Physics 610 Problem Set 1

Due: Wednesday, April 23, 2014

1. Calculate the length of the first four drift tubes in a linac for the following parameters: starting kinetic energy 100 keV, energy gain per gap 1 MeV, RF frequency 7 MHz. Perform the calculations for protons and for electrons and compare the results. Assume the accelerating gaps to be very short compared to the drift tubes.

2. The LHC has a circumference of 26659 m. Each proton beam in the LHC has an energy of 7 TeV.

   a) What is the effective bending field in the LHC?

   b) The actual magnetic field of a dipole magnet is 8.3 Tesla. What percentage of the LHC contains dipole magnets?

   c) There are 1232 dipole magnets in the LHC. How long is each one?

3. The average number $\overline{n}$ of ionizing collisions suffered by a fast particle of mass m, charge ze and velocity $v = \beta c$ in traversing dx (g cm^{-2}) of a medium, resulting in energy transfers in the range E$'$ $\rightarrow$ E$'$ + dE$'$, is

$$\overline{n} = f(E')dE'dx = \frac{2\pi z^2 e^4 N_0 Z}{mv^2 A}\frac{dE'}{(E')^2}(1 - \frac{v^2}{c^2}\frac{E'}{E'_{max}})dx$$

   where N_0 ts Avogadro's number and Z and A are the atomic number and mass number of atoms of the medium, and the maximum transferable energy is $E'_{max} = 2mv^2/(l - \beta^2)$, with $\beta = v/c$. For individual particles, the distribution in the number n of collisions follows the Poisson law, so that $< (n - \overline{n})^2 >= \overline{n}$. If we multiply the above equation by $(E')^2$ and integrate, we obtain the mean squared deviation in energy loss, $\epsilon^2 =< (\Delta E$ - $\overline{\Delta E})^2 >$, about the mean value $\overline{\Delta E}$. Show that

$$\epsilon^2 = 0.6(mc^2)^2\gamma^2\frac{Z}{A}(1 - \frac{\beta^2}{2})\delta x$$

   Calculate the fractional rms deviation in energy loss, ϵ / $\overline{\Delta E}$, for protons of kinetic energy 500 MeV traversing (a) 0.1, (b) 1.0 and (c) 10 g cm^{-2} of plastic scintillator (Z/A = 1/2). Take dE/dx as 3 MeV g^{-1}cm^2

4. A narrow pencil beam of singly charged particles of very high momentum p, travelling along the x-axis, traverses a slab of material s radiation lengths in thickness. If ionization loss in the slab may be neglected, calculate the rms lateral spread of the beam in the y-direction, as it emerges from the slab.

   (Hint: Consider an element of slab of thickness dx at depth x, and find the contribution $(dy)^2$ that this element makes to the mean squared lateral deflection, then integrate over the slab thickness.)

   Use the formula you derive to compute the rms lateral spread of a beam of 10 GeV/c muons in traversing a 100 m pipe filled with (a) air and (b) helium, at NTP.

5. Show that, in a head-on collision of a beam of relativistic particles of energy E_1 with one of energy E_2, the square of the energy in the center-of-momentum frame is $4E_1E_2$ and that for a crossing angle θ between the beams this is reduced by a factor $(1+\cos\theta)/2$. Calculate the required energy of a proton colliding with a stationary nucleon to produce the same available kinetic energy as a head-on collision at the LHC of two 7 TeV protons.

6. A proton of momentum $\mathbf{p}$, large compared with its rest mass M, collides with a proton inside a target nucleus, with Fermi momentum $\mathbf{p}_f$. Find the available kinetic energy in the collision, as compared with that for a free-nucleon (stationary) target, when $\mathbf{p}$ and $\mathbf{p}_f$ are (a) parallel, (b) antiparallel, (c) orthogonal.

7. It was sometimes possible to differentiate between the tracks due to relativistic pions, protons and kaons (rest masses 140, 938 and 494 MeV respectively) in a bubble chamber by virtue of the high energy δ-rays produced. For a beam momentum of 5 GeV/c, what is the minimum δ-ray energy which must be observed to prove that it is produced by a pion rather than a kaon or proton? What is the probability of observing such a knock-on electron in 1 m of liquid hydrogen (density 0.06)?

8. An experiment searching for proton decay in the mode $p \to e^+ + \pi^0$ is carried out using a cubical tank of water as the proton source. Possible decays are to be detected via the Cerenkov light emitted when the electromagnetic showers from the decay products traverse the water.

   (a) How big should the tank be in order to contain such showers if they start in the center?

   (b) Estimate the total track length integral (TLI) of the showers from a decay event and hence the total number of photons emitted in the visible region ($\lambda = 400 - 700$ nm).

   (c) If the light is detected by means of an array of photomultipliers at the water surfaces, the effective optical transmission of the water is 50% and the photocathode efficiency is 20%, what fraction of the surface must be covered by photocathode to give an energy resolution of 5%?