Invariance Principles and Conservation Laws

- Outline
  - Translation and rotation
  - Parity
  - Charge Conjugation
  - Charge Conservation and Gauge Invariance
  - Baryon and lepton conservation
  - CPT Theorem
  - CP violation and T violation
  - Isospin symmetry

Conservation Rules

Conserved quantity	Interaction: strong	EM	weak
energy-momentum, charge, baryon number, lepton number	yes	yes	yes
CPT	yes	yes	yes
P (parity)	yes	yes	no
C (charge conjugation parity)	yes	yes	no
CP (or T)	yes	yes	10^{-3} violation
I (isospin)	yes	no	no

Discrete and Continuous Symmtries

- Continuous Symmetries
  - Space-time symmetries
  - Lorentz transformations
  - Poincare transformations
    - combined space-time translation and Lorentz T.
- Discrete symmetries
  - cannot be built up from succession of infinitesimally small transformations
- Other types of symmetries
  - Dynamical symmetries - of the equations of motion
  - Internal symmetries - such as spin, charge, color, or isospin

Conservation Laws

- Continuous symmetries lead to additive conservation laws:
  - energy, momentum

- Discrete symmetries lead to multiplicative conservation laws
  - parity, charge conjugation

Translation and Rotation

- Invariance of the energy of an isolated physical system under <u>space tranlations</u> leads to <u>conservation of linear momentum</u>

- Invariance of the energy of an isolated physical system under <u>spatial rotations</u> leads to <u>conservation of angular momentum</u>

- Noether's Theorem

E. Noether, "Invariante Varlationsprobleme", Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math-phys. Klasse (1918), 235-257;

English translation M. A. Travel, Transport Theory and Statistical Physics 1(3) 1971,183-207.

Parity

- Spatial inversion
  - $(x,y,z) \rightarrow (-x,-y,-z)$
  - discrete symmetry
- $P\,\psi(r) = \psi(-r)$
  - P is the parity operator
- $P^2\,\psi(r) = P\,\psi(-r) = \psi(r)$,
  - therefore $P^2 = 1$ and the parity of an eigen-system is 1 or -1

Parity

- Example, the spherical harmonics:

$$\boxed{Y_L^M}$$

- The spherical harmonics describe a state in a spherically symmetric potential with definite ang. momentum

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta e^{2i\phi}$$

- $p = (-1)^L$

Parity

- Parity is a <u>multiplicative</u> quantum number

- Composite system:
  - parity is equal to product of the parts
  - eg. two pions in an angular momentum L state

$$P = P(\pi_1) \cdot P(\pi_2) \cdot (-1)^L$$

- Intrinsic parity of particles:
  - proton and neutron (+1 by definition, could be -1)
    - parity of fermion and anti-fermion are opposite, and assignment of one is arbitrary
  - pion (-1, measured)

Pion spin and parity

- 1. Spin of the charged pion
  - $p + p \leftrightarrow \pi^+ + d$ $\quad (|M_{if}|^2 = |M_{fi}|^2)$
  - then, "detailed balance" gives:

    $\sigma(pp \rightarrow \pi^+ d) \sim (2s_\pi + 1)(2s_d + 1)\, p_\pi^2$

    $\sigma(\pi^+ d \rightarrow pp) \sim 1/2\, (2s_p + 1)^2\, p_p^2$

    (1/2 because protons are identical)
  - Measurements of σ's reveals $s_\pi = 0$

- 2. Spin of the neutral pion
  - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0 (see following)

Pion spin and parity

- 2. Spin of the neutral pion
  - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0
  - along the flight path of the γs in the π^0 rest frame, the total photon spin (S_z) must be 0 or 2
  - If $S_\pi = 1$, then S_z must be 0
  - If $S_\pi = 1$, $S_z = 0$, the two-photon amplitude must behave as $P_l^{m=0}(\cos\ \theta)$, which is antisymmetric under interchange (=180^0 rotation)
  - This corresponds to two right or left circularly polarized photons travelling in opposite directions, which must be symmetric by Bose statistics -> forbidden $S_\pi \neq 1$
  - Therefore, $S_\pi = 0$ or $S_\pi \geq 2$ (which is ruled out by pion production statistics - all three charges produced equally)

Parity

- Parity of the charged pion
  - the observation of the reaction

    $$\pi^- + d \rightarrow n + n$$

    is evidence that the charged pion has $p = -1$
- Capture takes place from an s-state, $L_i=0$ ($S_d=1$)
  - (X-ray emissions following capture)
- $J=1$, since $S_d=1$ and $S_\pi=0$
- In the two neutron system, L+S must be even by the antisymmetric requirement on identical fermions
  - thus, ${}^{(2S+1)}J_L = {}^3P_1$ state with $p = (-1)^L = -1$
  - since initial state is $p_d p_\pi (-1)^{L_i} = (+1)\, p_\pi$, $p_\pi = -1$

Parity

- Parity of the neutral pion

  $\pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-)$ (rare decay of the pion)

  - the planes of the pairs follow the E vectors of the internally converted photons ($\pi^0 \rightarrow \gamma\gamma$)
  - even system of two photons (bosons)

$$A \sim (\mathbf{E}_1 \cdot \mathbf{E}_2) \qquad (P=+1)$$

$$\text{or } A \sim (\mathbf{E}_1 \times \mathbf{E}_2) \cdot \mathbf{k} \qquad (P=-1)$$

  - Intensities or rates $\sim |A|^2$

$$|A|^2 \sim \cos^2 \phi$$

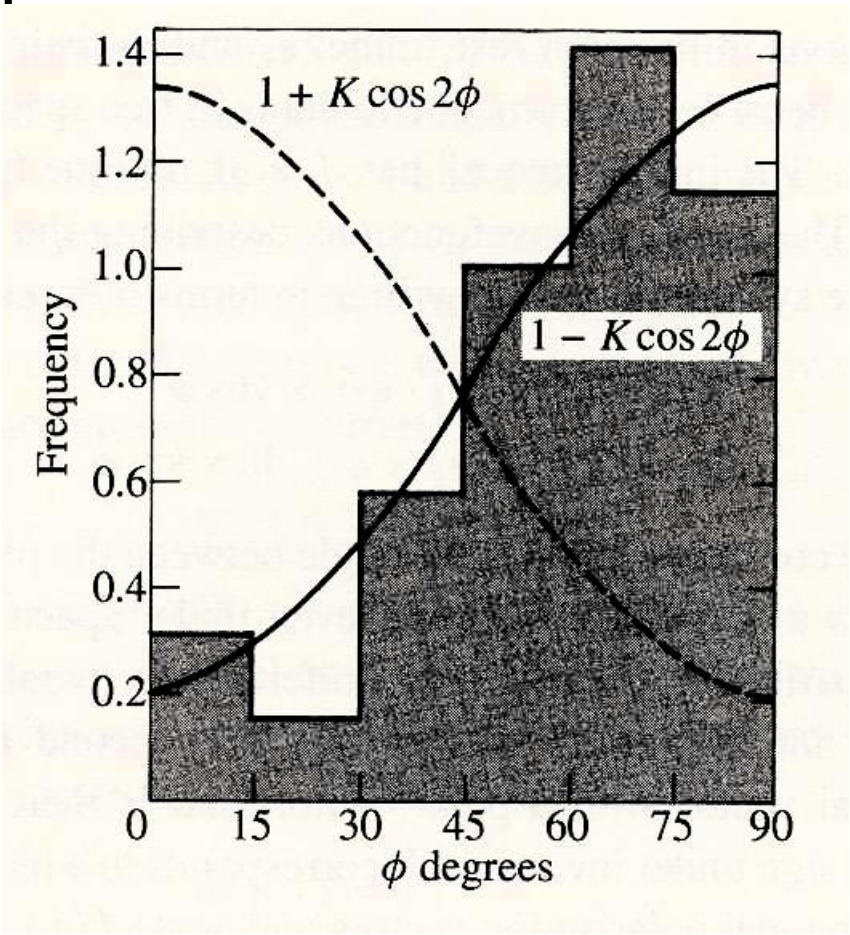
$$\text{or } |A|^2 \sim \sin^2 \phi$$

Parity

- Parity of the neutral pion

  $\pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-)$

  scalar(0^+) would follow dashed line, while pseudoscalar (0^-) would follow solid line



