Particle Physics Phenomenology

November 25, 2003
Quarks in Hadrons

- Charm and beauty ✓
- Quarkonium ✓
- Baryon decuplet
- Quark spin and color
- Baryon octet
- Pseudoscalar mesons
- Vector mesons
- Other tests of the quark model
- Mass relations and hyperfine splitting
- EM mass differences and isospin symmetry
- Baryon magnetic moments
- Heavy quark mesons
- The top quarks
Quark Model

- Patterns of observed particles led to proposal in early 1960's that hadrons were composed of quarks
  - u, d, and s (at that time)

- Were quarks real?
  - exhaustive searches for free quarks were unsuccessful

- With the discovery of “confined” quarks in the 1970's it was realized that quarks truly exist, but cannot be freed
Baryon Decuplet

- Lightest spin 3/2 baryons

Before the $\Omega^-$ was discovered it (and its properties) were predicted by this pattern
Discovery of the $\Omega^-$

Three, sequential decays of the strange quark

$\Omega^- = sss$
$\Xi^0 = ssu$
$\Lambda = sud$
Baryon Decuplet

- Notice the masses
  \[ M(\Delta) = 1232 \]
  \[ M(\Sigma) = 1384 \]
  \[ = M(\Delta) + 152 \]
  \[ M(\Xi) = 1533 \]
  \[ = M(\Sigma) + 149 \]
  \[ M(\Omega) = 1672 \]
  \[ = M(\Xi) + 139 \]

We see an orderly increase of mass with number of strange quarks
Group Theory

- Quarks are fundamental representations of the group SU(3)
  \[ 3 \otimes 3 = 6 \oplus 3 \]

\[ 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \]

1 is anti-symmetric under interchange of two quarks
10 is symmetric under interchange of two quarks
8's are mixed under interchange of two quarks
Quark Spin and Color

• Consider the $\Delta^{++}$
  - spin 3/2 (uuu)
  - therefore $u^\uparrow u^\uparrow u^\uparrow$

    • now, this appears to violate the Pauli principle
      - two or more identical fermions cannot exist in the same quantum state
    • resolution, another quantum number (color) and each of the u quarks have a different value:
      $u^\uparrow$(red)$u^\uparrow$(green)$u^\uparrow$(blue)

      and we have to anti-symmetrize the color
      $u^\uparrow u^\uparrow u^\uparrow$ (rgb-rbg+brg-bgr+gbr-grb)
Quark Spin and Color

- We also know from the rate of decay of the $\pi^0$ that there are three colors

\[ \Gamma(\pi^0 \rightarrow \gamma \gamma) = 7.73 \text{ eV} \ (N_c/3)^2 \]
\[ \Gamma(\text{observed}) = 7.76 \pm 0.6 \text{ eV} \]
\[ N_c = 2.99 \pm 0.12 \]

- Also the rate of $e^+e^- \rightarrow \text{hadrons}$ tells us $N_c = 3$
Baryon Octet

- Multiplet including the neutron and proton
  - lightest spin 1/2 baryons
  - wavefunction symmetric under simultaneous interchange of flavor and spin

- total wave-function must be anti-symmetric
  - (flavor)(spin)(color)(space)
  - color is anti-symmetric for all hadrons because they are color-neutral, singlet states
  - space is symmetric because L=0
  - \( \therefore \) (flavor)(spin) is symmetric
Baryon Octet

- Consider the proton (uud)
  - two quarks in the spin singlet state
    - $(\uparrow\downarrow-\downarrow\uparrow)/\sqrt{2}$ (anti-symmetric)
  - these quarks must also be in an anti-symmetric flavor state for an overall symmetric flavor-spin wavefunction
    - $(ud-du)/\sqrt{2}$ (anti-symmetric)

- Third Quark is spin up
  $(u^\uparrow d\downarrow - u\downarrow d^\uparrow - d^\uparrow u\downarrow + d\downarrow u^\uparrow) u^\uparrow$
Baryon Octet

