Lepton and Quark Scattering

- $e^+ e^- \rightarrow \mu^+ \mu^-$
- $e^+ e^-$ annihilation to hadrons ($e^+ e^- \rightarrow Q\bar{Q}$)
- Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$
- Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$
- Elastic lepton-nucleon scattering
- Deep inelastic scattering and partons
- Deep inelastic scattering and quarks
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon
- Sum rules
The “discovery” of quarks

- deep inelastic lepton-nucleon scattering revealed dynamical understanding of quark substructure

- leptoproduction of hadrons could be interpreted as elastic scattering of the lepton by a pointlike constituent of the nucleon, the quark

- theory of scattering of two spin-1/2, pointlike particles required
\( e^+ e^- \rightarrow \mu^+ \mu^- \)

- Dominated by single-photon exchange
- \( M_{if} = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2} \)
- \( \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} f(\theta) \)

- What about spins?
  - The conservation of helicity at high energy for the EM interaction means only LR and RL states will interact
\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

- Conservation of helicity
  - consider crossed diagrams
    - (recall, helicity is conserved in vector int. at high energy)
$e^+ e^- \rightarrow \mu^+ \mu^-$

- **Amplitude is** $d^J_{mm'}(\theta) = d^{1}_{1,1}(\theta) = (1+\cos \theta)/2$
  - if RL $\rightarrow$ RL
- **Amplitude is** $d^J_{mm'}(\theta) = d^{1}_{1,-1}(\theta) = (1-\cos \theta)/2$
  - if RL $\rightarrow$ LR
- $M^2 \sim [(1+\cos \theta)/2]^2 + [(1-\cos \theta)/2]^2 = (1+\cos^2 \theta)/2$
\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

- \[ \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) \]

- \[ \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) \]
  \[ = \frac{4\pi\alpha^2}{3s} \]
  \[ = 87 \text{nb} / s(\text{GeV}^2) \]
  (point cross section)
e^+ e^- → µ^+ µ^-

\[ f \sim a_{wk} a_{em} / a_{em}^2 \sim Gs/(4\pi\alpha) \sim 10^{-4}s \text{ (interference)} \]
e^+ e^- annihilation to hadrons
$e^+ e^- \text{ annihilation to hadrons}$

- $R = \sigma (e^+ e^- \rightarrow \text{hadrons}) / \sigma (\text{point})$
e\(^+\) e\(^-\) annihilation to hadrons

- \( R = \sigma (e^+ e^- \rightarrow \text{hadrons}) / \sigma(\text{point}) \)
  - consider \( e^+ e^- \rightarrow \text{hadrons} \) as \( e^+ e^- \rightarrow Q\bar{Q} \), summed over all quarks

- \( R = \sum e_i^2 / e^2 \)
  \[ = N_c ((1/3)^2 + (2/3)^2 + (1/3)^2 + (2/3)^2 + (1/3)^2 + \ldots) \]
  \[\text{d \hspace{1cm} u \hspace{1cm} s \hspace{1cm} c \hspace{1cm} b}\]
  \( N_c = 3 \)

- Therefore, \( R \) should increase by a well defined value as each flavor threshold is crossed
$e^+ e^-$ annihilation to hadrons

<table>
<thead>
<tr>
<th>Threshold</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>below c</td>
<td>2</td>
</tr>
<tr>
<td>charm</td>
<td>3 $1/3$</td>
</tr>
<tr>
<td>bottom</td>
<td>3 $2/3$</td>
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Threshold $R$
$e^+ e^- \rightarrow Q \bar{Q}$

If Q is spin 1/2

\[(1 + \cos^2 \theta)\]
e\textsuperscript{+} e\textsuperscript{-} annihilation to hadrons

- Constancy of $R$ indicates pointlike constituents

- Angular distributions of hadron jets prove spin 1/2 partons

- Values of $R$ consistent with partons of quarks expected charge and color quantum number
Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- **Mandelstam variables:**

  $s = - (k_1 + k_2)^2 = - (k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4$

