

# Physics 610

Adv Particle Physics

April 7, 2014

# Accelerators

History

Two Principles

Electrostatic

Cockcroft-Walton

Van de Graaff and tandem

Van de Graaff

Transformers

Cyclotron

Betatron

Linear Induction

Synchrocyclotron

Synchrotron

Phase Stability

Strong focusing

Betatron Oscillations

High-impedance Microwave

Devices

Superconducting Technology

X-band RF

Large Colliders

# Two Principle Approaches

- Electrostatic
  - particles traverse a difference in electric potential

or

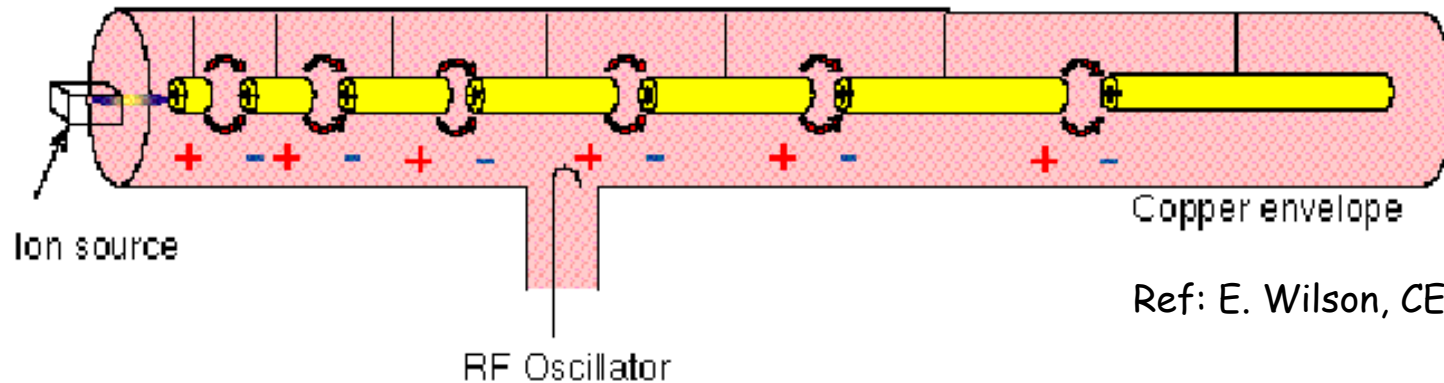
- Transformer
  - high-current, low-voltage circuit element used to supply energy to a high-voltage, low-current accelerating path

W.K.H. Panofsky and M. Breidenbach, Rev. Mod. Phys. 71, S122

# Transformer

- Electrostatic approaches fail above about 10 MeV
- Linear Accelerator
- Cyclotron
  - orbital period of nonrelativistic particles circulating in a uniform magnetic field is independent of energy
- Betatron
  - electrons become relativistic at moderate energies, and cyclotron fails

# RF Linear Accelerator



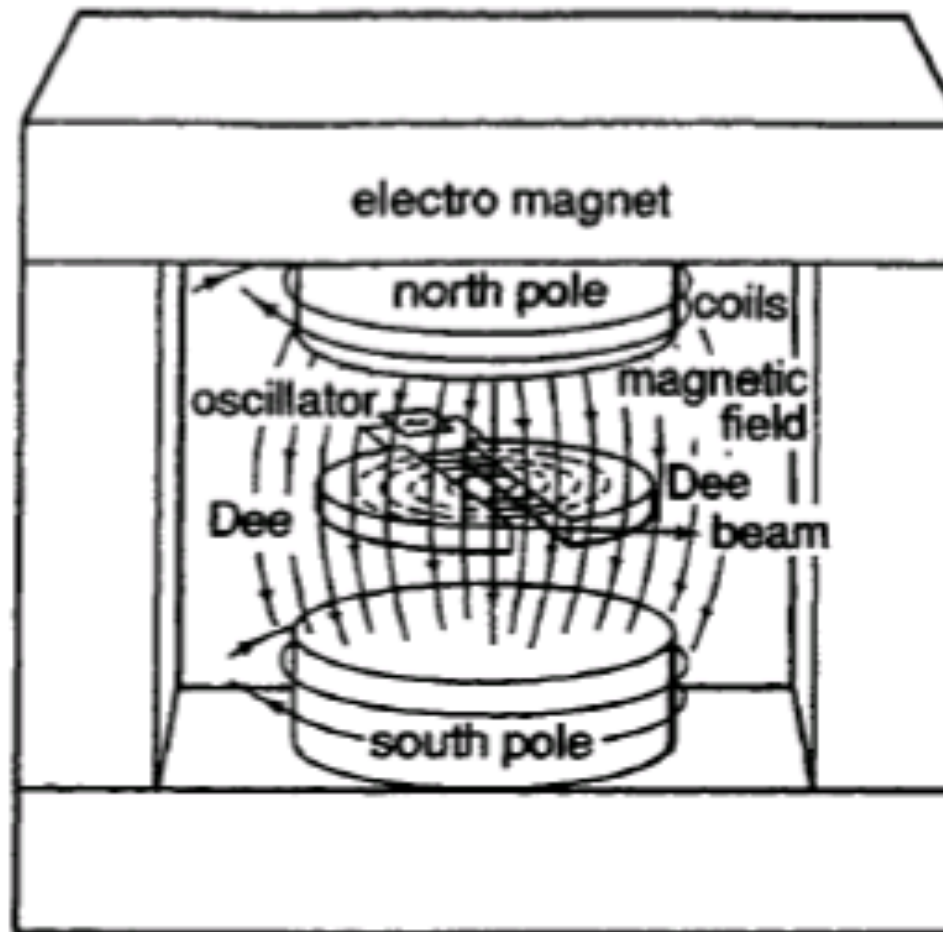
Ref: E. Wilson, CERN

- The early accelerators inspired the linear accelerator, where adding stages can increase energy reach
- Copper lined tubes in which oscillating fields are excited by radio transmitter (“drift tubes”)
- But in order to increase beam energy, length must be expanded
- Circular accelerators were invented to make higher energy possible in a more compact device

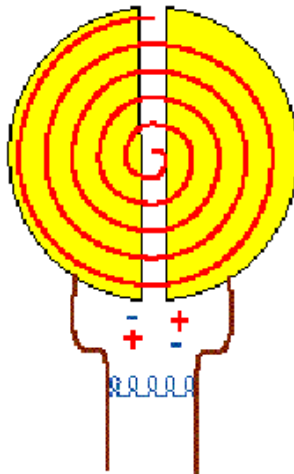
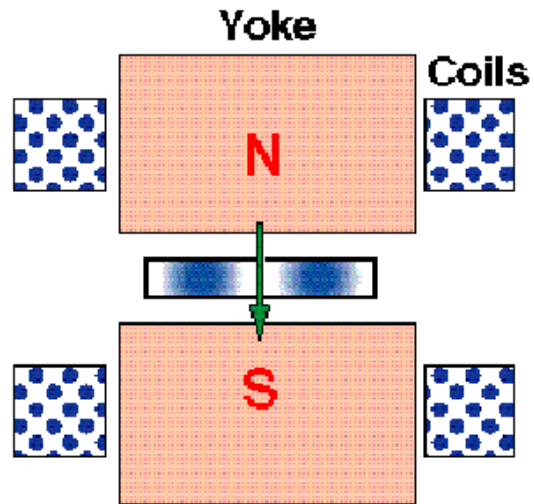
# Cyclotron

- Orbital period of nonrelativistic particles circulating in a uniform magnetic field is independent of energy  
 $f = qB / 2\pi m \approx 10^8 \text{ B(Tesla)}$  for electrons  
radio frequency
- Match the revolution frequency with a RF voltage across a gap
- Focussing by small radial decrease in magnetic field
  - results in decrease in orbital frequency
- 184-inch cyclotron at Berkeley for deuteron energies above 100 MeV

# Cyclotron

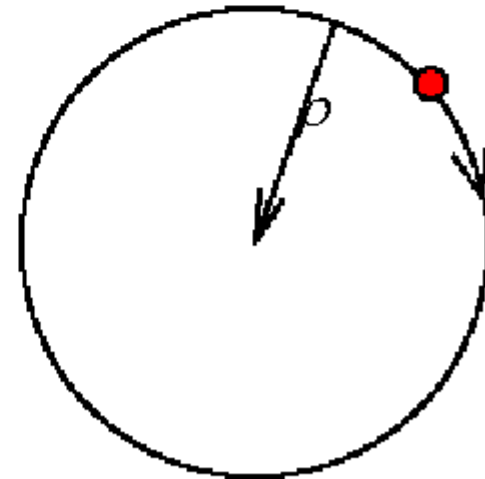


# Cyclotron



RF Driver

$$e\mathbf{v} \times \mathbf{B} = \frac{mv^2}{\rho}$$
$$B\rho = \frac{mv}{e} = \frac{p}{e}$$

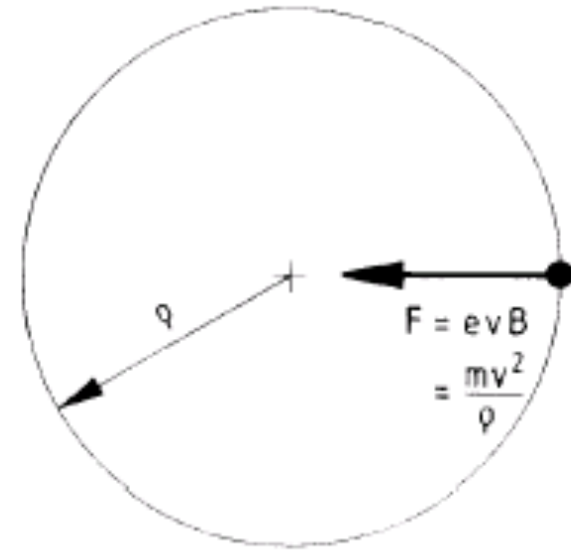


Ref: E. Wilson, CERN



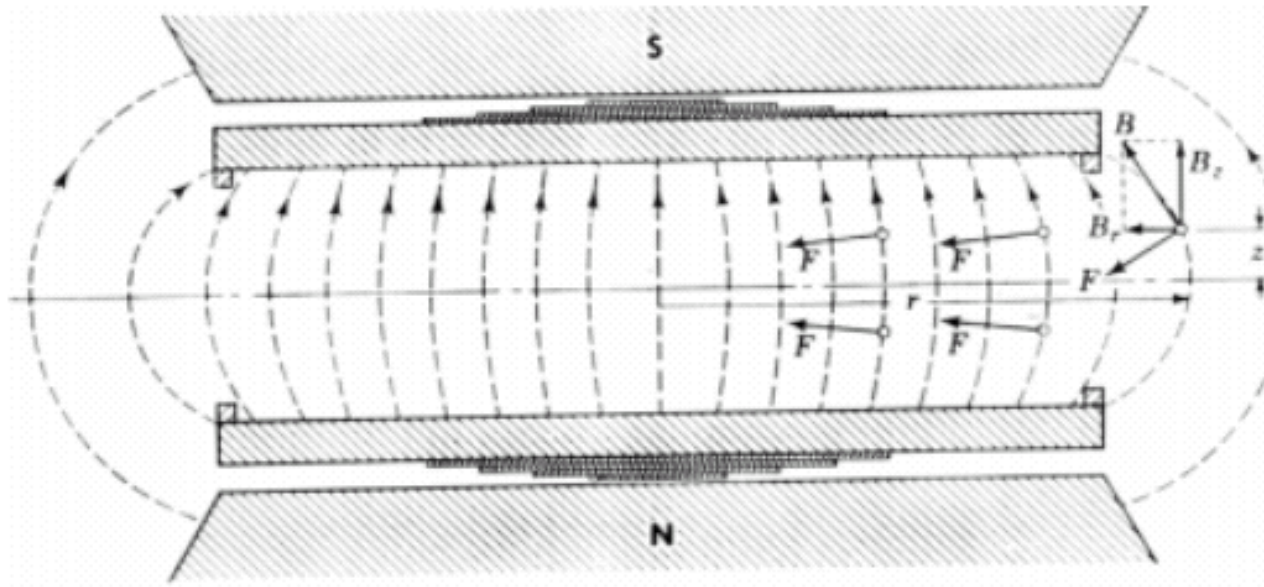
# Cyclotron

- Nonrelativistic acceleration
- Particles spiral out in uniform magnetic field,  $B$
- Balance of forces makes radius proportional to the velocity
- This means the period of rotation,  $P = 2\pi\rho/v$ , is a constant
- Focusing of orbits is needed to achieve the best currents
- Shims do this (see next page)
- The constancy of period breaks down for kinetic energies of about  $5\% mc^2$



# Cyclotron Focussing

- Shims lead to increased focussing
  - both vertical and horizontal
  - discovered more intense fields

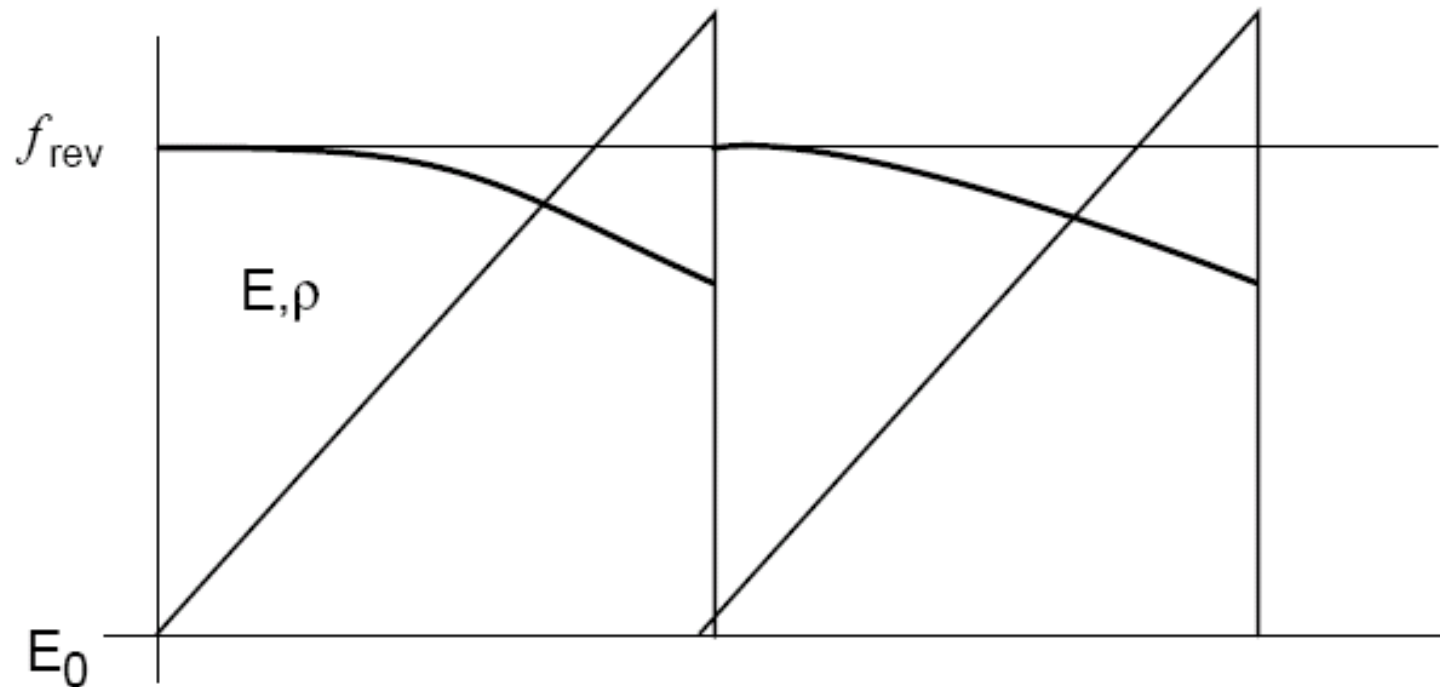


# Synchrocyclotron

- Problem: how to overcome the changing orbital period as energy increases
- One solution: add radial magnetic field gradient
  - new problem: destroys vertical focusing
- Second solution: change the RF frequency as particles circulate
  - another problem: particles must now be accelerated in bursts, not continuously
- Synchrocyclotron
  - Static magnetic field
  - RF frequency decreases to match the revolution frequency as function of energy