Parity of particles and antiparticles

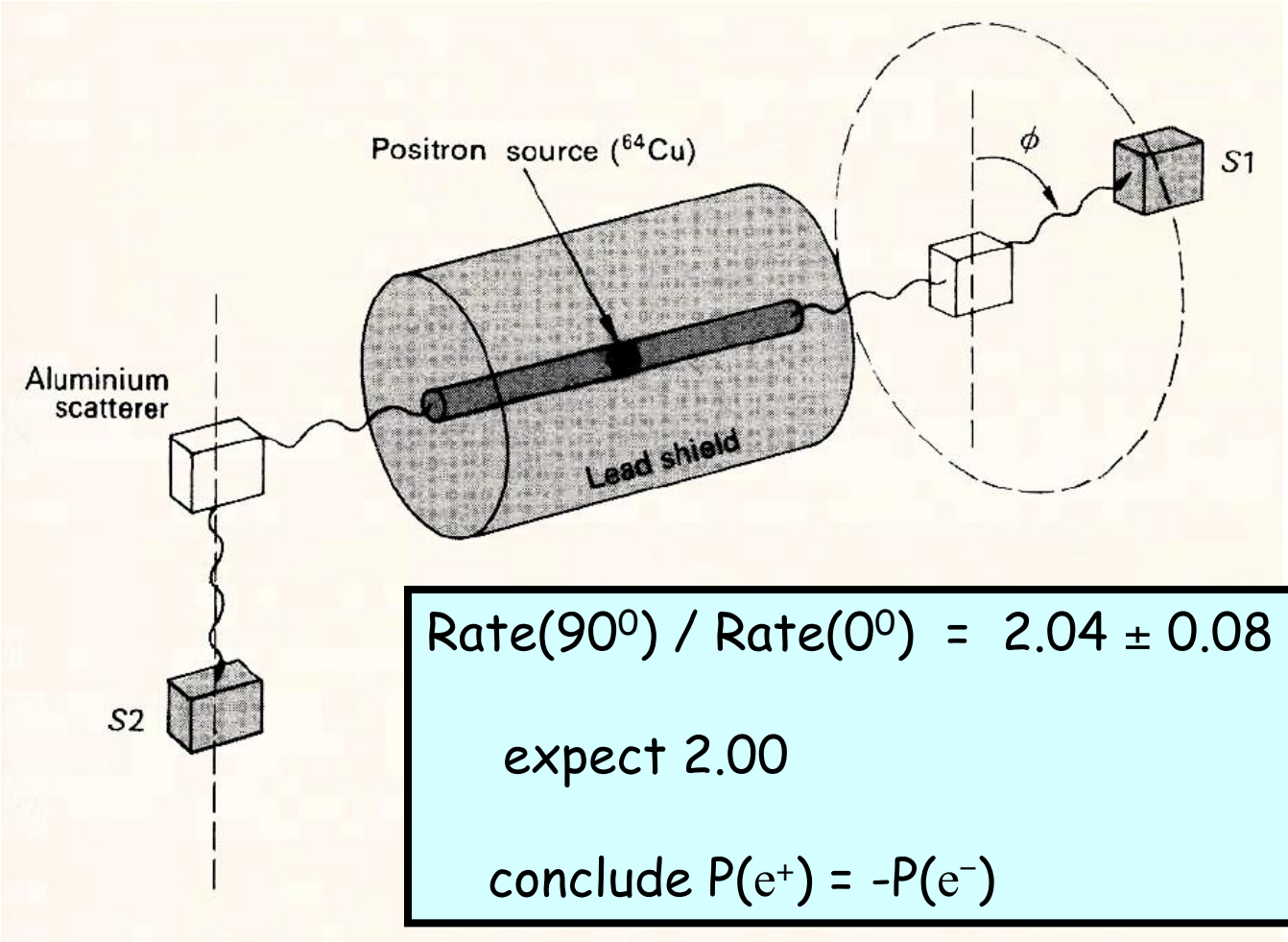
- Dirac theory required fermions and antifermions to have opposite intrinsic parity
- This was checked in decays of the spin singlet ground state of positronium

$$e^+e^- \left({}^{(2S+1)}J_L = {}^1S_0\right) \rightarrow 2\,\gamma$$

$$P = (+1)\,(-1)\,(-1)^0 = -1$$

- This is exactly the case of the π^0 decay
  - the photon polarizations must have a $\sin^2\phi$ form

Parity of particles and antiparticles



Rate(90^0) / Rate(0^0) = 2.04 ± 0.08

expect 2.00

conclude $P(e^+) = -P(e^-)$

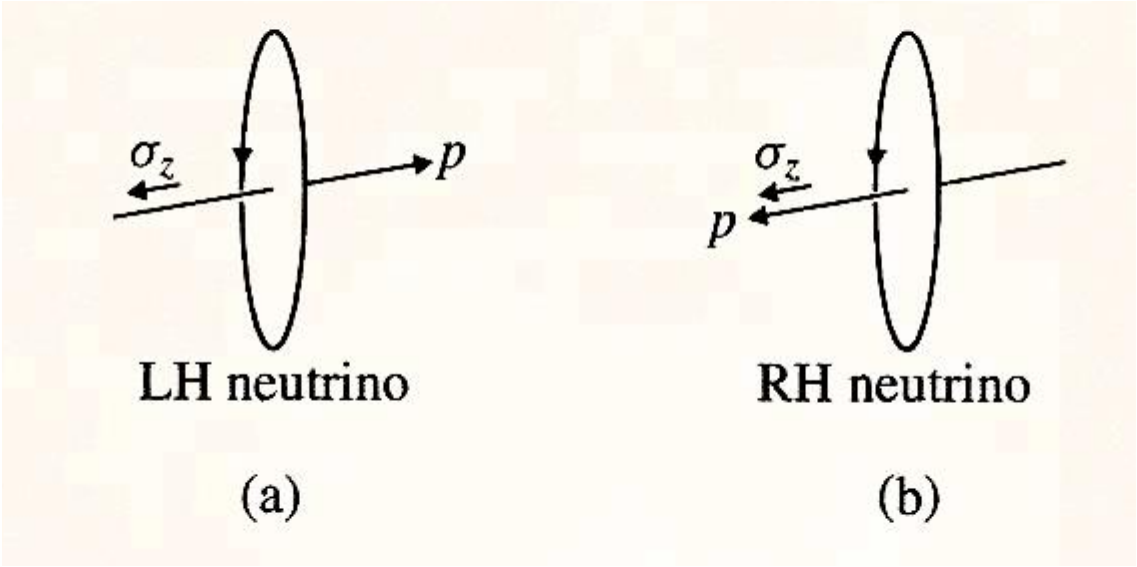
Parity of particles and antiparticles

- Two-particle systems
  - Fermion-antifermion
    - $p = (-1)^{L+1}$
    - eg, pion which is q-antiq with L=0 has p = -1
  - Boson-antiboson
    - same parity
    - $p = (-)^{L}$
    - eg. $\rho \rightarrow \pi^+ \ \pi^-$