  $t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4$

  $u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4$

  for $m = 0$

  $k = (p_x, p_y, p_z, iE)$
Electron-muon scattering, e⁻ µ⁺ → e⁻ µ⁺

- **Mandelstam variables:**
  
  \[ s = -(k_1 + k_2)^2 = -(k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4 \]
  
  suppose \( k_1 = (p, iE) \quad k_2 = (-p, iE) \)
  
  \[ s = -(p-p)^2 - (2iE)^2 = 4E^2 \quad \text{for } m = 0 \]

  \[ t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4 \]
  
  suppose \( k_3 = (p \cos \theta, p \sin \theta, 0, iE) \)
  
  \[ t = (p - p \cos \theta)^2 + (-p \sin \theta)^2^+ (E - E)^2 = 2p^2(1-\cos \theta) \]
  
  \[ = 4p^2 \sin^2 \theta/2 \]

  \[ u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4 = 4p^2 \cos^2 \theta/2 \]
Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- **Annihilation cross-section** ($e^+e^- \rightarrow \mu^+\mu^-$)
  \[
  \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2} \left( \frac{t^2 + u^2}{s^2} \right) = \frac{\alpha^2}{8p^2} \left[ \sin^4 \left( \frac{\theta}{2} \right) + \cos^4 \left( \frac{\theta}{2} \right) \right]
  \]
  \[
  = \frac{\alpha^2}{4s} \left[ 1 + \cos^2 \theta \right]
  \]

- **Now consider the crossed channel**, $e^- \mu^+ \rightarrow e^- \mu^+$

\[
2 \sin^2 \left( \frac{\theta}{2} \right) = 1 - \cos \theta
\]
\[
2 \cos^2 \left( \frac{\theta}{2} \right) = 1 + \cos \theta
\]
\[
s = 4p^2
\]
Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

crossed channel ($t \leftrightarrow u$)

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2} \left( \frac{s^2 + u^2}{t^2} \right)
\]

\[
= \frac{\alpha^2}{8p^2 \sin^4(\theta/2)} [1 + \cos^4(\theta/2)]
\]

So far, all of these expressions are for the center-of-momentum system

What about the laboratory frame?
Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Suppose the muon is a target at rest in the laboratory
  (Note: this is not practical since the lifetime of the muon is 2.2 microseconds)
- $\gamma$ is the boost from cms to lab
- $p$ and $\theta$ are the projectile parameters in the cms

- In the lab:
  
  \[ E_\mu = \gamma (p - \beta p \cos \theta) \quad \text{(scattered muon)} \]
  \[ = \gamma p (1 - \cos \theta) \]
  \[ E_e = \gamma (p + \beta p) = 2\gamma p \quad \text{(incident electron)} \]
  \[ \gamma = E_\mu / E_e = (1 - \cos \theta) / 2 \]

  \[ \text{so } \cos^2 \theta / 2 = (1 + \cos \theta) / 2 = 1 - \gamma \]
  \[ d\Omega = 2\pi d(\cos \theta) = 4\pi \, dy \]
Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2\sin^4(\theta/2)} \left[1 + \cos^4(\theta/2)\right]
\]

\[
\frac{d\sigma}{4\pi dy} = \frac{\alpha^2}{q^4 / 2p^2} \left[1 + (1-y)^2\right]
\]

\[
\frac{d\sigma}{dy} = \frac{2\pi \alpha^2 s}{q^4} \left[1 + (1-y)^2\right]
\]

As $y \to 0$, we recover the Rutherford formula.

\[
d\Omega = 4\pi dy \\
\cos^2 \theta/2 = 1-y \\
q^2 = t = 4p^2\sin^2 \theta/2
\]
Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$

Consider only the charged-current reaction

$$M(\nu_e e^- \rightarrow \nu_e e^-) = \frac{(g/\sqrt{2})^2}{(q^2 + M_w^2)}$$

$$= \frac{g^2}{2M_w^2} \text{ for } q^2 \ll M_w^2$$