# Synchrocyclotron

The variation of revolution and RF frequencies as a function of time in a synchro-cyclotron

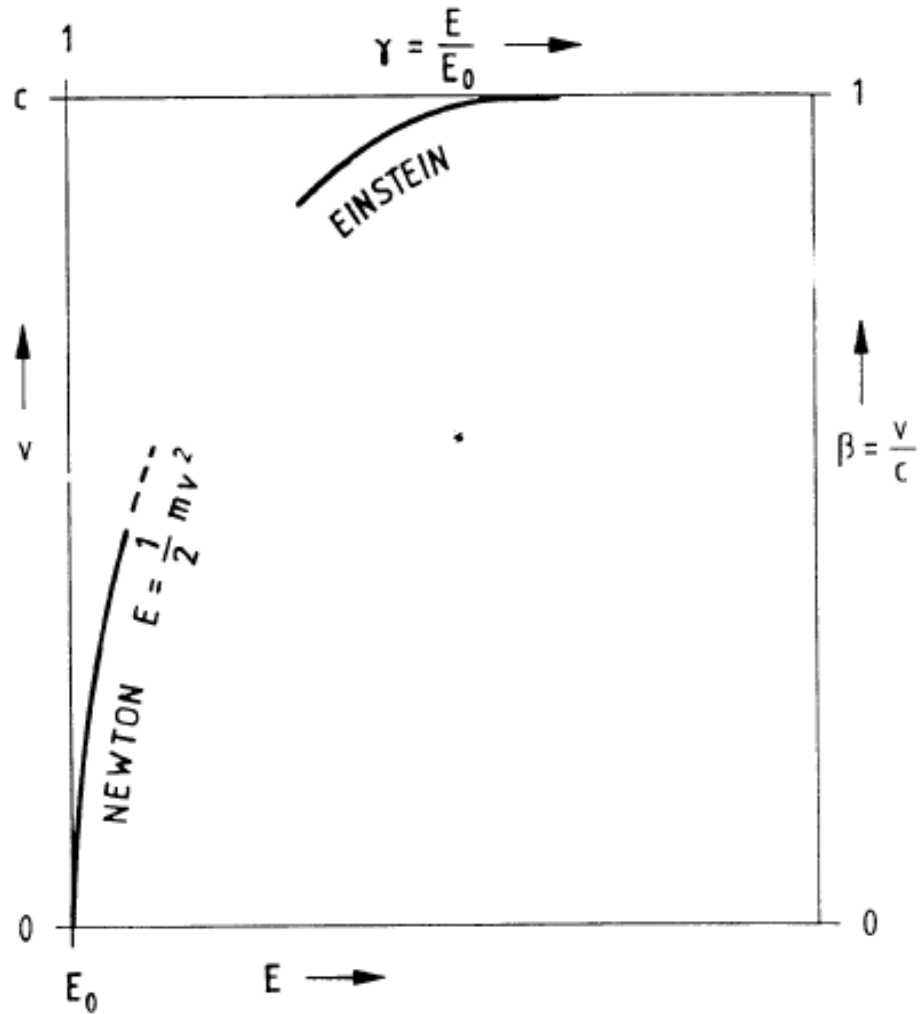


# Betatron

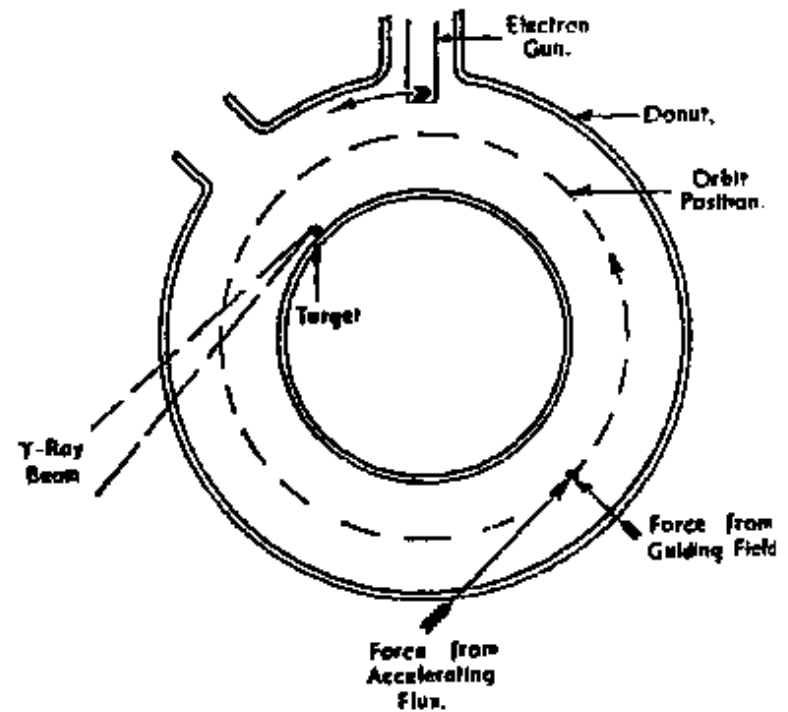
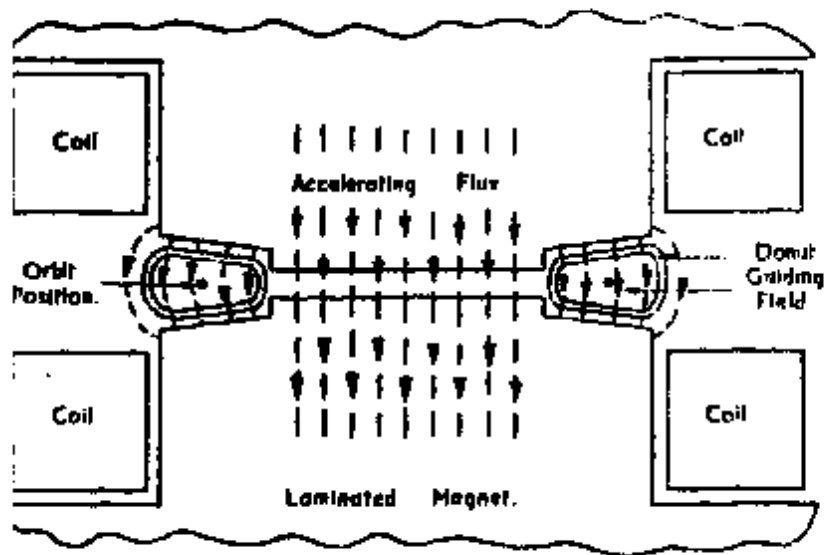
- Electrons become relativistic at moderate energies, and the cyclotron concept fails (period no longer constant)
- Energy of electrons in circular orbit increased by the induced E field from an increasing flux in a central iron core
- Particles are kept in single circular orbit (not spirally out as in cyclotron)
- Limited at 300 MeV due to radiation losses which cannot be compensated

# Relativistic effect

Increase of velocity with energy



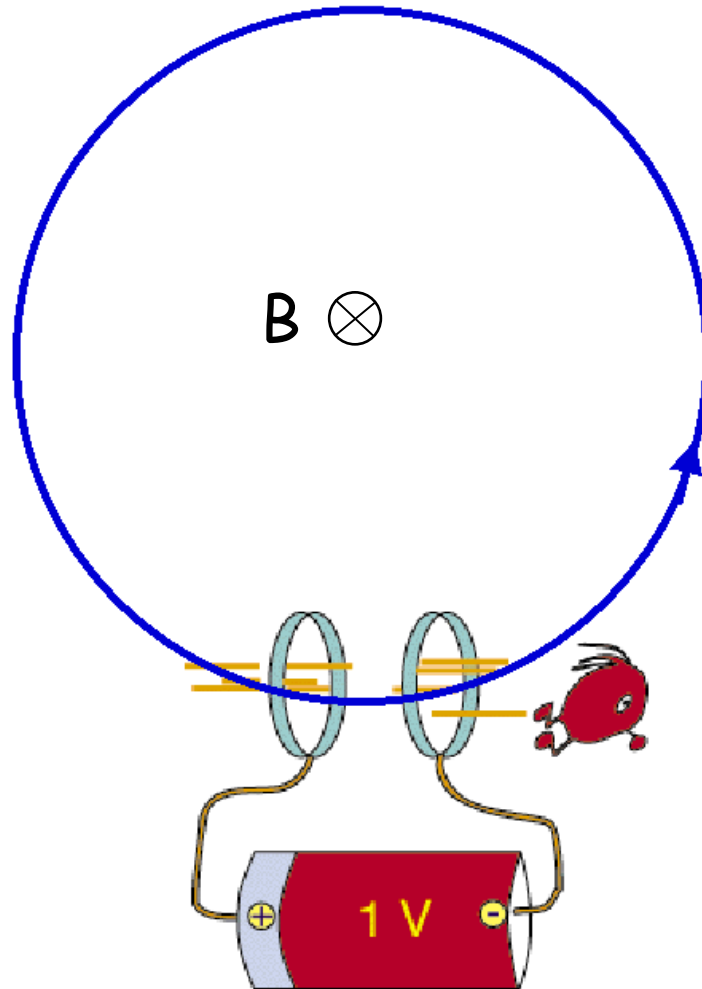
# Betatron



$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Ref: E. Wilson, CERN

# Simple Circular Accelerator

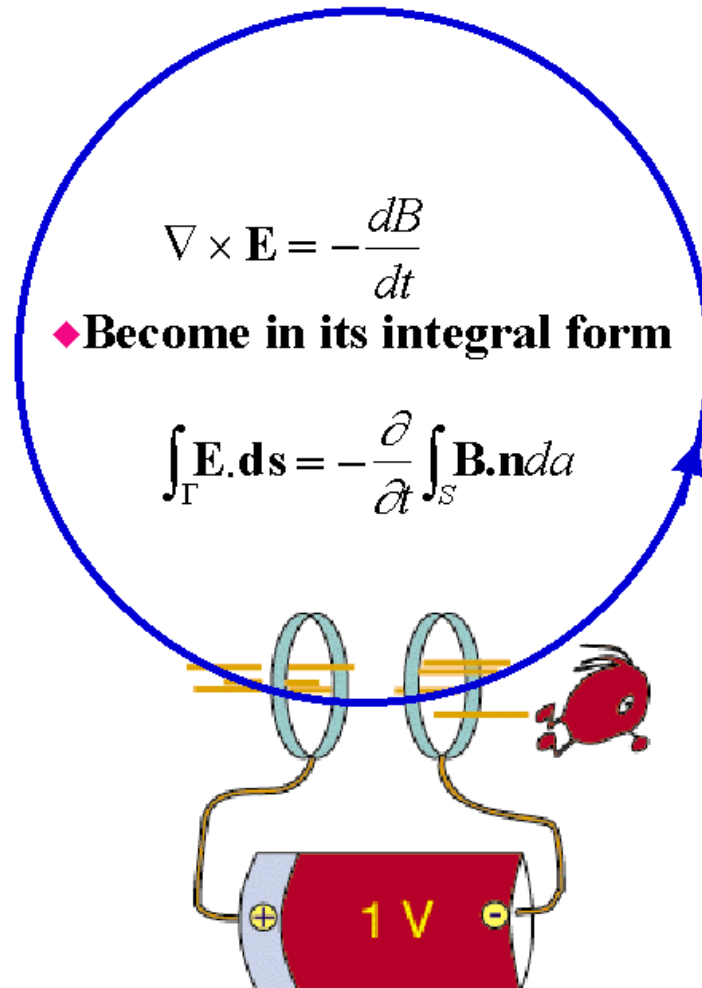


- Circular orbits determined by magnetic field
- Acceleration on each revolution
- Will this work?

