Physics 610 Problem Set 2

Due: Monday, May 19, 2014

1. The Friedmann equation describes the temporal development of a homogeneous and isotropic distribution of matter:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N \rho}{3} - \frac{Kc^2}{R^2} + \frac{\Lambda}{3}$$

   (a.) Assuming the Universe is described by this equation, show that distances in a flat Universe expand as $t^{2/3}$ during the matter dominated era.

   (b.) Determine the time dependence of expansion for a dark energy dominated, flat Universe.

   (c.) Show that in a flat, radiation dominated universe, the temperature of the Universe will fall inversely as the square-root of time.

   (d.) Calculate the time dependency of the temperature of the Universe during a flat, matter dominated era.

2. It is estimated that dark vacuum energy today contributes approximately 0.68 to the closure parameter Ω. At what value of the redshift parameter and at what age of the universe would vacuum energy have been less than 10^{-4} of the energy density of radiation?

3. Estimate the age of a flat universe (K = 0) if radiation is neglected and it is presently made up of matter with $\Omega_m = 0.32$ and vacuum energy with $\Omega_\Lambda = 0.68$.

4. The spectrum of black body photons of energy E = pc = hν is given by the BoseEinstein (BE) distribution, describing the number of photons per unit volume in the momentum interval p $\rightarrow$ p + dp. Including $g_\gamma = 2$ as the number of spin substates of the photon, this is

$$N(p)dp = \frac{p^2 dp}{\pi^2 \hbar^3 [exp(E/kT) - 1]} \left(\frac{g_\gamma}{2}\right)$$

   Calculate the mean quantum energy and the corresponding wavelength of the cosmic microwave photons for a temperature of T = 2.725 K. The original discovery of cosmic microwave radiation was made with receivers tuned to 7.3 cm wavelength. What fraction of the photons would have wavelengths in excess of 7.3 cm?

5. Light arrives at the Earth from two far-off, equidistant regions of the sky separated by angle θ_0. Suppose that light started out shortly after the matter-radiation decoupling time, $t_d \sim 4 \times 10^5$ yr. Show that if the regions had been causally connected at $t < t_d$, the angle $\theta_0 < t_d T_d/(t_0 T_0)$, where $kT_d \sim 0.3$ eV measures the decoupling temperature, $kT_0 \sim 1$ meV is the present temperature and t_0 is the age of the universe ($t_0 >> t_d$). Hence show that (in the absence of a preliminary inflationary stage) regions separated by more than about 1° in the sky apparently could not have been in thermal equilibrium.

6. Assume that the universe is flat, with $\Omega_m(0)$=0.32, $\Omega_\Lambda(0)$=0.68. What is the numerical value of the acceleration or deceleration with respect to the Earth, of a galaxy at redshift z =0.03? Compare this with the local acceleration (g) due to the Earth's gravity. Neglect the "peculiar velocity" of the Earth with respect to the Hubble flow and assume $H_0 = 70$ km s^{-1} Mpc^{-1}.

7. In a flat universe, with Ω_m= 0.32 and Ω_Λ= 0.68, at what value of z will the acceleration/deceleration be zero? Such a universe which is neither accelerating nor decelerating is often said to be 'coasting'.

8. EXTRA CREDIT!!

   The primordial helium mass fraction, with $m_{He} \simeq 4m_H$, can be expressed by

$$Y = \frac{4N_{He}}{4N_{He} + N_H},$$

   and is related to the primordial neutron-to-proton ratio, $r = N_n/N_p$, by

$$Y = \frac{2r}{1+r}$$

   Calculate the expected ratio of primordial helium to hydrogen, for the general case of n light neutrino flavors, where n = 3, 4, 5, 6, ... Show that each additional neutrino flavor will increase the He/H ratio and calculate the increase for each case.