      $1^- \rightarrow 0^- \ 0^-$, L=1

Tests of parity conservation

- Strong and EM interactions conserve parity, but weak interactions do not



- LH neutrino observed, but RH neutrino is not
  - this is a maximally violated symmetry

Tests of parity conservation

- In interactions dominated by the strong or the EM interaction, some parity violation may be observed due to the small contribution of the weak int. to the process

$$H = H_{strong} + H_{EM} + H_{weak}$$

- In nuclear transitions, for example, the degree of parity violation will be of the order of the ratio of the weak to the strong couplings ($\sim 10^{-7}$)

Tests of parity conservation

- Examples of nuclear transitions
  - fore-aft asymmetry in gamma emission

$$^{19}F^* \rightarrow {}^{19}F + \gamma \text{ (110 keV)}$$

$$J^P: \quad 1/2^- \rightarrow 1/2^+$$

$$\Delta \sim 10^{-4}$$

  - very narrow decay

$$^{16}O^* \rightarrow {}^{12}C + \alpha$$

$$J^P: \quad 2^- \rightarrow 2^+$$

$$\Gamma = 10^{-10} \text{ eV} \quad (\text{note } {}^{16}O^* \rightarrow {}^{16}O + \gamma, \Gamma = 3 \times 10^{-3} \text{ eV})$$

Charge Conjugation Invariance

- Charge conjugation reverses sign of charge and magnetic moment, leaving other coord unchanged
- Good symmetry in strong and EM interactions:
  - eg. mesons in $p + \bar{p} \rightarrow \pi^+ + \pi^- + \ldots\ldots$
- Only neutral bosons are eigenstates of C
- The charge conjugation eigenvalue of the γ is -1:
  - EM fields change sign when charge source reverses sign
- Since $\pi^0 \rightarrow \gamma\,\gamma$, $C|\pi^0\rangle = +1$
- Consequence of C of π^0 and γ, and C-cons. in EM int,

  $\pi^0 \rightarrow \gamma\,\gamma\,\gamma$ forbidden
  - experiment: $\pi^0 \rightarrow \gamma\,\gamma\,\gamma \,/\, \pi^0 \rightarrow \gamma\,\gamma < 3 \times 10^{-8}$

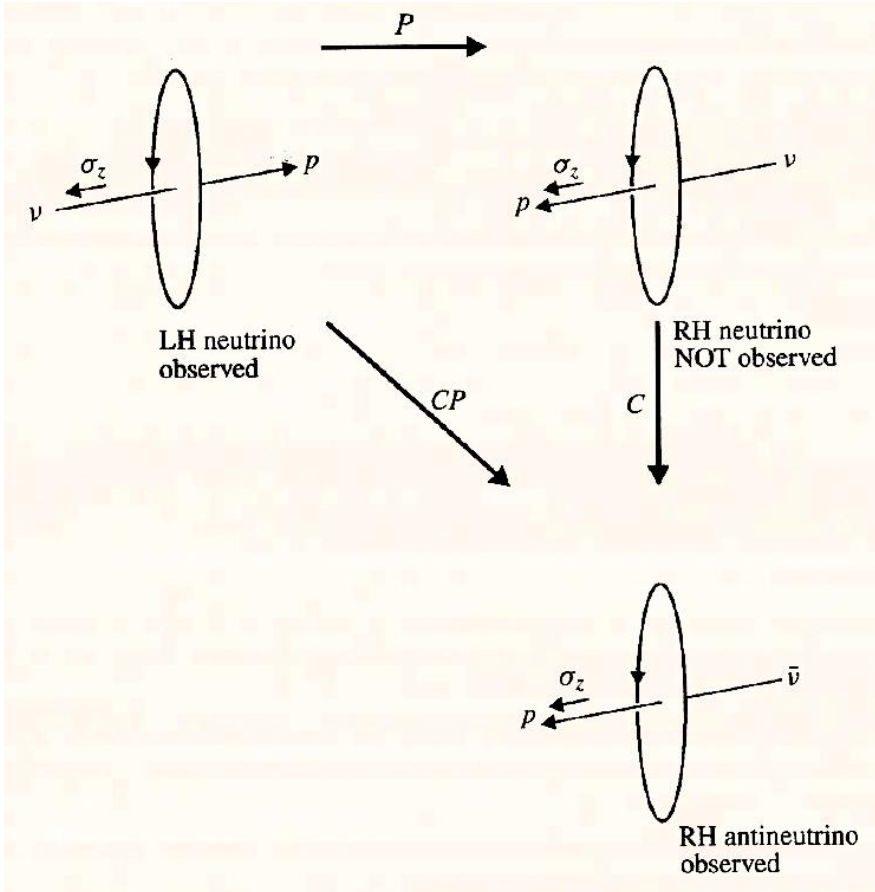
Charge Conjugation Invariance

CHARGE CONJUGATION (*C*) INVARIANCE

$\Gamma(\pi^0 \to 3\gamma)/\Gamma_{total}$		$<3.1 \times 10^{-8}$, CL = 90%
η *C*-nonconserving decay parameters		
$\pi^+\pi^-\pi^0$ left-right asymmetry		$(0.09^{+0.11}_{-0.12}) \times 10^{-2}$
$\pi^+\pi^-\pi^0$ sextant asymmetry		$(0.12^{+0.10}_{-0.11}) \times 10^{-2}$
$\pi^+\pi^-\pi^0$ quadrant asymmetry		$(-0.09 \pm 0.09) \times 10^{-2}$
$\pi^+\pi^-\gamma$ left-right asymmetry		$(0.9 \pm 0.4) \times 10^{-2}$
$\pi^+\pi^-\gamma$ parameter β (*D*-wave)		-0.02 ± 0.07 (S = 1.3)
$\Gamma(\eta \to \pi^0\gamma)/\Gamma_{total}$		$<9 \times 10^{-5}$, CL = 90%
$\Gamma(\eta \to 2\pi^0\gamma)/\Gamma_{total}$		$<5 \times 10^{-4}$, CL = 90%
$\Gamma(\eta \to 3\pi^0\gamma)/\Gamma_{total}$		$<6 \times 10^{-5}$, CL = 90%
$\Gamma(\eta \to 3\gamma)/\Gamma_{total}$		$<1.6 \times 10^{-5}$, CL = 90%
$\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma_{total}$	[a]	$<4 \times 10^{-5}$, CL = 90%
$\Gamma(\eta \to \pi^0 \mu^+ \mu^-)/\Gamma_{total}$	[a]	$<5 \times 10^{-6}$, CL = 90%
$\Gamma(\omega(782) \to \eta\pi^0)/\Gamma_{total}$		$<2.1 \times 10^{-4}$, CL = 90%
$\Gamma(\omega(782) \to 2\pi^0)/\Gamma_{total}$		$<2.1 \times 10^{-4}$, CL = 90%
$\Gamma(\omega(782) \to 3\pi^0)/\Gamma_{total}$		$<2.3 \times 10^{-4}$, CL = 90%
asymmetry parameter for $\eta'(958) \to \pi^+\pi^-\gamma$ decay		-0.03 ± 0.04
$\Gamma(\eta'(958) \to \pi^0 e^+ e^-)/\Gamma_{total}$	[a]	$<1.4 \times 10^{-3}$, CL = 90%
$\Gamma(\eta'(958) \to \eta e^+ e^-)/\Gamma_{total}$	[a]	$<2.4 \times 10^{-3}$, CL = 90%
$\Gamma(\eta'(958) \to 3\gamma)/\Gamma_{total}$		$<1.0 \times 10^{-4}$, CL = 90%
$\Gamma(\eta'(958) \to \mu^+\mu^-\pi^0)/\Gamma_{total}$	[a]	$<6.0 \times 10^{-5}$, CL = 90%