Ref: E. Wilson, CERN



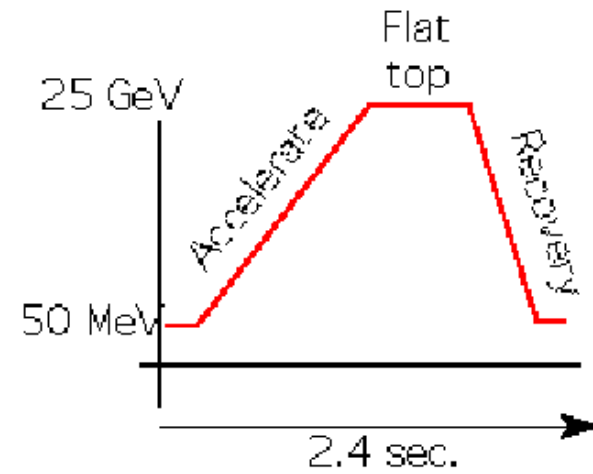
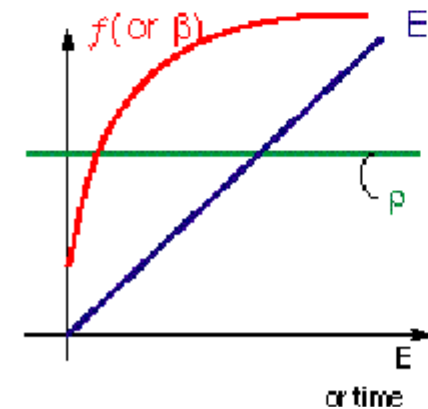
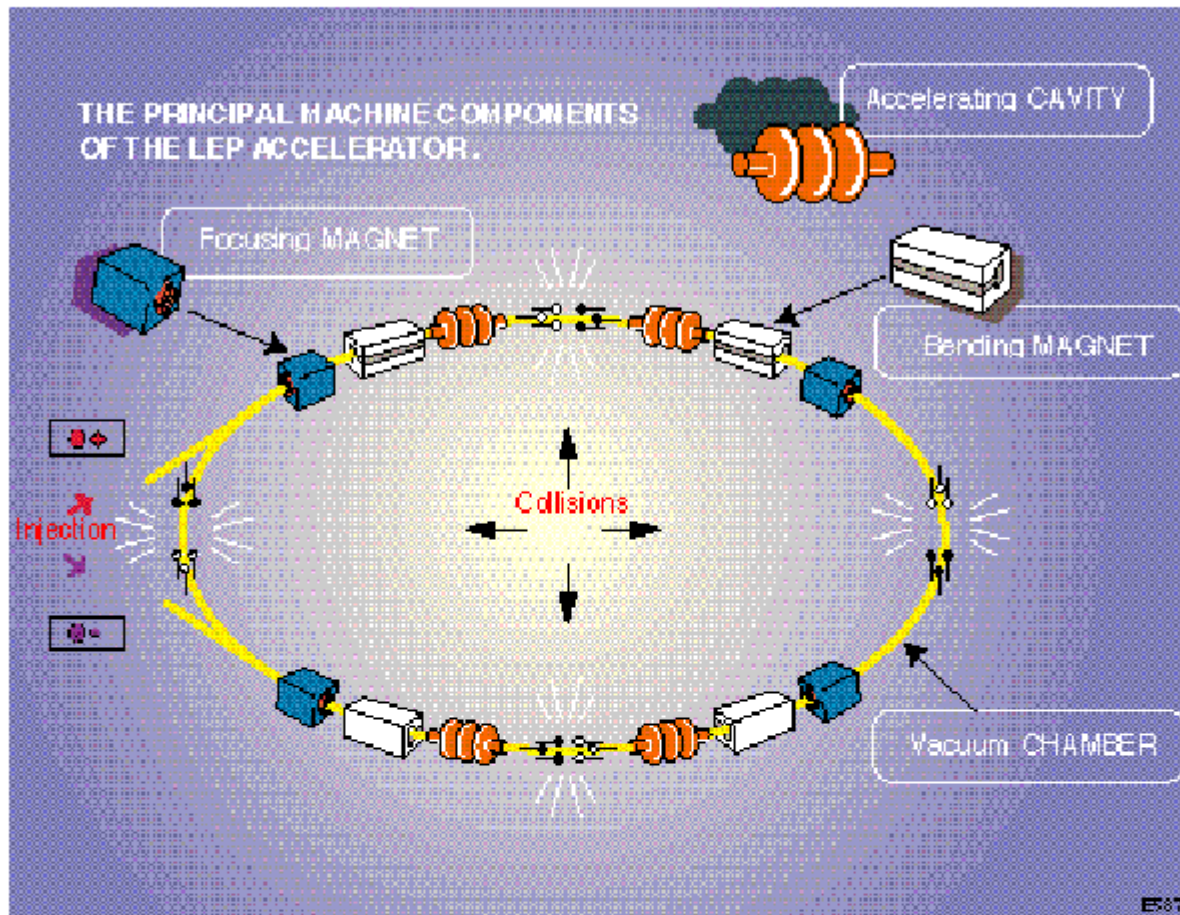
# Simple Circular Accelerator



- There can be no acceleration without time dependent magnetic field
- The converse: time-dependent flux may accelerate particles

Ref: E. Wilson, CERN

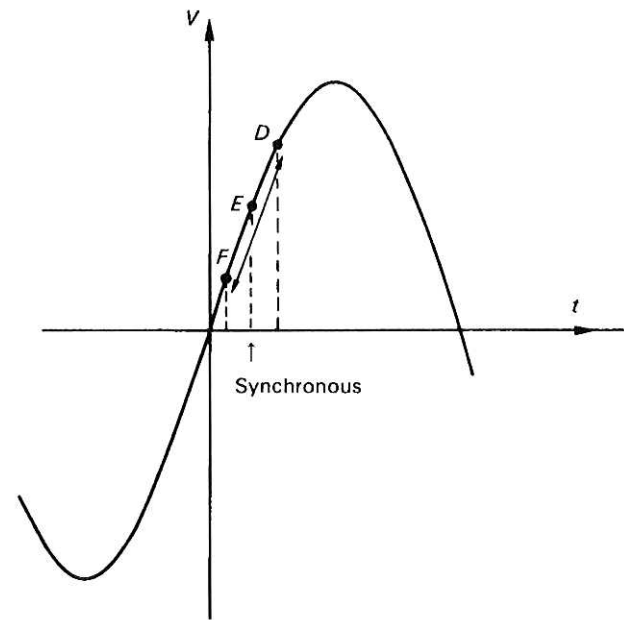
# Synchrotron



Ref: E. Wilson, CERN

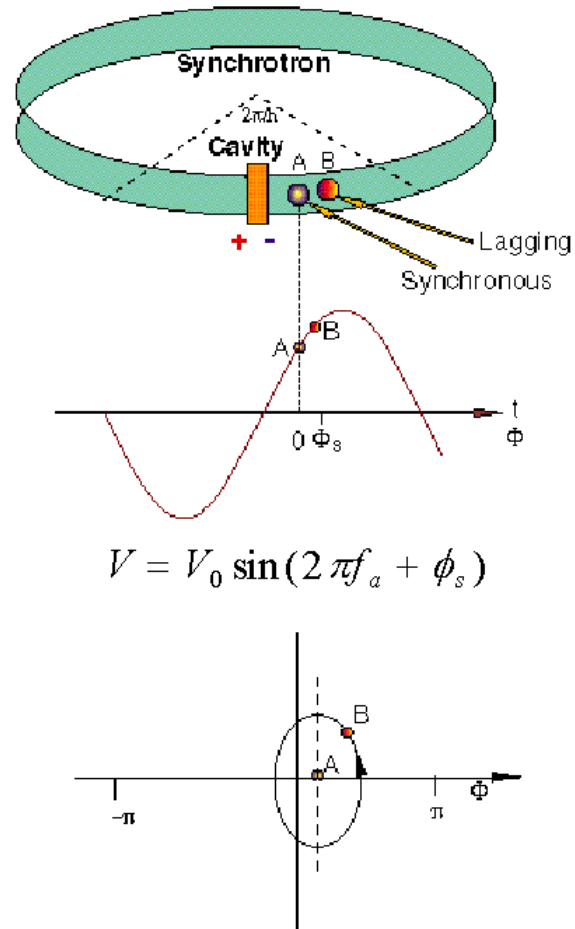
# Phase Stability

- In circular accelerator, phase stably locked by synchronizing the phase of the RF voltage (rising or falling) as particle crosses an accelerating gap
- Synchrotron oscillations about a stable phase results
  - synchrotron oscillations  
(longitudinal oscillations)



# Phase Stability

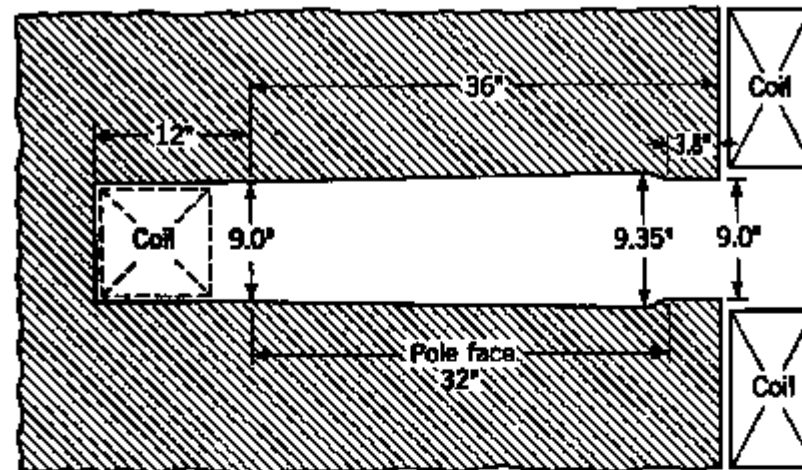
- Particles are trapped in an “RF bucket”



Ref: E. Wilson, CERN

# Weak focusing

- Vertical focussing from the curvature of the field lines when the field falls off with radius
- Horizontal focusing from the curvature of the path
- The negative field gradient defocuses horizontally and must not be so strong as to cancel curvature effect

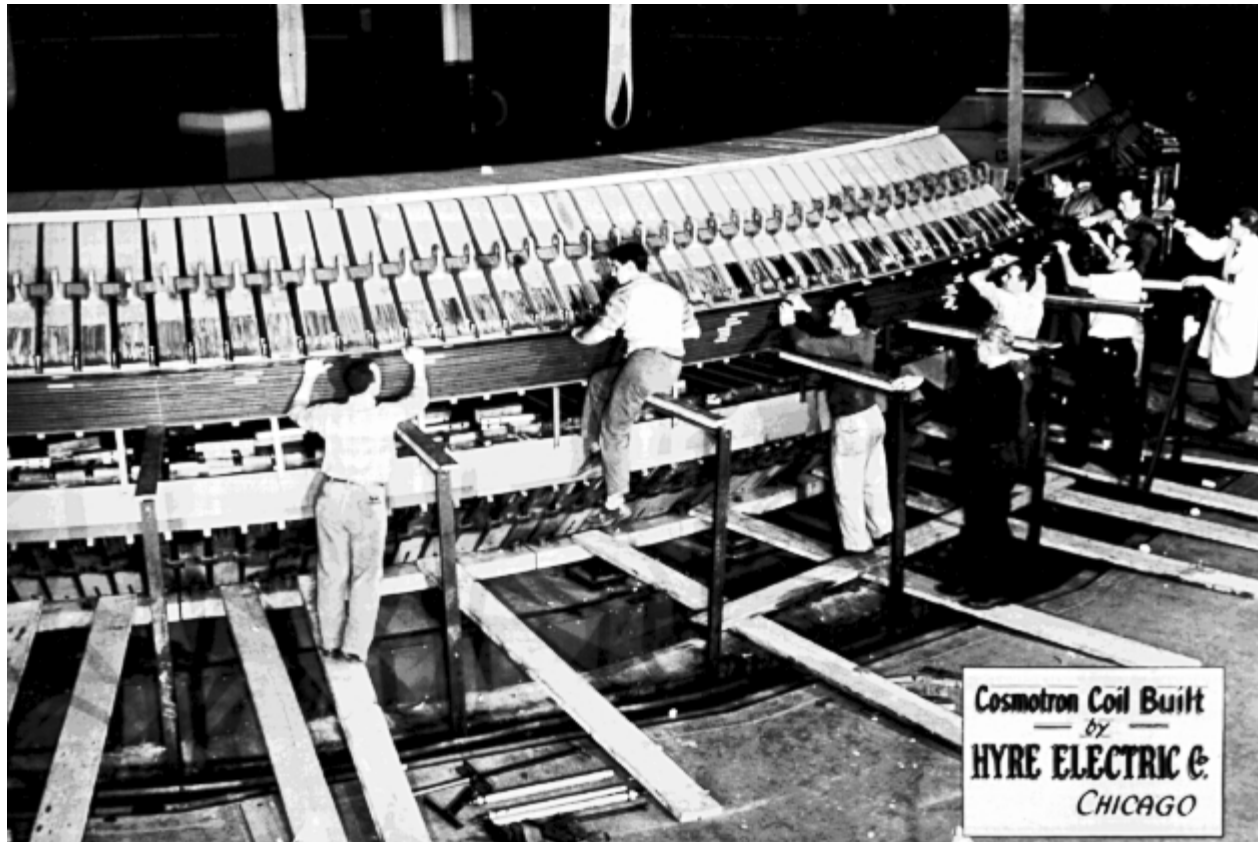


## The Cosmotron magnet

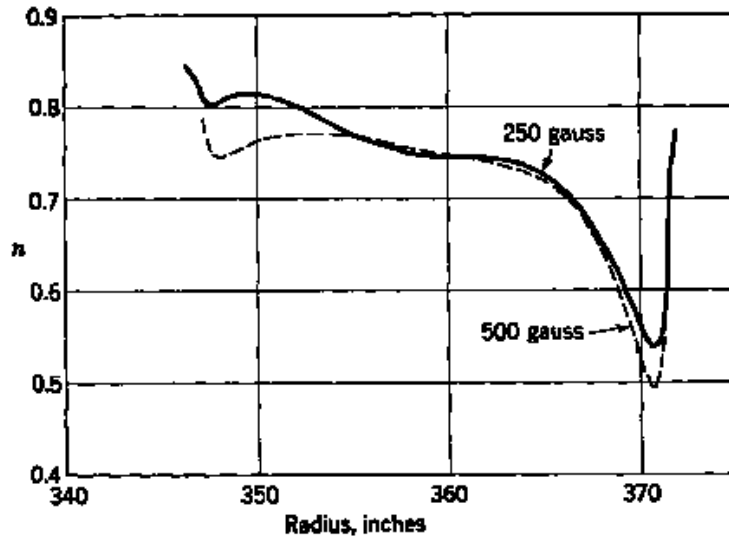


Ref: E. Wilson, CERN

# Cosmotron



# Weak focusing



$$B_z = B_0 \left( 1 - n \frac{\Delta r}{r_0} \right) \quad n = - \frac{r}{B} \frac{\partial B}{\partial r}$$

