Physics 610 Problem Set 3

Due: Wednesday, June 11, 2014, 2 pm

1. The power radiated by a gravitational radiation source with time dependent moment of inertia I and angular frequency ω is

$$P_{grav} = \frac{32GI^2\omega^6}{5c^5}$$

For a binary system of equal masses M with an orbital radius of r, orbiting in the x-y plane, the momenta of inertia, I_{ii}, are

$$I_{xx} = 2Mr^2cos^2(\omega t) = Mr^2cos(2\omega t)$$

$$I_{yy} = 2Mr^2sin^2(\omega t) = Mr^2sin(2\omega t)$$

The power radiated is therefore

$$P_{grav} = \frac{32G(Mr^2)^2\omega^6}{5c^5} = \frac{32GM^2r^4\omega^6}{5c^5}$$

Calculate the power radiated by the Moon in the Earth's orbit due to gravitational radiation. (the Earth's mass = 6×10^{24} kg, the Moon's mass = 7.4×10^{22} kg, the mean Earth-Moon distance = 3.8×10^5 km, orbital period of the Moon = 27.3 days).

2. The binary pulsar PSR 1913 + 16, discovered by Hulse and Taylor in 1975, is the most convincing evidence for the existence of gravitational radiation. This system of two neutron stars, one of which is a pulsar with period 0.059 s. has an orbital period of 7.8 hours. The observed fractional rate of decrease in the period (τ) of the binary system is found to be:

$$\frac{d\tau}{dt} = -(2.409 \pm 0.005) \times 10^{-12}$$

Use the formulae of gravitational radiation to check this result, assuming a circular orbit and equal masses for the members of the binary star system.

3. (a.) Show equation for the Ricci tensor, $R_{\mu\nu}$, in terms of the perturbations, $h_{\mu\nu}$, results from the Einstein field equations for that vacuum ($T_{\mu\nu} = 0$)

$$2R_{\mu\nu} = \partial_\rho\partial_\mu h^\mu_\nu + \partial_\nu\partial_\mu h^\mu_\rho - \Box h_{\nu\rho} - \partial_\rho\partial_\nu h^\mu_\mu$$

(b.) Show that in the case of traceless ($h^\mu_\mu = 0$) and transverse ($\partial_\mu h^{\mu\nu} = \partial_\mu h^{\nu\mu} = 0$) perturbations $h^{\mu\nu}$, this reduces to the simple wave-equation

$$\Box h^{\mu\nu} = 0.$$

4. Suppose that you have at your disposal a device for measuring lengths with the same accuracy as LIGO. How far away would an object have to be to produce an uncertainty of 1 mm in the distance determination?