Charge Conjugation Invariance

- Weak interaction
  - respects neither C nor P, but CP is an approximate symmetry of the weak interaction



Charge Conservation Invariance

ELECTRIC CHARGE (Q)

$e \rightarrow \nu_e \gamma$ and astrophysical limits	[u]	$>4.6 \times 10^{26}$ yr, CL = 90%
$\Gamma(n \rightarrow p \nu_e \overline{\nu}_e)/\Gamma_{\text{total}}$		$<8 \times 10^{-27}$, CL = 68%

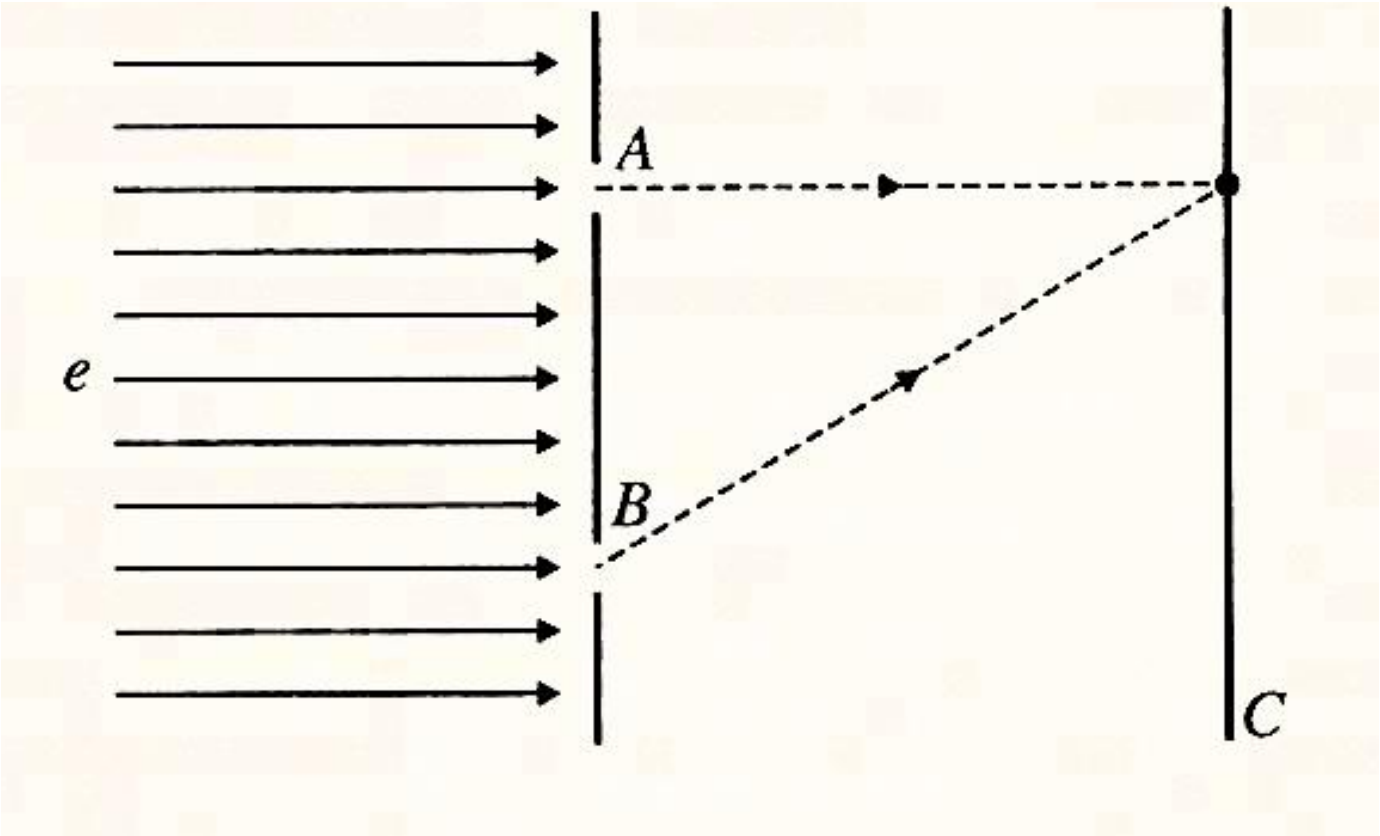
[u] This is the best limit for the mode $e^- \rightarrow \nu\gamma$. The best limit for "electron disappearance" is 6.4×10^{24} yr.

Charge Conservation and Gauge Invariance

- It is possible to described charged particles by wavefuntions with phases chosen arbitrarily at different times and places, provided:
  - the charges couple to a long-range field (the EM field) to which the same local gauge transformation is applied
  - <u>charge is conserved</u>

Charge Conservation and Gauge Invariance

- The pattern at C results from the interference of the electrons coming through slits A and B. If we allow an arbitrary phase for each slit, the pattern at C is altered.



Charge Conservation and Gauge Invariance

- After adding an arbitrary phase (θ), the wave function of the electron will be

$$\psi = e^{i(\mathbf{p}.\mathbf{x}-Et+e\theta)} = e^{i(px+e\theta)}$$

- Now the gradient of the phase is

$$\frac{\partial}{\partial x} i(px + e\theta) = i(p + e\frac{\partial \theta}{\partial x})$$

- If the phase depends on location (it is local), the electron interaction with the EM field will also alter the pattern.

$$\psi = e^{i(px-eAx)} \qquad A = (A, i\phi)$$

- Then, the space-time gradient of the phase becomes

$$\frac{\partial}{\partial x} i(px - eAx + e\theta) = i(p - eA + e\frac{\partial \theta}{\partial x})$$

- If $A \rightarrow A + \frac{\partial \theta}{\partial x}$ the pattern is unaltered
- This is an example of a <u>local gauge transformation.</u>

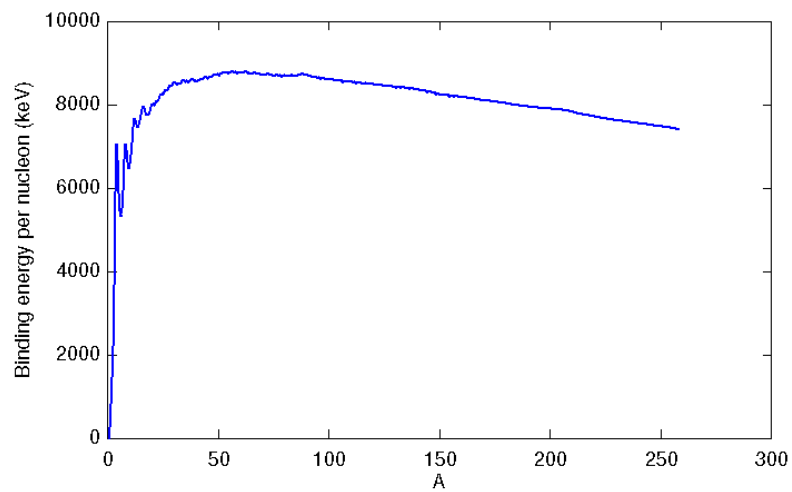
Charge Conservation and Gauge Invariance

- It is possible to described charged particles by wavefuntions with phases chosen arbitrarily at different times and places (local gauge transformation), provided:
  - the charges couple to a long-range field (the EM field) to which the same local gauge transformation is applied
  - <u>charge is conserved</u>

- This property of <u>local gauge symmetry</u> is a critical ingredient in a field theory that will be <u>renormalizable</u>, such as QED, or the EW theory, leading to cross-sections and decay rates that are finite and calculable to all orders in the coupling constant.