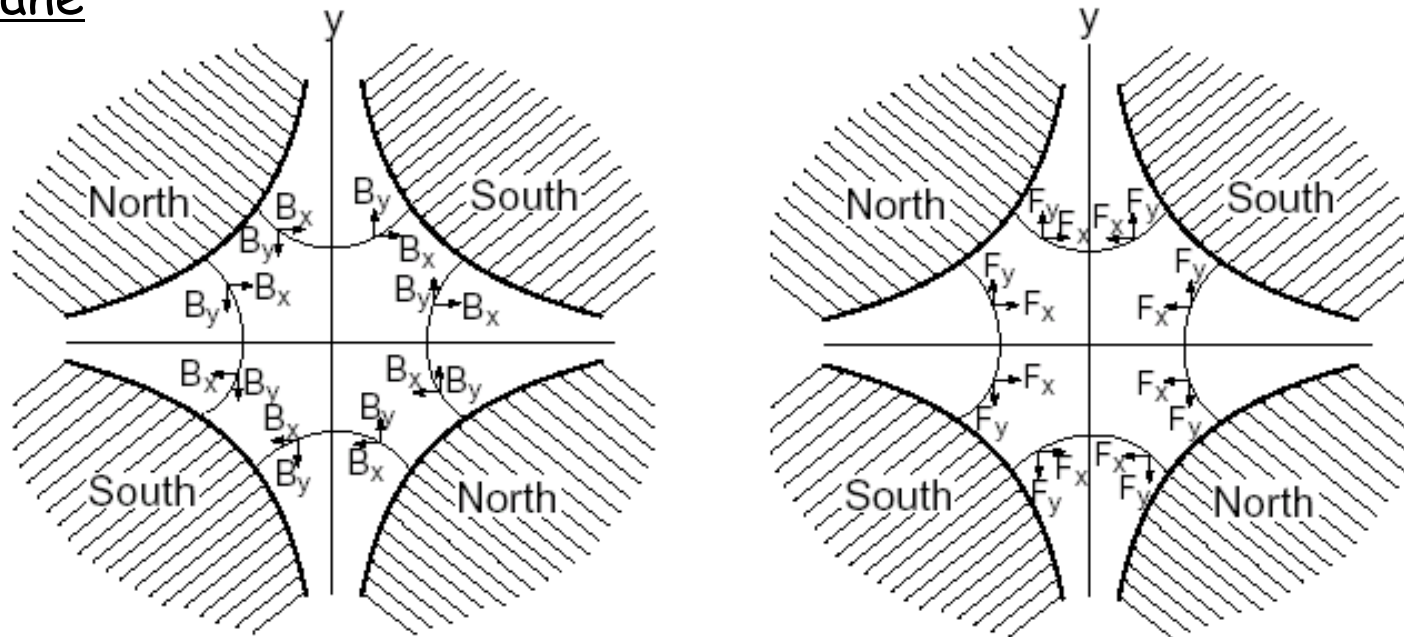
$$f_r = f_0 (1 - n)^{1/2} \quad f_z = f_0 n^{1/2}$$

Stable if:  $0 < n < 1$  (weak)

Ref: E. Wilson, CERN

# Quadrupole

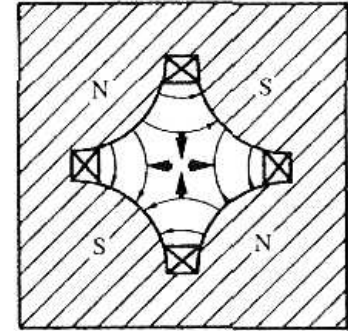
- The principal focusing element used today in synchrotrons is the quadrupole magnet
- The quadrupole shown below would focus in the horizontal plane positive particles coming out or negative particles going in. Such a quadrupole is defocusing in the vertical plane





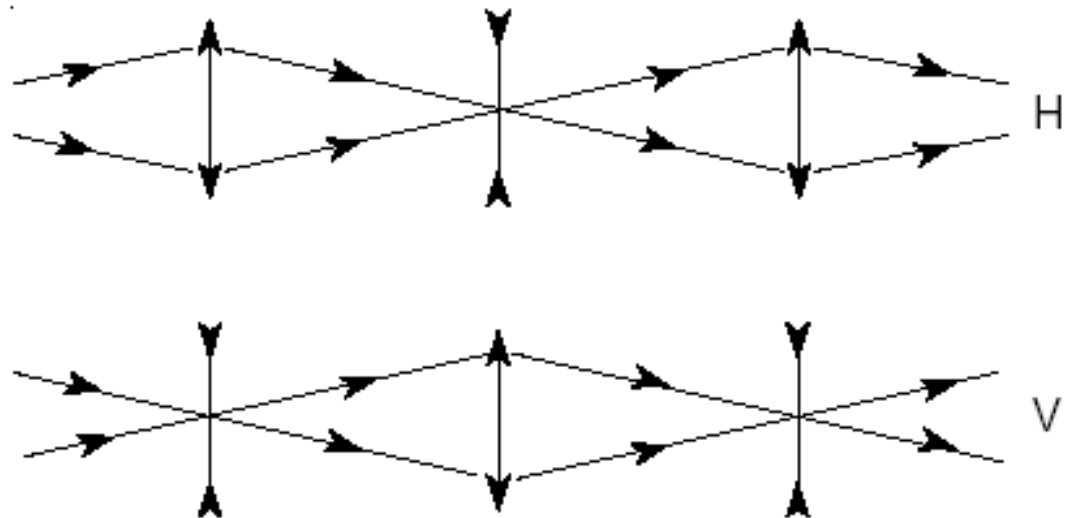
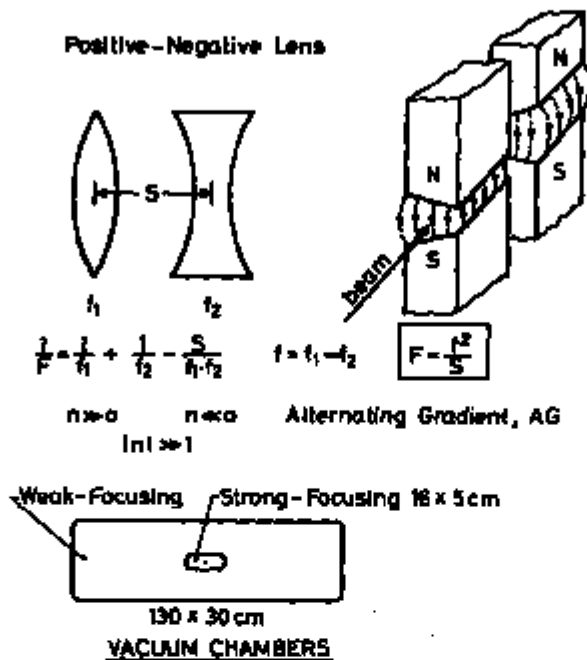
# Strong focusing

- Alternating diverging and focusing lenses separated by finite distance results in net focusing
- Magnetic quadrupole focuses in one plane and defocuses in orthogonal plane
- Alternating quadrupoles focus in both planes
  - much stronger than focussing of solenoids of radial magnetic gradients (dipoles)
  - decreases aperture required for stability
  - greatly extends energy range of acceleration



# Strong Focusing

- Ray diagrams showing the contained trajectory in an alternating gradient optical system
- An alternating pattern of lenses which are convex in one plane and concave in the other will transport rays which pass through the centres of defocusing lenses. The upper diagram shows the horizontal motion and the lower shows the vertical



Ref: E. Wilson, CERN

# Betatron Oscillations

## 21.3.1. *Betatron oscillations:*

Present-day high-energy accelerators employ alternating gradient focussing provided by quadrupole magnetic fields [1]. The equations of motion of a particle undergoing oscillations with respect to the design trajectory are

$$x'' + K_x(s)x = 0 \quad , \quad y'' + K_y(s)y = 0 \quad , \quad (21.5)$$

with

$$x' \equiv dx/ds \quad , \quad y' \equiv dy/ds \quad (21.6)$$

$$K_x \equiv B'/(B\rho) + \rho^{-2} \quad , \quad K_y \equiv -B'/(B\rho) \quad (21.7)$$

$$B' \equiv \partial B_y / \partial x \quad . \quad (21.8)$$

The independent variable  $s$  is path length along the design trajectory. This motion is called a *betatron* oscillation because it was initially studied in the context of that type of accelerator. The functions  $K_x$  and  $K_y$  reflect the transverse focussing—primarily due to quadrupole fields except for the radius of curvature,  $\rho$ , term in  $K_x$  for a synchrotron—so each equation of motion resembles that for a harmonic oscillator but with spring constants that are a function of position. No terms relating to synchrotron oscillations appear, because their time scale is much longer and in this approximation play no role.

from Particle Data Group

# Betatron Oscillations

These equations have the form of Hill's equation and so the solution in one plane may be written as

$$x(s) = A\sqrt{\beta(s)} \cos(\psi(s) + \delta), \quad (21.9)$$

where  $A$  and  $\delta$  are constants of integration and the phase advances according to  $d\psi/ds = 1/\beta$ . The dimension of  $A$  is the square root of length, reflecting the fact that the oscillation amplitude is modulated by the square root of the amplitude function. In addition to describing the envelope of the oscillation,  $\beta$  also plays the role of an 'instantaneous'  $\lambda$ . The wavelength of a betatron oscillation may be some tens of meters, and so typically values of the amplitude function are of the order of meters rather than on the order of the beam size. The beam optics arrangement generally has some periodicity and the amplitude function is chosen to reflect that periodicity. As noted above, at the interaction point a small value of the amplitude function is desired, and so the focussing optics is tailored in the neighborhood to provide a suitable  $\beta^*$ .

The number of betatron oscillations per turn in a synchrotron is called the *tune* and is given by

$$\nu = \frac{1}{2\pi} \oint \frac{ds}{\beta}. \quad (21.10)$$

# Betatron Oscillations

- betatron oscillations

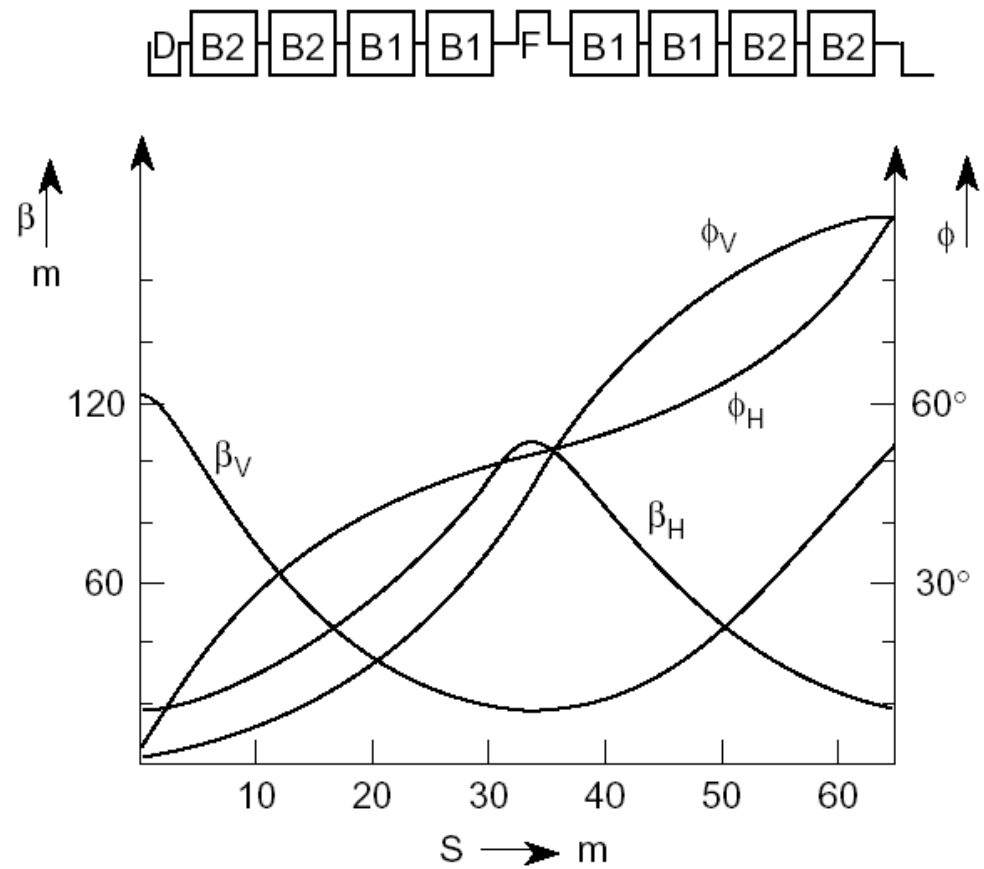
$$x(s) = A\sqrt{\beta(s)} \cos(\psi(s) + \delta),$$

- phase ( $\psi$ ) advances as  $1/\beta = d\psi/ds$
- $\beta^*$  (wavelength at IP, want to minimize)

- tune:  $\nu = \frac{1}{2\pi} \oint \frac{ds}{\beta}$

# FODO cells

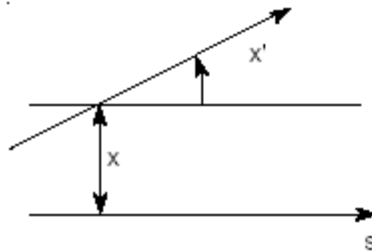
- Quadrupole magnets alternate with a the lattice of bending magnets
- Structure is called FODO.
- The envelope of oscillations follows the function  $\beta(s)$
- $\beta$  has the dimensions of length, but is not the physical beam size.



One cell of the CERN SPS representing 1/108 of the circumference.

# Particle trajectories

The beam size can be expressed in terms of two quantities, the transverse emittance,  $\varepsilon$ , and the amplitude function,  $\beta$ .

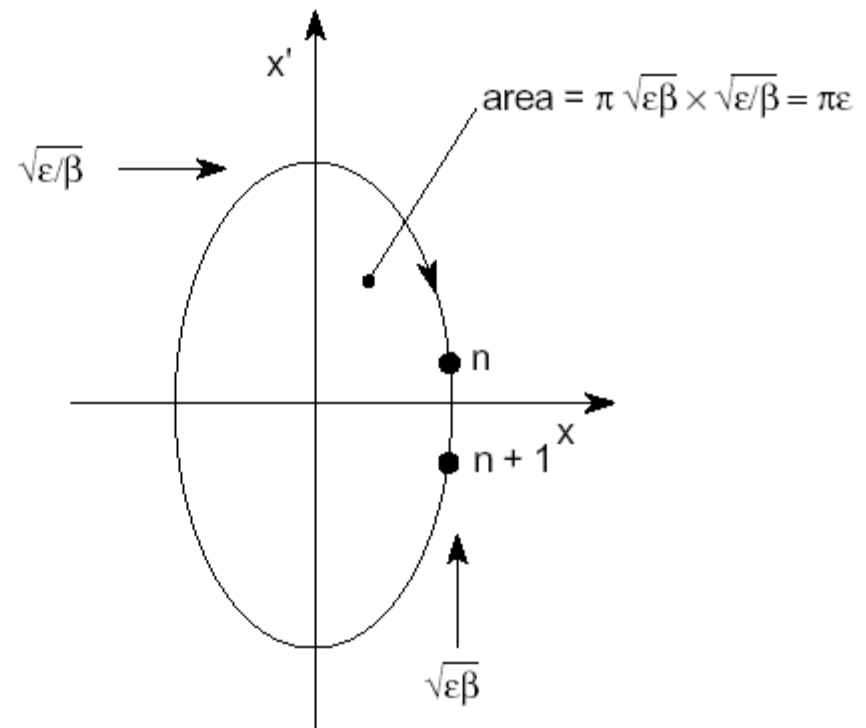


- The coordinates  $(x, x')$  of particles in the beam will fall in the range:

$$-\sqrt{\beta\varepsilon} \leq x \leq \sqrt{\beta\varepsilon}$$

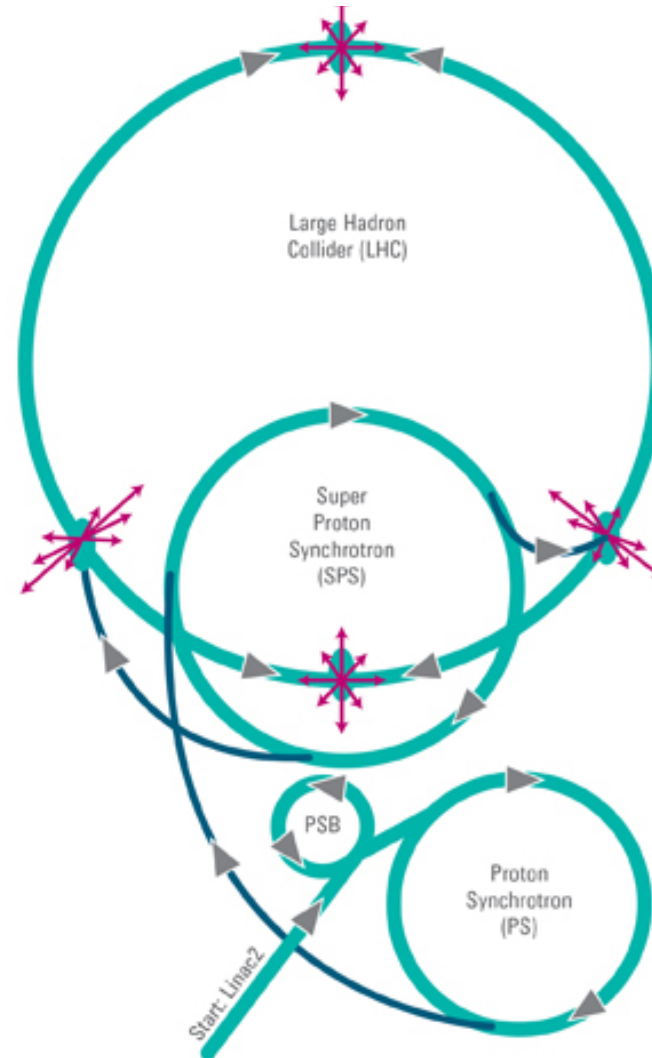
$$-\sqrt{\varepsilon/\beta} \leq x' \leq \sqrt{\varepsilon/\beta}$$