- Now symmetrize by making cyclic permutation
  
  \[
  (2u^\uparrow u^\uparrow d^\downarrow + 2d^\downarrow u^\uparrow u^\uparrow + 2u^\uparrow d^\downarrow u^\uparrow \\
  - u^\downarrow d^\uparrow u^\uparrow - u^\uparrow u^\downarrow d^\uparrow - u^\downarrow u^\uparrow d^\uparrow \\
  - d^\uparrow u^\downarrow u^\uparrow - u^\uparrow d^\uparrow u^\downarrow - d^\uparrow u^\uparrow u^\downarrow ) / \sqrt{18}
  \]
Baryon Octet
Baryon Octet

- Notice the masses
  \[ M(N) = 939 \]
  \[ M(\Sigma) = 1193 \]
  \[ = M(N) + 254 \]
  \[ M(\Lambda) = 1116 \]
  \[ = M(N) + 177 \]
  \[ M(\Xi) = 1318 \]
  \[ = M(\Sigma) + 125 \]
  \[ = M(\Lambda) + 202 \]

Pattern is more complicated than for decuplet
Light Pseudoscalar Mesons

- Combine a quark and an antiquark
  - \( 3 \otimes \bar{3} = 8 \oplus 1 \)
Light Pseudoscalar Mesons

- Combine a quark and an antiquark
  \[ 3 \otimes \overline{3} = 8 \oplus 1 \]

### The strangeless mesons

<table>
<thead>
<tr>
<th>$I$</th>
<th>$I_3$</th>
<th>Wavefunction</th>
<th>$Q/e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$u\bar{d} = \pi^+$</td>
<td>+1</td>
</tr>
<tr>
<td>1</td>
<td>−1</td>
<td>$-\bar{u}d = \pi^-$</td>
<td>−1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$\frac{1}{\sqrt{2}} (d\bar{d} - u\bar{u}) = \pi^0$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$\frac{1}{\sqrt{2}} (d\bar{d} + u\bar{u}) = \eta$</td>
<td>0</td>
</tr>
</tbody>
</table>

This table ignores the strange quarks
Light Pseudoscalar Mesons

- **C** operation on quarks
  - (Condon-Shortley convention)
### Light Pseudoscalar Mesons

<table>
<thead>
<tr>
<th>( I )</th>
<th>( I_3 )</th>
<th>( S )</th>
<th>Meson</th>
<th>Quark combination</th>
<th>Decay</th>
<th>Mass, MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( \pi^+ )</td>
<td>( u\bar{d} )</td>
<td>( \pi^\pm \to \mu\nu )</td>
<td>140</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>( \pi^- )</td>
<td>( d\bar{u} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( \pi^0 )</td>
<td>( \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u}) )</td>
<td>( \pi^0 \to 2\gamma )</td>
<td>135</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>+1</td>
<td>( K^+ )</td>
<td>( u\bar{s} )</td>
<td>( K^+ \to \mu\nu )</td>
<td>494</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>+1</td>
<td>( K^0 )</td>
<td>( d\bar{s} )</td>
<td>( K^0 \to \pi^+\pi^- )</td>
<td>498</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>-1</td>
<td>( K^- )</td>
<td>( \bar{u}s )</td>
<td>( K^- \to \mu\nu )</td>
<td>494</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>-1</td>
<td>( \bar{K}^0 )</td>
<td>( \bar{d}s )</td>
<td>( \bar{K}^0 \to \pi^+\pi^- )</td>
<td>498</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \eta_8 )</td>
<td>( \frac{1}{\sqrt{6}}(d\bar{d} + u\bar{u} - 2s\bar{s}) )</td>
<td>( \eta \to 2\gamma )</td>
<td>549</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \eta_0 )</td>
<td>( \frac{1}{\sqrt{3}}(d\bar{d} + u\bar{u} + s\bar{s}) )</td>
<td>( \eta' \to \eta\pi\pi \to 2\gamma )</td>
<td>958</td>
</tr>
</tbody>
</table>
Light Pseudoscalar Mesons