Baryon Conservation

- Baryon number is conserved to a very high level:
  - If it didn't, what would prevent $p \rightarrow e^+ \pi^0$?
- Does an underlying long range field lead to this?
  - Equivalency of <u>gravitational</u> and <u>intertial</u> mass established at the 10^{-12} level
  - Since the nuclear binding energy differs with nuclei, the ratio of mass to number of nucleons is not constant, providing a potential source for a difference between gravitational and inertial mass



Baryon Conservation

- Baryon conservation is established to a very low level of violation:
  - $\tau(p \rightarrow e^{+}\pi^{0}) > 8 \times 10^{33}$ yr
- This is a remarkably precise symmetry, yet we know of no long-range field that could cause it
- We have reason to suspect it is violated at some level:
  - baryon dominance in the Universe today

Baryon Conservation

BARYON NUMBER

$\Gamma(Z \to pe)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p\mu)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(\tau^- \to \bar{p}\gamma)/\Gamma_{total}$		$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \bar{p}\pi^0)/\Gamma_{total}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \bar{p}2\pi^0)/\Gamma_{total}$		$<3.3 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \bar{p}\eta)/\Gamma_{total}$		$<8.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \bar{p}\pi^0\eta)/\Gamma_{total}$		$<2.7 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \Lambda\pi^-)/\Gamma_{total}$		$<7.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \bar{\Lambda}\pi^-)/\Gamma_{total}$		$<1.4 \times 10^{-7}$, CL = 90%
$\Gamma(D^0 \to pe^-)/\Gamma_{total}$	[q]	$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \bar{p}e^+)/\Gamma_{total}$	[r]	$<1.1 \times 10^{-5}$, CL = 90%
p mean life	[s]	$>2.1 \times 10^{29}$ years, CL = 90%

A few examples of proton or bound neutron decay follow. For limits on many other nucleon decay channels, see the Baryon Summary Table.

$\tau(N \to e^+\pi)$	> 158 (n), > 8200 (p) $\times 10^{30}$ years, CL = 90%
$\tau(N \to \mu^+\pi)$	> 100 (n), > 6600 (p) $\times 10^{30}$ years, CL = 90%
$\tau(N \to e^+K)$	> 17 (n), > 150 (p) $\times 10^{30}$ years, CL = 90%
$\tau(N \to \mu^+K)$	> 26 (n), > 120 (p) $\times 10^{30}$ years, CL = 90%
limit on $n\bar{n}$ oscillations (free n)	$>0.86 \times 10^8$ s, CL = 90%

Lepton Conservation

- Direct searches have not been fruitful:
  - eg. $\tau(^{76}Ge \rightarrow {}^{76}Se + 0\nu + e^- + e^-) > 10^{26}$ yr
    "neutrinoless double beta decay"

- Yet we do observe neutrino oscillations:
  - (eg. $\nu_e \rightarrow \nu_\mu$)
  - these are small transitions

Lepton Conservation

$\Gamma(Z \to e^{\pm}\mu^{\mp})/\Gamma_{\text{total}}$	[l]	$<1.7 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to e^{\pm}\tau^{\mp})/\Gamma_{\text{total}}$	[l]	$<9.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to \mu^{\pm}\tau^{\mp})/\Gamma_{\text{total}}$	[l]	$<1.2 \times 10^{-5}$, CL = 95%
$\sigma(e^+e^- \to e^{\pm}\tau^{\mp}) / \sigma(e^+e^- \to \mu^+\mu^-)$		$<8.9 \times 10^{-6}$, CL = 95%
$\sigma(e^+e^- \to \mu^{\pm}\tau^{\mp}) / \sigma(e^+e^- \to \mu^+\mu^-)$		$<4.0 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^-$ conversion		
$\sigma(\mu^- {}^{32}\text{S} \to e^- {}^{32}\text{S}) / \sigma(\mu^- {}^{32}\text{S} \to \nu_\mu {}^{32}\text{P}^*)$		$<7 \times 10^{-11}$, CL = 90%
$\sigma(\mu^- \text{Ti} \to e^- \text{Ti}) / \sigma(\mu^- \text{Ti} \to \text{capture})$		$<4.3 \times 10^{-12}$, CL = 90%
$\sigma(\mu^- \text{Pb} \to e^- \text{Pb}) / \sigma(\mu^- \text{Pb} \to \text{capture})$		$<4.6 \times 10^{-11}$, CL = 90%
limit on muonium $\to$ antimuonium conversion $R_g = G_C / G_F$		<0.0030, CL = 90%
$\Gamma(\mu^- \to e^-\nu_e\bar{\nu}_\mu)/\Gamma_{\text{total}}$	[m]	$<1.2 \times 10^{-2}$, CL = 90%
$\Gamma(\mu^- \to e^-\gamma)/\Gamma_{\text{total}}$		$<1.2 \times 10^{-11}$, CL = 90%
$\Gamma(\mu^- \to e^-e^+e^-)/\Gamma_{\text{total}}$		$<1.0 \times 10^{-12}$, CL = 90%
$\Gamma(\mu^- \to e^-2\gamma)/\Gamma_{\text{total}}$		$<7.2 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^-\gamma)/\Gamma_{\text{total}}$		$<3.3 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^-\gamma)/\Gamma_{\text{total}}$		$<4.4 \times 10^{-8}$, CL = 90%

CPT Invariance

- Very general assumptions in quantum field theory:
  - spin-statistics relation: integer and half-integer spin particles obey Bose and Fermi statistics
  - Lorentz invariance

  lead to the CPT Theorem:
  - <u>all interactions</u> are invariant under the successive operation of C (=charge conjugation), P (=parity operation), and T (= time reversal)

- Masses, lifetimes, moments, etc. of particles and antiparticles must be identical

CPT Invariance

- Tests of the CPT Theorem

$(m_{W^+} - m_{W^-}) / m_{\text{average}}$	-0.002 ± 0.007
$(m_{e^+} - m_{e^-}) / m_{\text{average}}$	$<8 \times 10^{-9}$, CL = 90%
$\lvert q_{e^+} + q_{e^-} \rvert / e$	$<4 \times 10^{-8}$
$(g_{e^+} - g_{e^-}) / g_{\text{average}}$	$(-0.5 \pm 2.1) \times 10^{-12}$
$(\tau_{\mu^+} - \tau_{\mu^-}) / \tau_{\text{average}}$	$(2 \pm 8) \times 10^{-5}$
$(g_{\mu^+} - g_{\mu^-}) / g_{\text{average}}$	$(-0.11 \pm 0.12) \times 10^{-8}$
$(m_{\tau^+} - m_{\tau^-}) / m_{\text{average}}$	$<2.8 \times 10^{-4}$, CL = 90%
$2\,(m_t - m_{\bar{t}}) / (m_t + m_{\bar{t}})$	0.022 ± 0.022
$(m_{\pi^+} - m_{\pi^-}) / m_{\text{average}}$	$(2 \pm 5) \times 10^{-4}$
$(\tau_{\pi^+} - \tau_{\pi^-}) / \tau_{\text{average}}$	$(6 \pm 7) \times 10^{-4}$
$(m_{K^+} - m_{K^-}) / m_{\text{average}}$	$(-0.6 \pm 1.8) \times 10^{-4}$
$(\tau_{K^+} - \tau_{K^-}) / \tau_{\text{average}}$	$(0.10 \pm 0.09)\%$ (S = 1.2)
$K^\pm \to \mu^\pm \nu_\mu$ rate difference/average	$(-0.5 \pm 0.4)\%$
$K^\pm \to \pi^\pm \pi^0$ rate difference/average	[1] $(0.8 \pm 1.2)\%$

CPT Invariance (cont.)