- $\beta$  is a property of the accelerator
- $\varepsilon$  is a property of the beam



# CERN's LHC Complex

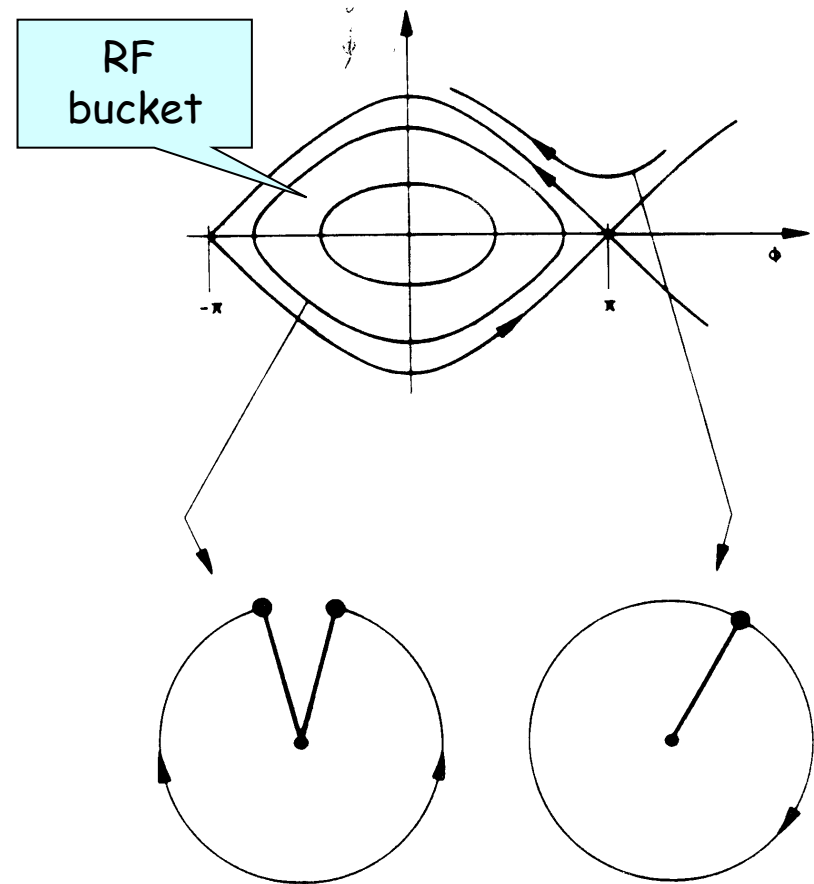
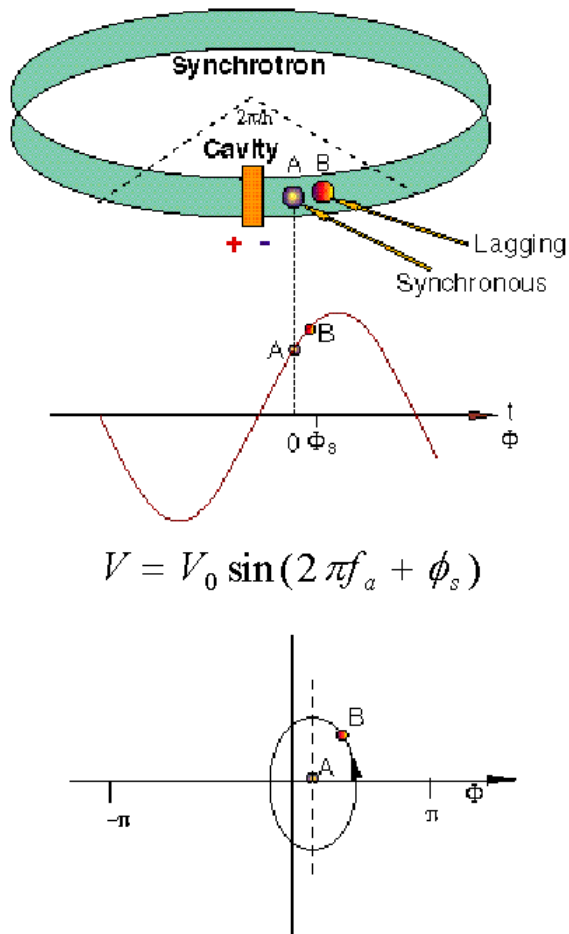
Protons start in Linac 2  
raises energy to 50 MeV  
Proton Synchrotron Booste,  
accelerates to 1.4 GeV  
Proton Synchrotron  
boosts energy to 25 GeV  
Super Proton Synchrotron,  
accelerates to 450 GeV  
LHC  
energy reaches 7 TeV





# Circular Accelerator Theory - Stability

- Longitudinal stability (synchrotron oscillations)



# Circular Accelerator Theory - Stability

- Betatron oscillations (transverse oscillations)

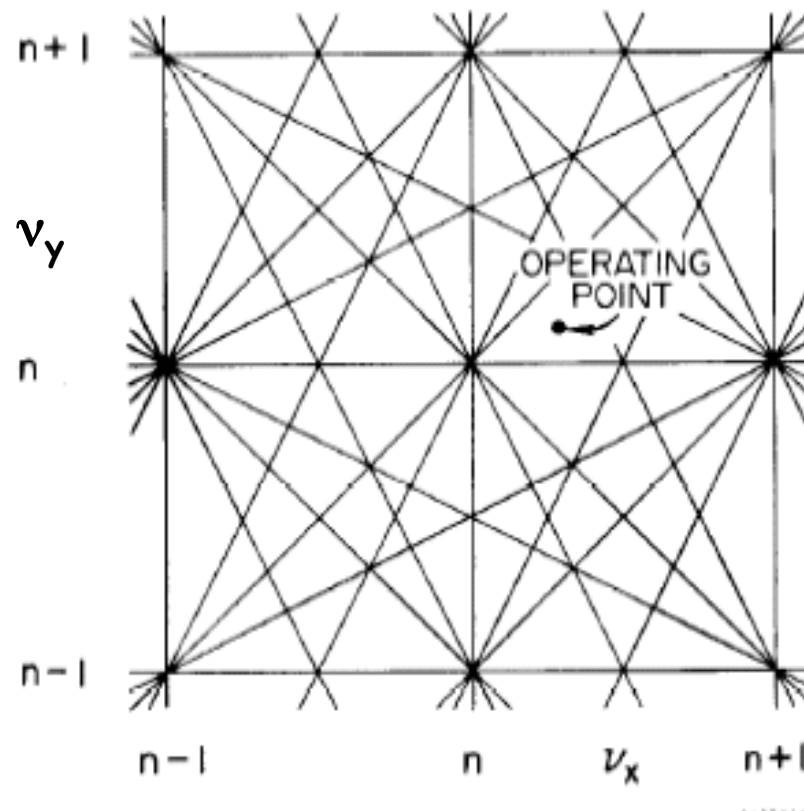
$$x(s) = A\sqrt{\beta(s)} \cos(\psi(s) + \delta), \quad 1/\beta = d\psi/ds$$

- Tune:  $\nu = \frac{1}{2\pi} \oint \frac{ds}{\beta}$

- Resonances (particularly critical for storage rings):
  - If  $\nu$  is an integer, tiny perturbations will drive beam out of stable orbit
    - eg. the smallest imperfection in the guide field  
(and there will surely be at least one!)  
will act as a perturbation which is synchronous with the  
oscillation frequency.
  - More generally, a resonance occurs when  $m\nu_x + n\nu_y = r$ , for integers  $m$ ,  $n$ , and  $r$ .

# Storage Ring - Stability

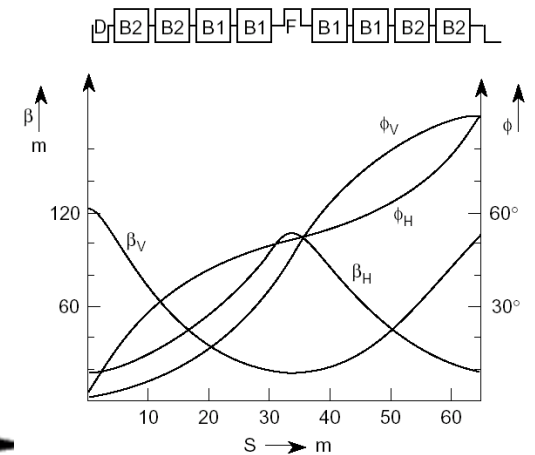
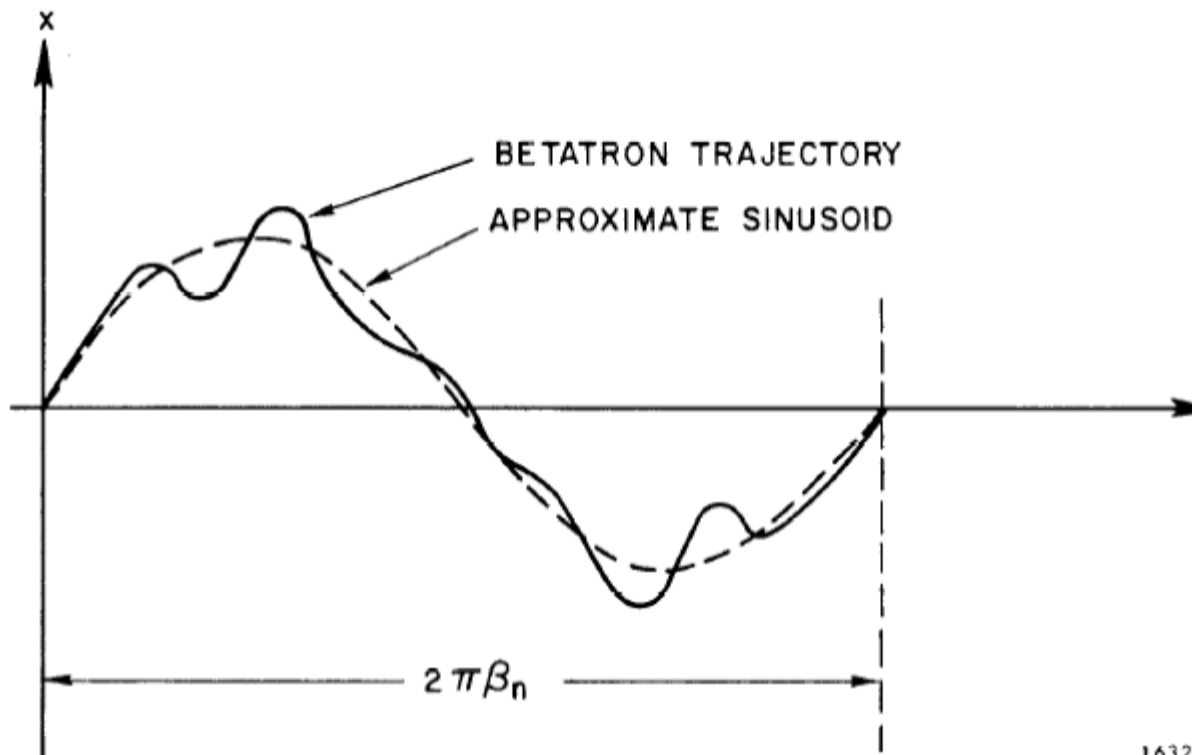
- A resonance occurs when for certain “tunes”  $mv_x + nv_y = r$ , for integers  $m$ ,  $n$ , and  $r$ .
- Significant effects are usually observed for small integers
- The operating point of a storage ring is specified by giving both  $\nu_x$  and  $\nu_y$  and must be chosen to avoid the serious resonances.



# Storage Ring - Typical Orbit

$$x = A \cos(s/\beta_n + \theta)$$

$$\oint \frac{ds}{\beta} = \frac{L}{\beta_n}$$



1632A15

# Storage Ring - Tune Shift

- The field gradient in the storage ring is never the ideal of the design.
- These gradient errors change the function of the betatron oscillations from the ideal orbit.
- And the betatron number is changed from its nominal value  $\nu$ , to another value,  $\nu + \Delta\nu$ . ( $\Delta\nu$  is called the tune shift)
- The tune shift  $\Delta\nu$  must be controlled to keep the operating point away from resonances.