Gell-Mann Okubo Mass Formula

\[ 4(M_K)^2 = M_\pi^2 + 3 M_\eta^2 \]

\[ 0.988 \text{ GeV}^2 = 0.924 \text{ GeV}^2 \]
The light vector mesons

<table>
<thead>
<tr>
<th>Mass, MeV</th>
<th>Dominant decay mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>776</td>
<td>$\rho \rightarrow 2\pi$</td>
</tr>
<tr>
<td>892</td>
<td>$K^* \rightarrow K\pi$</td>
</tr>
<tr>
<td>783</td>
<td>$\omega \rightarrow 3\pi$</td>
</tr>
<tr>
<td>1019</td>
<td>$\phi \rightarrow K\bar{K}$</td>
</tr>
</tbody>
</table>
# The light vector mesons

## $\phi(1020)$ Decay Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_1$</td>
<td>$K^+ K^-$</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>$K^0_L K^0_S$</td>
</tr>
<tr>
<td>$\Gamma_3$</td>
<td>$\rho \pi + \pi^+ \pi^- \pi^0$</td>
</tr>
<tr>
<td>$\Gamma_4$</td>
<td>$\rho \pi$</td>
</tr>
<tr>
<td>$\Gamma_5$</td>
<td>$\pi^+ \pi^- \pi^0$</td>
</tr>
<tr>
<td>$\Gamma_6$</td>
<td>$\eta \gamma$</td>
</tr>
<tr>
<td>$\Gamma_7$</td>
<td>$\pi^0 \gamma$</td>
</tr>
<tr>
<td>$\Gamma_8$</td>
<td>$e^+ e^-$</td>
</tr>
<tr>
<td>$\Gamma_9$</td>
<td>$\mu^+ \mu^-$</td>
</tr>
<tr>
<td>$\Gamma_{10}$</td>
<td>$\eta e^+ e^-$</td>
</tr>
<tr>
<td>$\Gamma_{11}$</td>
<td>$\pi^+ \pi^-$</td>
</tr>
</tbody>
</table>

## $\omega(782)$ Decay Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_1$</td>
<td>$\pi^+ \pi^- \pi^0$</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>$\pi^0 \gamma$</td>
</tr>
<tr>
<td>$\Gamma_{3}$</td>
<td>$\pi^+ \pi^-$</td>
</tr>
<tr>
<td>$\Gamma_4$</td>
<td>$\eta \gamma$</td>
</tr>
<tr>
<td>$\Gamma_5$</td>
<td>$\pi^0 e^+ e^-$</td>
</tr>
<tr>
<td>$\Gamma_6$</td>
<td>$\pi^0 \mu^+ \mu^-$</td>
</tr>
<tr>
<td>$\Gamma_7$</td>
<td>$e^+ e^-$</td>
</tr>
<tr>
<td>$\Gamma_{8}$</td>
<td>(excluding $\pi^0 \gamma$)</td>
</tr>
</tbody>
</table>

$(49.4 \pm 0.7)\% \quad (33.6 \pm 0.6)\% \quad (15.5 \pm 0.6)\% \quad \ldots$
The light vector mesons

- “ideal mixing”
  \[ \phi = (\phi_0 - \sqrt{2} \phi_8) / \sqrt{3} = s\bar{s} \]
  \[ \omega = (\phi_8 + \sqrt{2} \phi_0) / \sqrt{3} = (d\bar{d} + u\bar{u}) \]
The light vector mesons

(a) $\phi \rightarrow s \bar{s} \rightarrow K^- \bar{u} \rightarrow u \bar{s} \rightarrow K^+$

(b) $\omega \rightarrow u \bar{u} \rightarrow \pi^+ \bar{d} \rightarrow d \pi^0$

(c) $\phi \rightarrow s \bar{s} \rightarrow \pi^+ \bar{d} \rightarrow d \pi^0 \bar{d} \rightarrow d \pi^-$

Suppressed like $J/\psi$, because of unconnected quark lines (OZI rule)