phase difference $\phi_{00} - \phi_{+-}$		$(0.2 \pm 0.4)^{\circ}$
$\mathrm{Re}(\frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}) - \frac{A_L}{2}$		$(-3 \pm 35) \times 10^{-6}$
$A_{CPT}(D^0 \to K^-\pi^+)$		0.008 ± 0.008
$\lvert m_p - m_{\overline{p}} \rvert / m_p$	[k]	$<2 \times 10^{-9}$, CL = 90%
$(\lvert \frac{q_{\overline{p}}}{m_p} \rvert - \frac{q_p}{m_p}) / \frac{q_p}{m_p}$		$(-9 \pm 9) \times 10^{-11}$
$\lvert q_p + q_{\overline{p}} \rvert / e$	[k]	$<2 \times 10^{-9}$, CL = 90%
$(\mu_p + \mu_{\overline{p}}) / \mu_p$		$(-0.1 \pm 2.1) \times 10^{-3}$
$(m_n - m_{\overline{n}}) / m_n$		$(9 \pm 6) \times 10^{-5}$
$(m_\Lambda - m_{\overline{\Lambda}}) / m_\Lambda$		$(-0.1 \pm 1.1) \times 10^{-5}$ (S = 1.6)
$(\tau_\Lambda - \tau_{\overline{\Lambda}}) / \tau_\Lambda$		-0.001 ± 0.009
$(\tau_{\Sigma^+} - \tau_{\overline{\Sigma}^-}) / \tau_{\Sigma^+}$		$(-0.6 \pm 1.2) \times 10^{-3}$
$(\mu_{\Sigma^+} + \mu_{\overline{\Sigma}^-}) / \mu_{\Sigma^+}$		0.014 ± 0.015
$(m_{\Xi^-} - m_{\overline{\Xi}^+}) / m_{\Xi^-}$		$(-3 \pm 9) \times 10^{-5}$
$(\tau_{\Xi^-} - \tau_{\overline{\Xi}^+}) / \tau_{\Xi^-}$		-0.01 ± 0.07
$(\mu_{\Xi^-} + \mu_{\overline{\Xi}^+}) / \lvert \mu_{\Xi^-} \rvert$		$+0.01 \pm 0.05$
$(m_{\Omega^-} - m_{\overline{\Omega}^+}) / m_{\Omega^-}$		$(-1 \pm 8) \times 10^{-5}$
$(\tau_{\Omega^-} - \tau_{\overline{\Omega}^+}) / \tau_{\Omega^-}$		0.00 ± 0.05

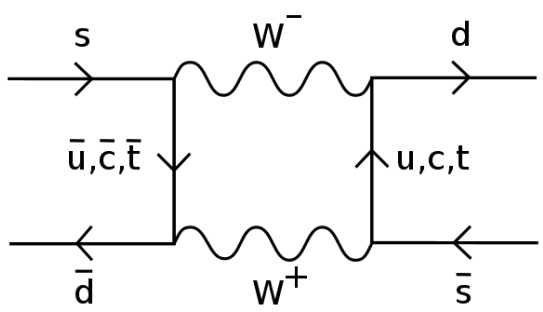
CP Violation

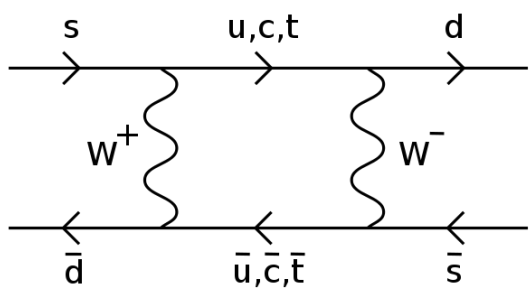
- Until 1964 it was believed that CP symmetry was respected in the weak interaction
  - P and C were known to be separately violated
- A small team led by Cronin and Fitch studying the long-lived neutral kaon discovered that it violated CP with a rate of 2×10^{-3}

$$K_L^0 = \sqrt{\tfrac{1}{2}}(|K^0\rangle - \overline{|K^0\rangle})$$

- $K_L^0 \rightarrow \pi\ \pi\ \pi$ (> 99 %)
- CP = -1 $\rightarrow$ CP = -1
- $K_L^0 \rightarrow \pi\ \pi$ (0. 2 %)

  CP = -1 $\rightarrow$ CP = +1





CP Violation

- The Standard Model of CP violation explains its origin from the CP-violating phase in the mixing matrix of the six quark flavors

- Experiments at PEPII (BaBar) at SLAC and KEKB (Belle) at KEK have confirmed the Standard Model explanation with measurements of CP violation in B decays

T Violation

- The <u>CPT Theorem</u> dictates that if CP is violated, T must also be violated so that CPT can be invariant

		Effect of T	Effect of P
• position	r	r	$-r$
• momentum	p	$-p$	$-p$
• spin	σ $= (r \times p)$	$-\sigma$	σ
• electric field	E $(= -\nabla V)$	E	$-E$
• magnetic field	B	$-B$	B
• mag. dip. mom.	$\sigma \cdot B$	$\sigma \cdot B$	$\sigma \cdot B$
• el. dipl. mom.	$\sigma \cdot E$	$-\sigma \cdot E$	$-\sigma \cdot E$
• long. pol.	$\sigma \cdot p$	$\sigma \cdot p$	$-\sigma \cdot p$
• trans. pol.	$\sigma \cdot (p_1 \times p_2)$	$-\sigma \cdot (p_1 \times p_2)$	$\sigma \cdot (p_1 \times p_2)$

T Violation

- Transverse polarization

		Effect of T	Effect of P
• trans. pol.	$\sigma \cdot (p_1 \times p_2)$	$-\sigma \cdot (p_1 \times p_2)$	$\sigma \cdot (p_1 \times p_2)$

- $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$
- $n \rightarrow p + e^- + \overline{\nu}_e$