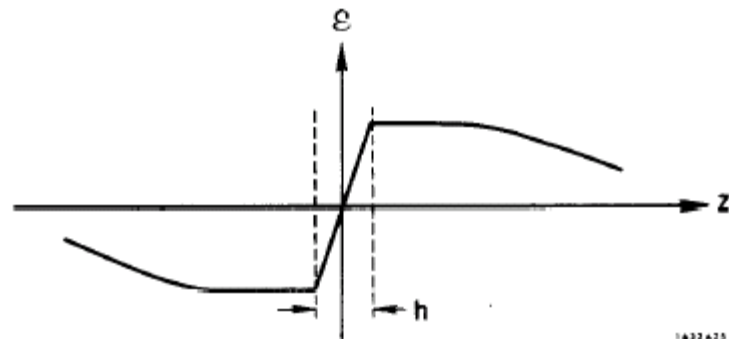
$$2\pi \Delta\nu = - \oint \frac{\Delta\beta(s)}{\beta^2} ds$$

- BEAM-BEAM TUNE SHIFT

$$\Delta\nu_y = \frac{r_e}{2\pi} \frac{N_B \beta_z^*}{\gamma \sigma_z (\sigma_x + \sigma_z)}$$

$$\Delta\nu_x = \frac{r_e}{2\pi} \frac{N_B \beta_x^*}{\gamma \sigma_x (\sigma_z + \sigma_x)}$$

$$r_e = \frac{e^2}{4\pi \epsilon_0 m c^2}$$



# Luminosity

- If two bunches containing  $n_1$  and  $n_2$  particles collide with frequency  $f$ , then the luminosity is

$$\mathcal{L} = f \frac{n_1 n_2}{4\pi\sigma_x\sigma_y}$$

- But recall beam size is a function of the transverse emittance,  $\epsilon$ , and the  $\beta$  function

$$\epsilon = \pi\sigma^2/\beta$$

- So

$$\mathcal{L} = f \frac{n_1 n_2}{4\sqrt{\epsilon_x \beta_x^* \epsilon_y \beta_y^*}}$$

# Luminosity Lifetime

$$\mathcal{L} = f \frac{n_1 n_2}{4 \sqrt{\epsilon_x \beta_x^* \epsilon_y \beta_y^*}}$$

Storage ring luminosity lifetime is limited by:

- interactions of the two beams at the IPs
- beam size blowup due to intra-beam scattering
- reduction in the beam intensity due to rest gas scattering
- beam size reduction due to synchrotron radiation damping
- beam size blowup due to the non-linear beam-beam interactions

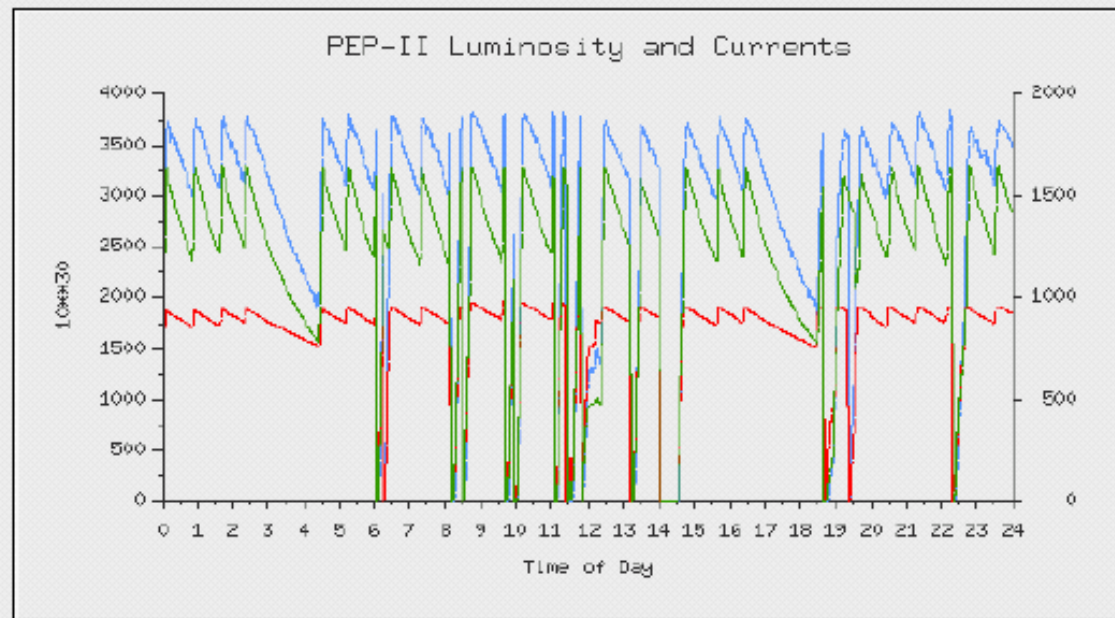
Different effects will dominate  
in different colliders

Future Super-B Factory  
(requires continuous injection)

Lifetime Contribution	HER	LER
Luminosity lifetime (min)	15	58
Vacuum lifetime (min)	100	30
Touschek lifetime (min)	300	30
Beam-beam tune shift lifetime (min)	10	10
Dynamic aperture lifetime (min)	20	20
Overall lifetime (min)	4.4	4.1

# Luminosity Lifetime - PEP II

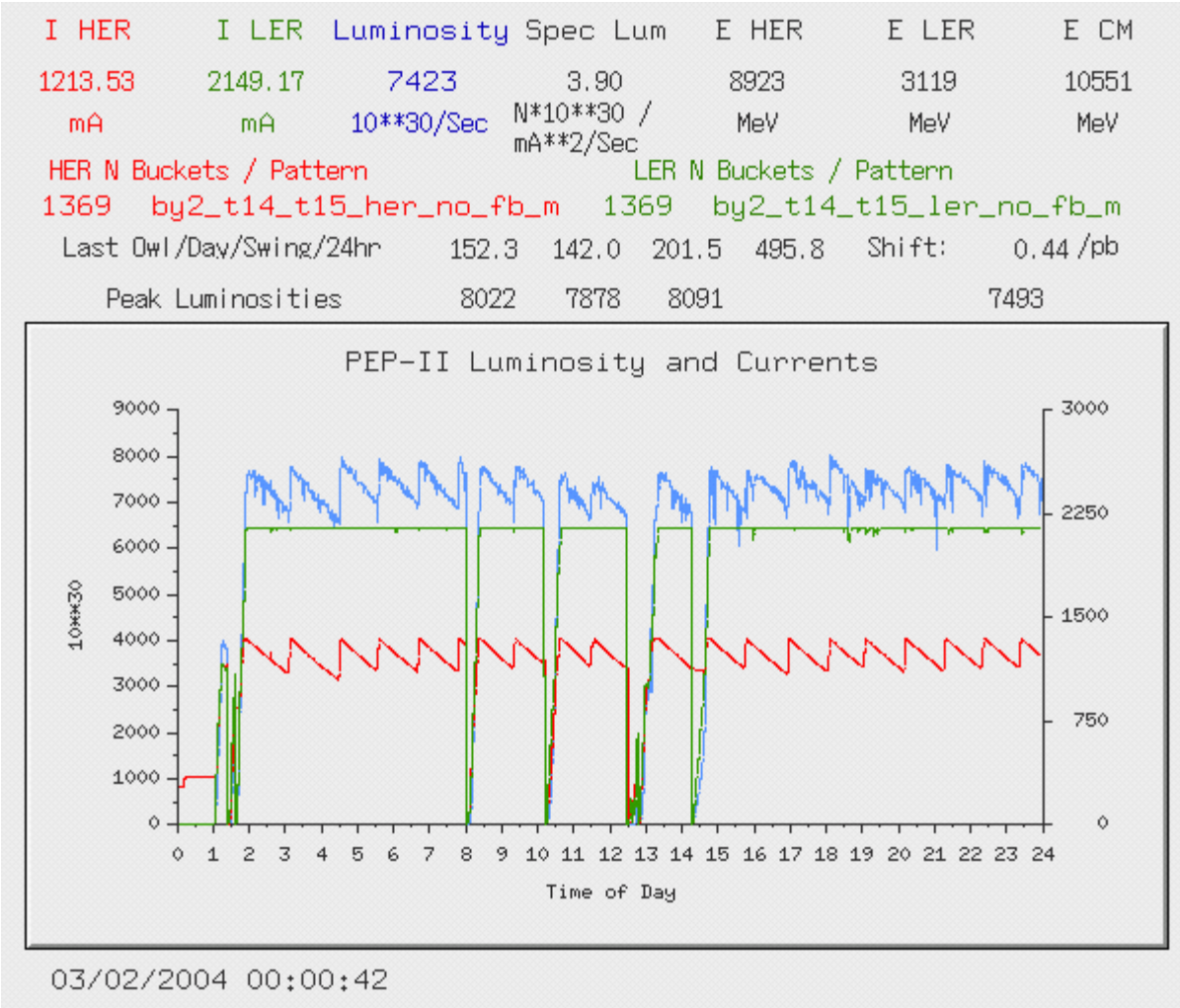
I HER	I LER	Luminosity	Spec Lum	E HER	E LER	E CM
918.67		3504	2.71	8992	8120	10598
mA	mA	10**30/Sec	N*10**30 / mA**2/Sec	MeV	MeV	MeV
HER N Buckets / Pattern				LER N Buckets / Pattern		
993	by3_trains_of_28			993	by3_trains_of_28	
Last Owl/Day/Swing/24hr		81.9	58.0	68.3	201.2	Shift: 0.21 /pb
Peak Luminosities		3845	3872	3882		3555



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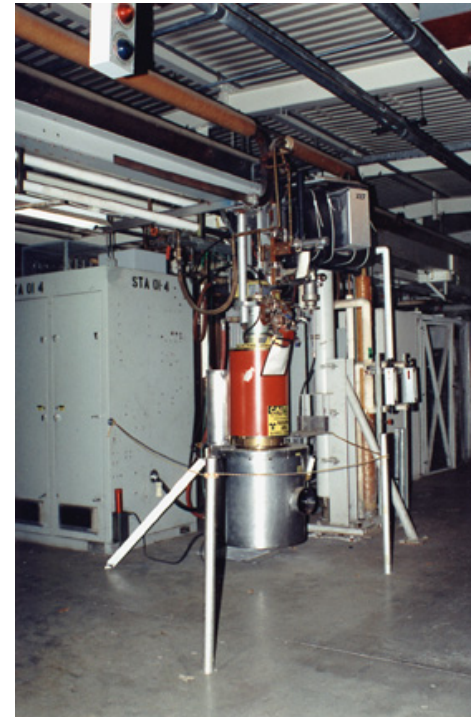


# Luminosity Lifetime - continuous injection



# High-impedance Microwave Devices

- EM cavities were invented as a way to generate high voltage at moderate input power
  - Amplifiers
  - Oscillators
  - Cavities
  - Disk-loaded waveguides

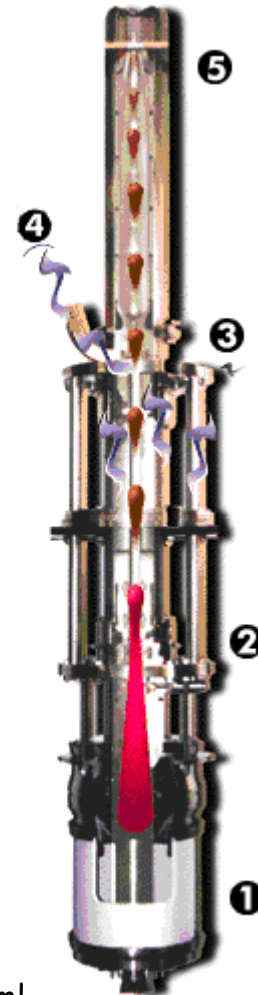


# Klystrons

- The klystron was invented at Stanford in 1937. The klystron served as an oscillator in radar receivers during WW II. After the war, however, very high-power klystrons were built at Stanford for use in the first linear accelerators. This opened the way for the use of klystrons not only in accelerators and radar, but also in UHF-TV, satellite communications, and industrial heating.
- Klystrons are high-vacuum devices based on the interaction of a well-focused pencil electron beam with a number of microwave cavities that it traverses, which are tuned at or near the operating frequency of the tube. The principle is conversion of the kinetic energy in the beam, imparted by a high accelerating voltage, to microwave energy. Conversion takes place as a result of the amplified RF input signal, causing the beam to form "bunches." These give up their energy to the high level induced RF fields at the output cavity. The amplified signal is extracted from the output cavity through a vacuum window.

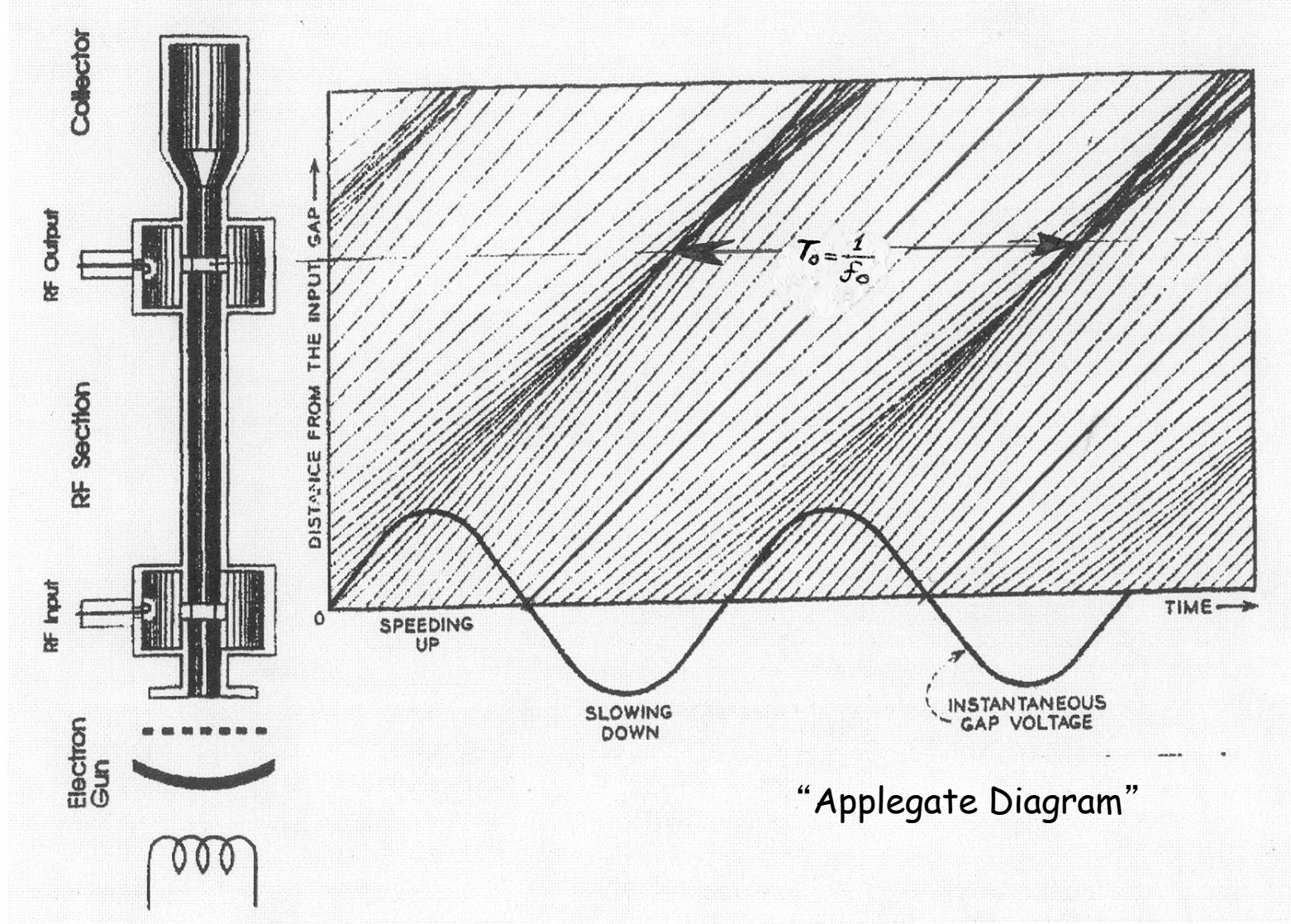
# Klystron

The electron gun (1) produces a flow of electrons.  
The bunching cavities (2) regulate the speed of the electrons so that they arrive in bunches at the output cavity.  
The bunches of electrons excite microwaves in the output cavity (3) of the klystron.  
The microwaves flow into the waveguide (4), which transports them to the accelerator.  
The electrons are absorbed in the beam stop (5).



