents should be about as smart as (good) economists, e.g. model agents as econometricians.

• Stability of RE under private agent learning is not automatic. Monetary policy must be designed to ensure both determinacy and stability under learning.

• Policymakers may need to use policy to guide expectations. Under learning there is the possibility of persistent deviations from RE, hyperinflation, and deflationary spirals with stagnation. Appropriate monetary and fiscal policy design can minimize these risks.