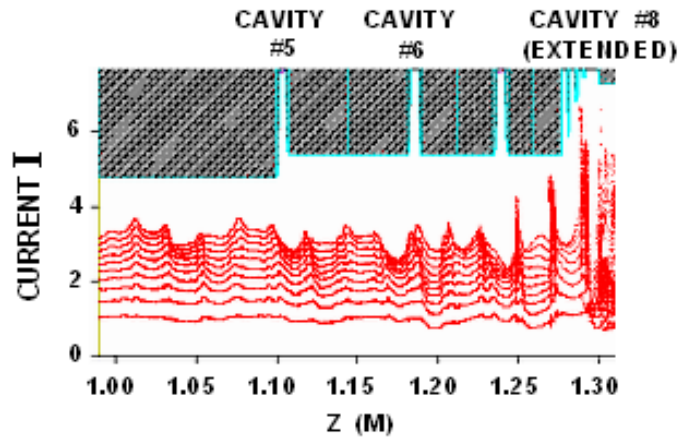
<http://www2.slac.stanford.edu/vvc/accelerators/klystron.html>

# Klystron

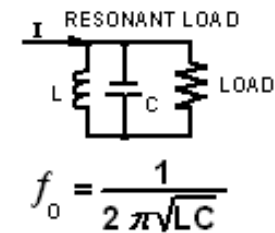
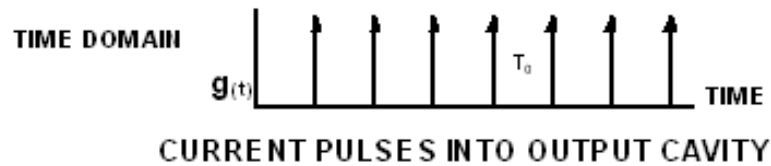


“Applegate Diagram”

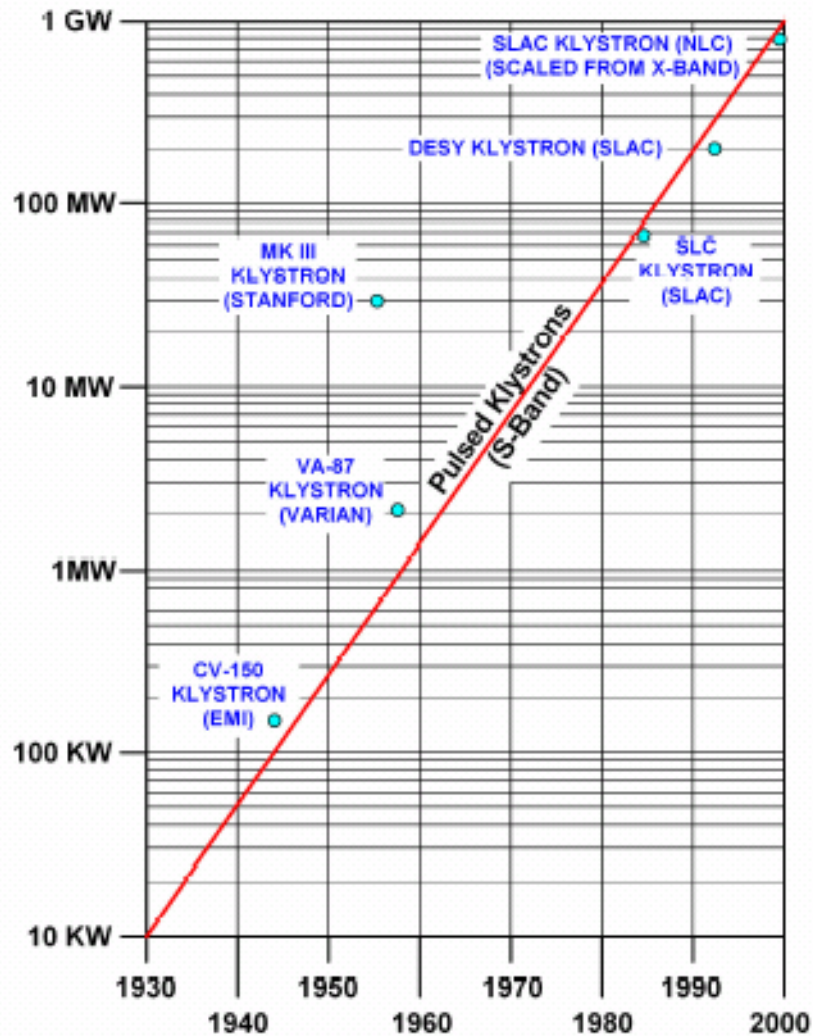
# Klystron



MAGIC SIMULATION (OUTPUT SECTION OF 8-CAVITY PPM KLYSTRON)



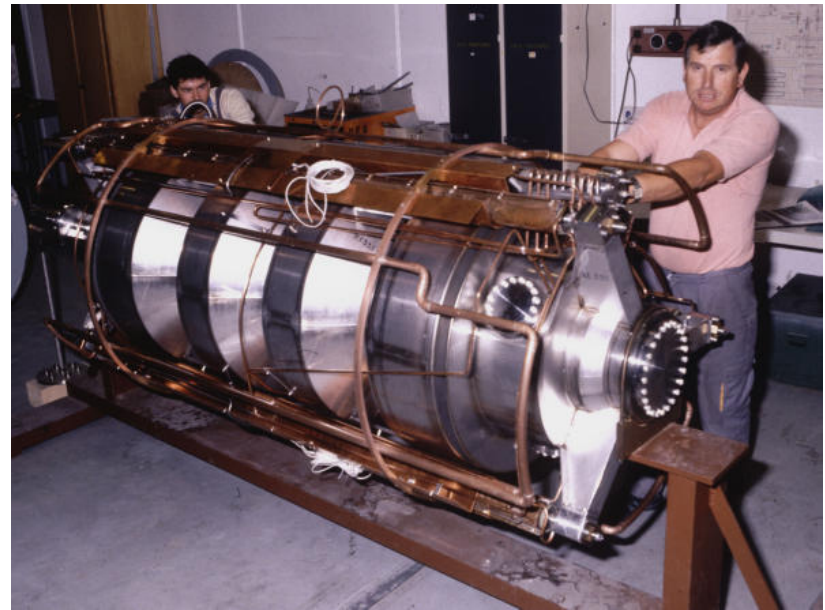
# Klystron



X-Band:  
75 Megawatts at 11.4 GHz,  
with 1.6 microsecond pulses

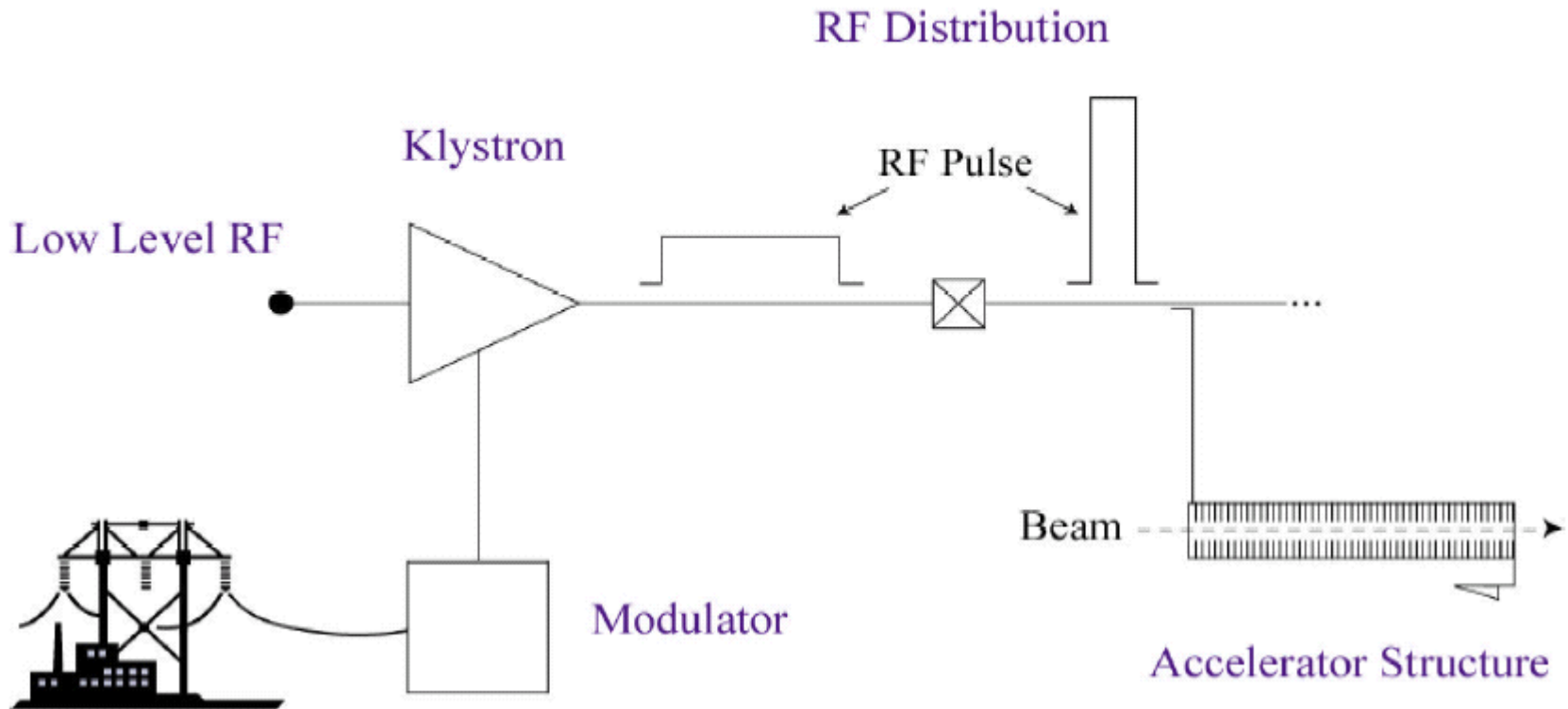
# Superconducting Technology

- Superconducting materials added to the technology base of accelerators
- Niobium-titanium
  - multistrand cables
  - Niobium-tin for higher fields, but brittle
- Niobium coatings inside RF cavities
  - now practical and reliable



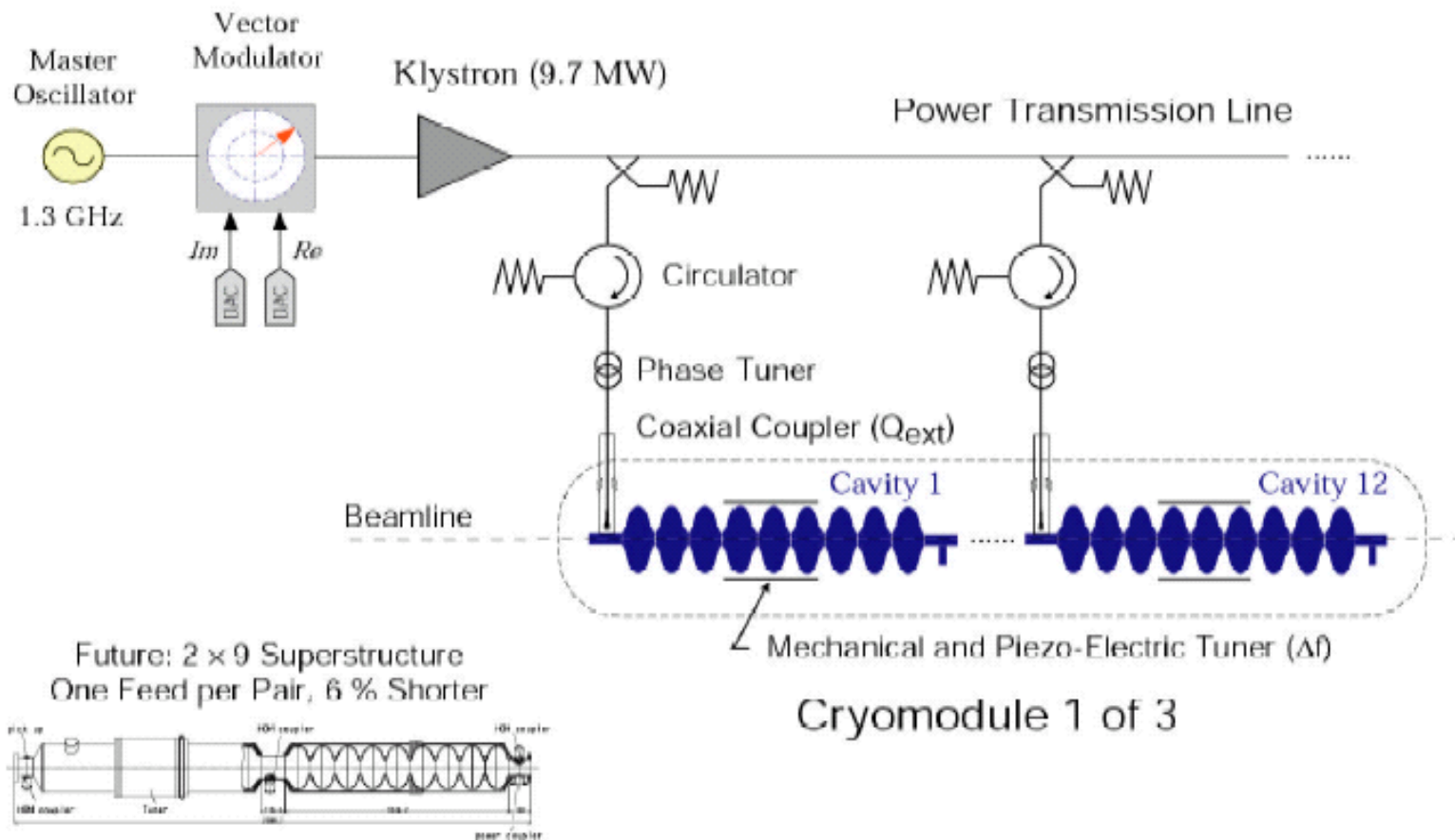


# Simplified RF System Accelerator Layout



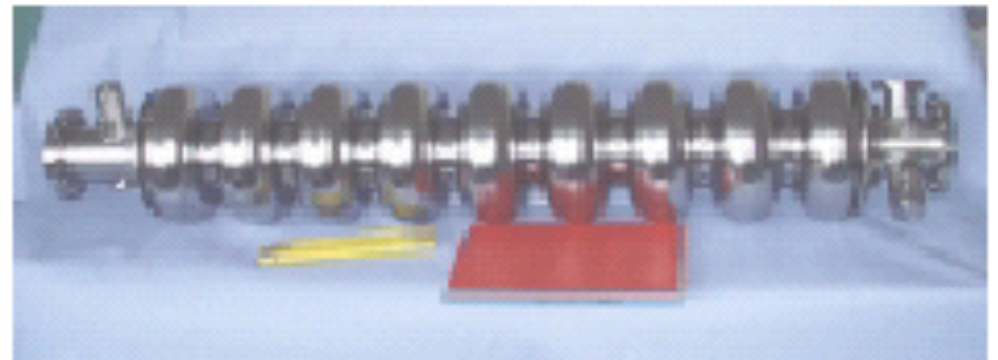
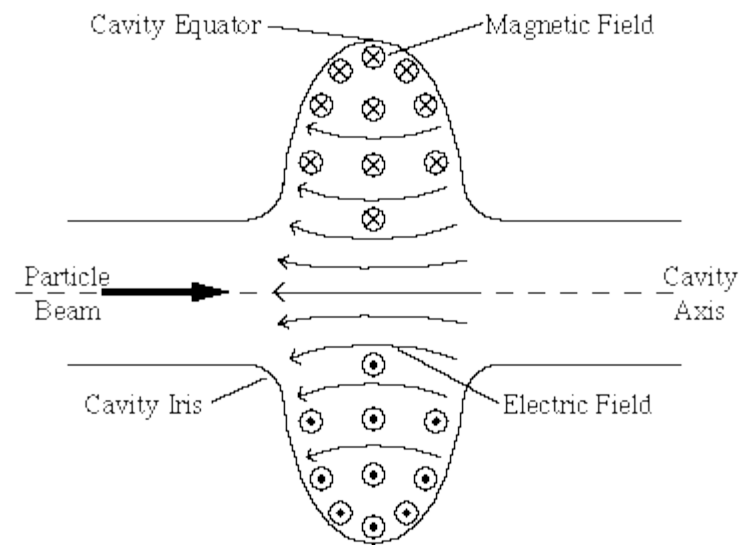
# Superconducting RF System

Length = 50 m, Filling Fraction with Quads = 75%



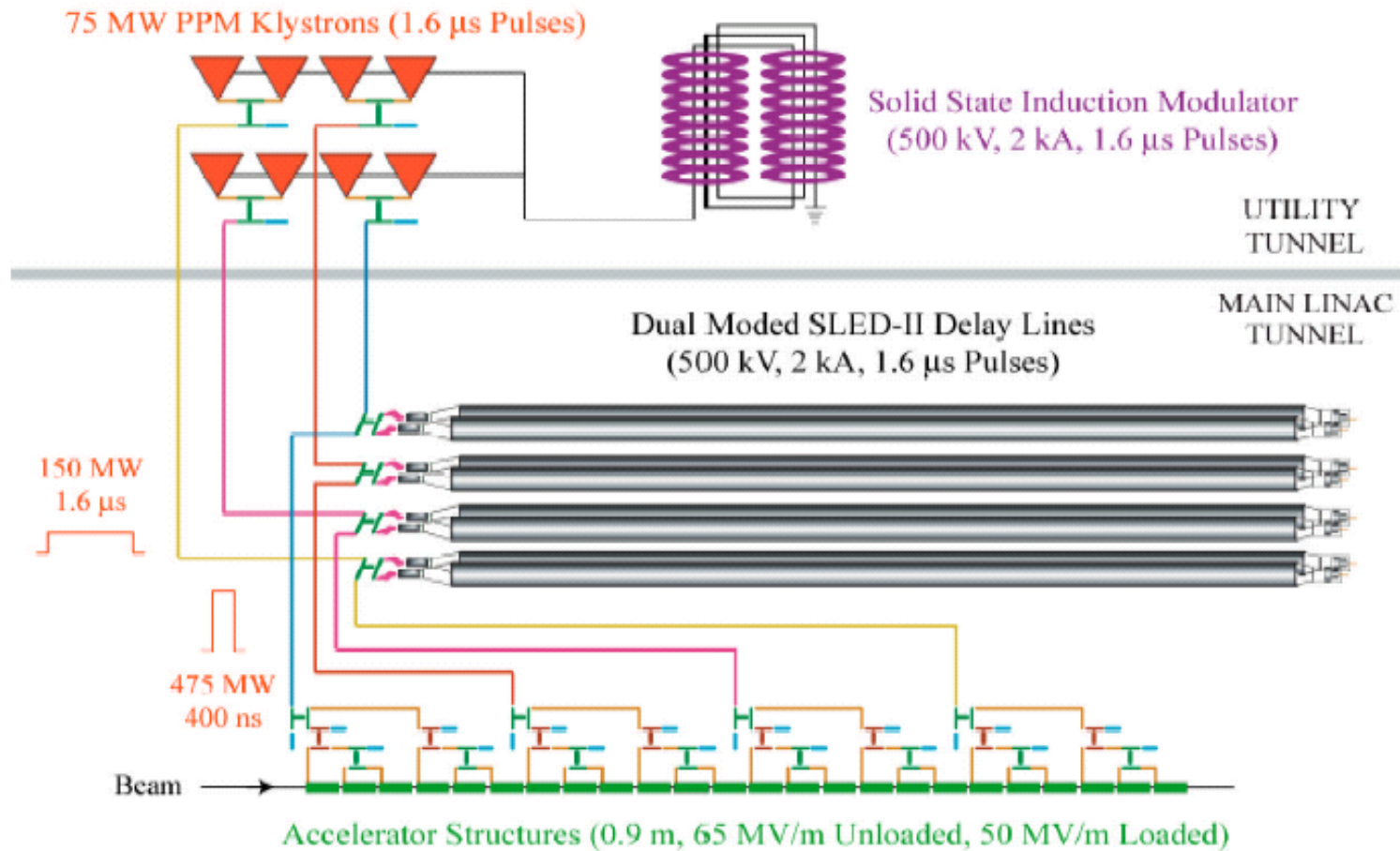
# Superconducting RF Accelerating Structure

- Typical cylindrically symmetric cavity, showing the fundamental, or lowest RF frequency, mode (TM<sub>010</sub>)
- The electric field is roughly parallel to the beam axis, and decays to zero radially upon approach to the cavity walls.
- Boundary conditions demand that the electric surface be normal to the metal surface.
- The peak surface electric field is located near the iris, or region where the beam tube joins the cavity.
- The magnetic field is azimuthal, with the highest magnetic field located near the cavity equator. The magnetic field is zero on the cavity axis.

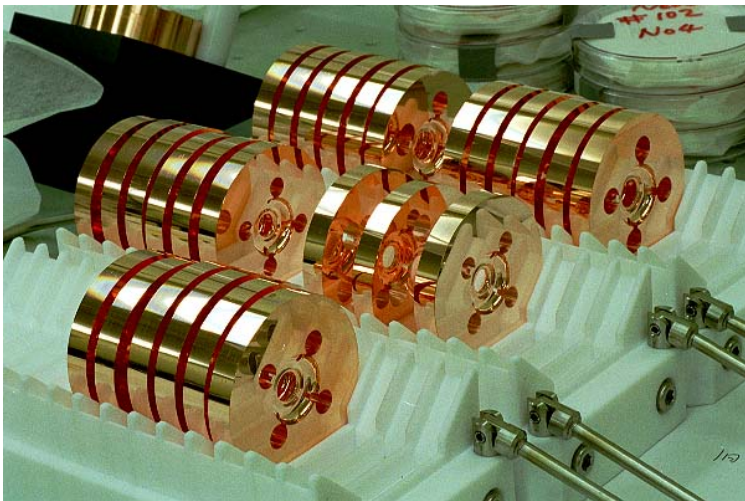
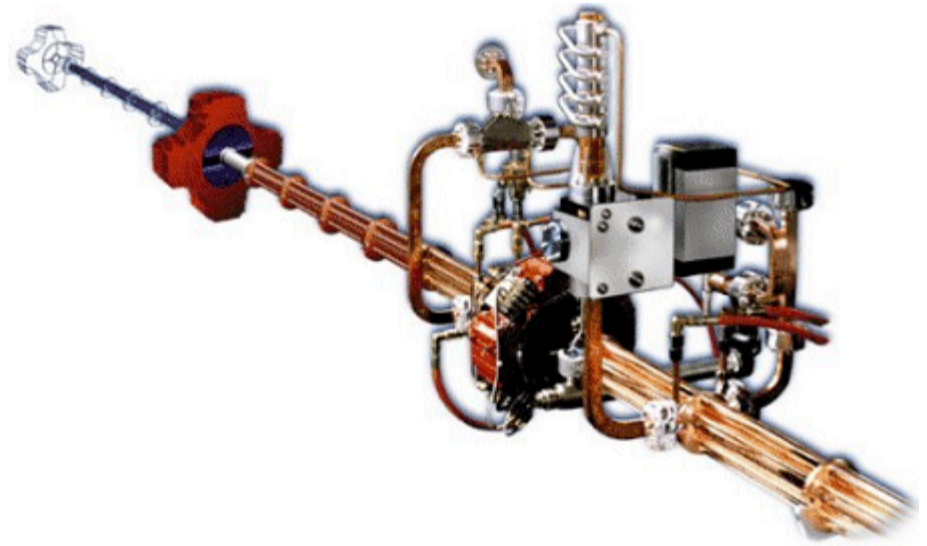
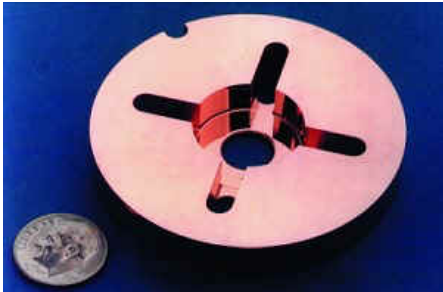


# X-Band (warm) RF System

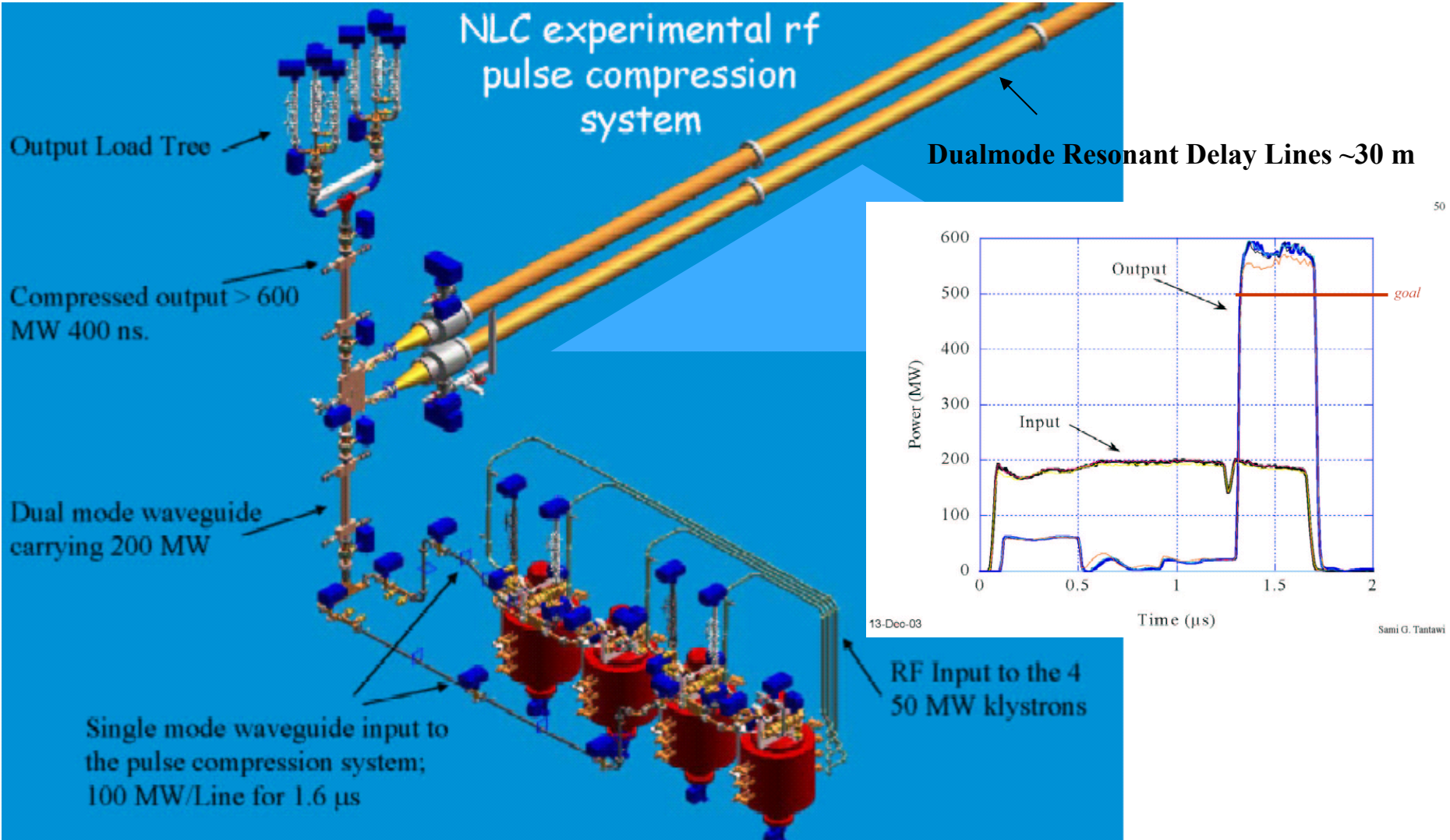
## NLC/JLC-X Linac RF Unit



# X-Band (warm) Accelerating Structure



# X-Band Pulse Compression Achieved - 2003



# CLIC

## CLIC Drive Beam Power Source and 30 GHz Accelerator