  - T-violating contributions below 10^{-3}

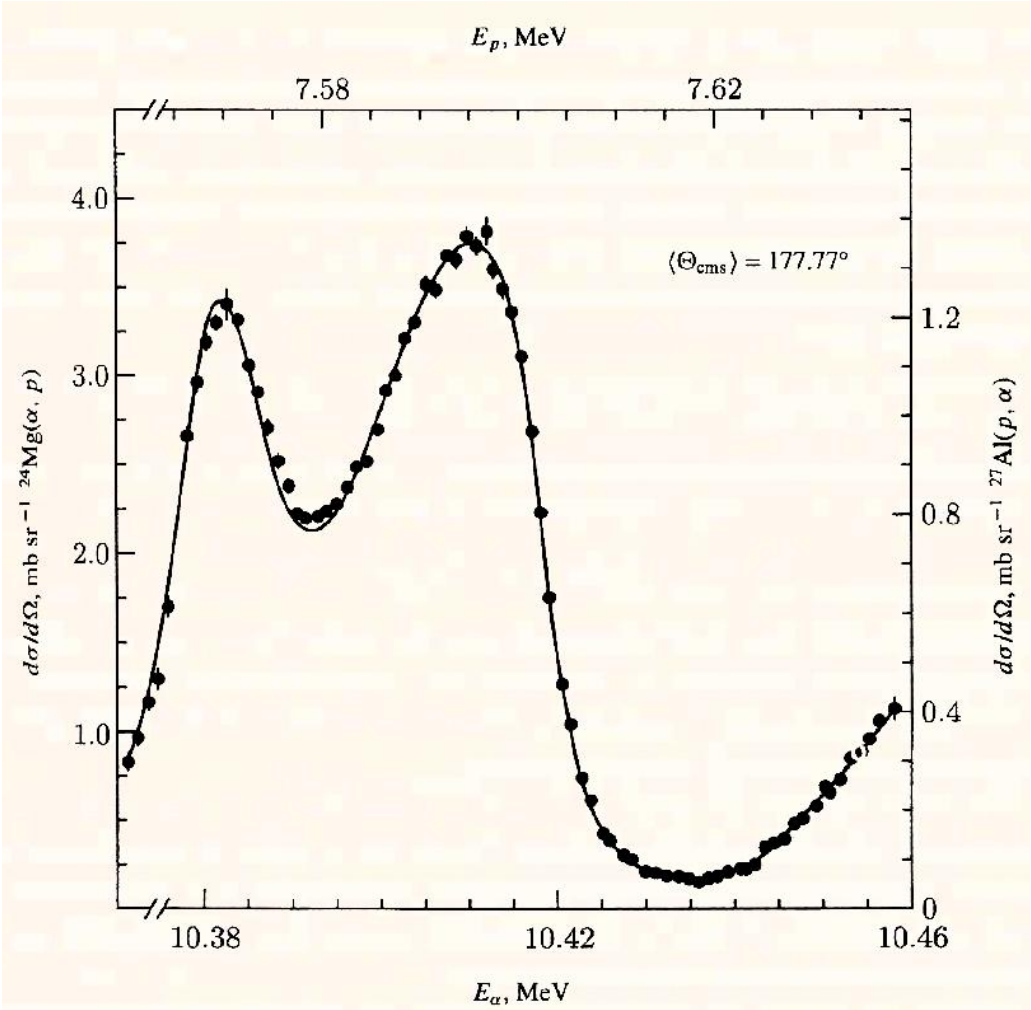
T Violation

- Detailed Balance

$p + {}^{27}Al \leftrightarrow \alpha + {}^{24}Mg$

curve and points

T violation
$< 5 \times 10^{-4}$



Neutron Electric Dipole Moment

- Most sensitive tests of T violation come from searches for the neutron and electron dipole moments
  - electron $0.07 \pm 0.07 \times 10^{-26}$ e-cm
  - neutron $< 0.29 \times 10^{-25}$ e-cm
- Note, the existence of these electric dipole moments would also violate parity

Neutron Electric Dipole Moment

- How big might we expect the EDM to be?
  - EDM = (charge x length) x T-violating parameter

    length ~ 1/Mass

    assume weak interaction - $\sim g^2 / M_W^2$

    $\sim e^2 M_N / M_W^2 \sim 4\pi/137\ (0.94\ \text{GeV}) / (80\text{GeV})^2$

    $\sim 10^{-5}\ \text{GeV}^{-1}$ [200 MeV-fm]

    $\sim 10^{-6}$ fm $\sim 10^{-19}$ cm

    T-violating parameter $\sim 10^{-3}$ (like K^0)

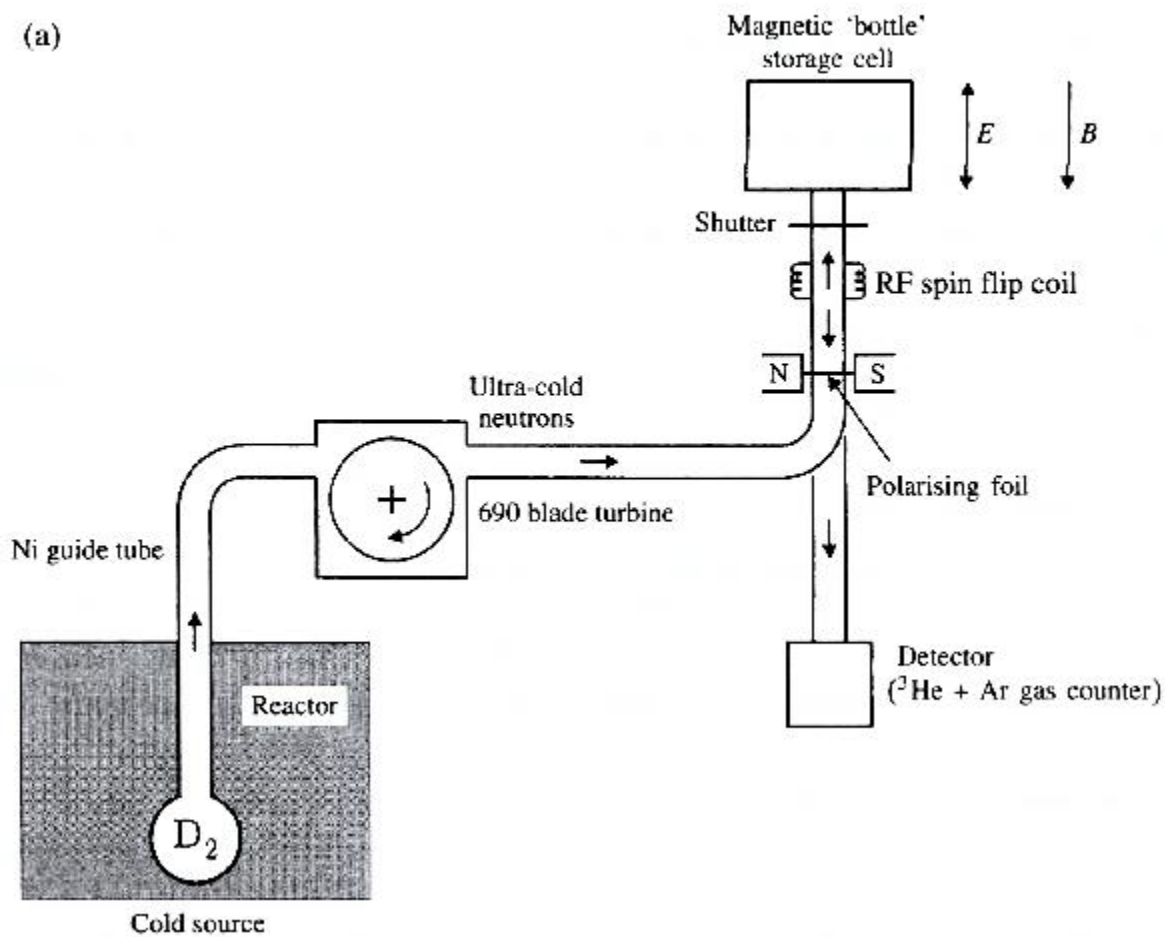
Then:

EDM $\sim e\,(10^{-19}\ \text{cm})(10^{-3})$

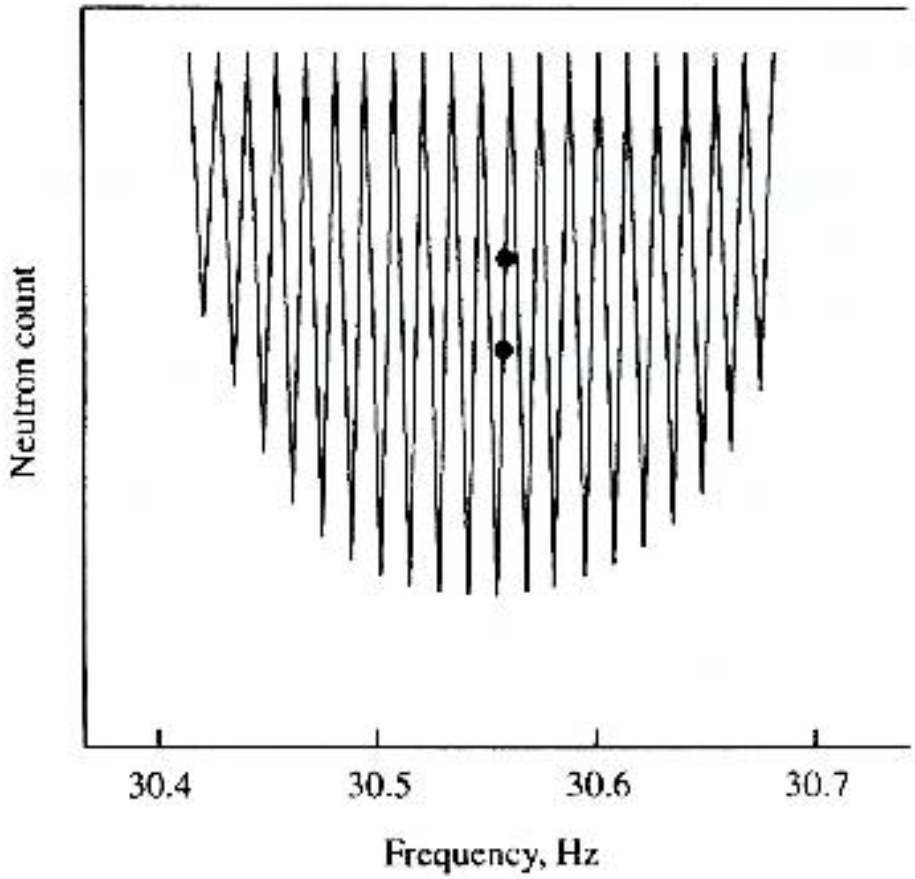
$\sim 10^{-22}$ e-cm

Experiment: Neutron EDM $< 0.29 \times 10^{-25}$ e-cm

Neutron Electric Dipole Moment



Neutron Electric Dipole Moment



- Neutron EDM $< 0.63 \times 10^{-25}$ e-cm

T Violation

TIME REVERSAL (T) INVARIANCE

e electric dipole moment	$(0.07 \pm 0.07) \times 10^{-26}$ e cm
μ electric dipole moment	$(-0.1 \pm 0.9) \times 10^{-19}$ e cm
μ decay parameters	
transverse e^+ polarization normal to plane of μ spin, e^+ momentum	$(-2 \pm 8) \times 10^{-3}$
p electric dipole moment	$<0.54 \times 10^{-23}$ e cm
n electric dipole moment	$<0.29 \times 10^{-25}$ e cm, CL = 90%

Isospin Symmetry

- The nuclear force of the neutron is nearly the same as the nuclear force of the proton
- 1932 - Heisenberg - think of the proton and the neutron as two charge states of the nucleon
- In analogy to spin, the nucleon has isospin 1/2

  $I_3 = 1/2$ proton

  $I_3 = -1/2$ neutron

  $Q = (I_3 + 1/2)\, e$

- Isospin turns out to be a conserved quantity of the strong interaction

Isospin Symmetry

- The notion that the neutron and the proton might be two different states of the same particle (the nucleon) came from the near equality of the n-p, n-n, and p-p nuclear forces (once Coulomb effects were subtracted)

- Within the quark model, we can think of this symmetry as being a symmetry between the u and d quarks:

p = d u u	I_3 (u) = $\frac{1}{2}$
n = d d u	I_3 (d) = -1/2

Isospin Symmetry

- Example from the meson sector:
  - the pion (an isospin triplet, I=1)

$\pi^+ = u\bar{d}$ $(I_3 = +1)$

$\pi^- = d\bar{u}$ $(I_3 = -1)$

$\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$ $(I_3 = 0)$

The masses of the pions are similar,

$M(\pi^\pm) = 140$ MeV $M(\pi^0) = 135$ MeV,

reflecting the similar masses of the u and d quark

Isospin in Two-nucleon System

- Consider the possible two nucleon systems
  - pp $I_3 = 1 \Rightarrow I = 1$
  - pn $I_3 = 0 \Rightarrow I = 0$ or 1
  - nn $I_3 = -1 \Rightarrow I = 1$

- This is completely analogous to the combination of two spin 1/2 states
  - p is $I_3 = 1/2$
  - n is $I_3 = -1/2$

Isospin in Two-nucleon System

- Combining these doublets yields a triplet plus a singlet

  $2 \otimes 2 = 3 \oplus 1$

- Total wavefunction for the two-nucleon state:

  $\psi(\text{total}) = \phi(\text{space})\ \alpha(\text{spin})\ \chi(\text{isospin})$

- Deuteron = pn

  spin = 1 $\Rightarrow$ α is symmetric

  L = 0 $\Rightarrow$ ϕ is symmetric $[(-1)^L]$

  Therefore, Fermi statistics required χ be antisymmetric

  I = 0, singlet state without related pp or nn states

Isospin in Two-nucleon System

- Consider an application of isospin conservation
  - (i) $p + p \rightarrow d + \pi^+$ (isospin of the final state is 1)
  - (ii) $p + n \rightarrow d + \pi^0$ (isospin of the final state is 1)

  - initial state:
    - (i) $I = 1$
    - (ii) $I = 0$ or 1 (CG coeff say 50%, 50%)

  - Therefore $\sigma(ii)/\sigma(i) = 1/2$
  - which is experimentally confirmed

Isospin in Pion-nucleon System

- Consider pion-nucleon scattering:
  - three reactions are of interest for the contributions of the I=1/2 and I=3/2 amplitudes

    (a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)

    (b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)

    (c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

  - $\sigma \propto \langle \psi_f | H | \psi_i \rangle^2 = M_{if}^2$

    $M_1 = \langle \psi_f (1/2) | H_1 | \psi_i (1/2) \rangle$

    $M_3 = \langle \psi_f (3/2) | H_3 | \psi_i (3/2) \rangle$

Clebsch-Gordon Coefficients

Notation:	J	J	$\ldots$
	M	M	$\ldots$
m_1	m_2	Coefficients	
m_1	m_2		
$\vdots$	$\vdots$		

$1 \times 1/2$

			3/2					
			+3/2	3/2	1/2			
+1	+1/2		1	+1/2	+1/2			
		+1	−1/2	1/3	2/3	3/2	1/2	
		0	+1/2	2/3	−1/3	−1/2	−1/2	
				0	−1/2	2/3	1/3	3/2
				−1	+1/2	1/3	−2/3	−3/2
						−1	−1/2	1

Isospin in Pion-nucleon System

- (a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)
  (b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)
  (c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

- (a) is purely I=3/2
  - $\sigma_a = K\,|M_3|^2$
- (b) is a mixture of I = 1/2 and 3/2

$$|\psi_i\rangle = |\psi_f\rangle = \sqrt{1/3}\,|\chi(3/2,-1/2)\rangle - \sqrt{2/3}\,|\chi(1/2,-1/2)\rangle$$

$$\sigma_b = K\,|\langle\psi_f|H_1+H_3|\psi_i\rangle|^2$$
$$= K\,|(1/3)M_3 + (2/3)M_1|^2$$

Isospin in Pion-nucleon System

- (a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)
  (b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)
  (c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

- (c) is a mixture of I = 1/2 and 3/2

$$|\psi_i\rangle = \sqrt{1/3}\,|\chi(3/2,-1/2)\rangle - \sqrt{2/3}\,|\chi(1/2,-1/2)\rangle$$

$$|\psi_f\rangle = \sqrt{2/3}\,|\chi(3/2,-1/2)\rangle + \sqrt{1/3}\,|\chi(1/2,-1/2)\rangle$$

$$\sigma_c = K\,|\langle\psi_f|H_1+H_3|\psi_i\rangle|^2$$

$$= K\,|\sqrt{(2/9)}M_3 - \sqrt{(2/9)}M_1|^2$$

Therefore:

$$\sigma_a : \sigma_b : \sigma_c = |M_3|^2 : (1/9)|M_3 + 2M_1|^2 : (2/9)|M_3 - M_1|^2$$

Isospin in Pion-nucleon System

$\sigma_a : \sigma_b : \sigma_c = |M_3|^2 : (1/9)|M_3 + 2M_1|^2 : (2/9)|M_3 - M_1|^2$

Limiting situations:

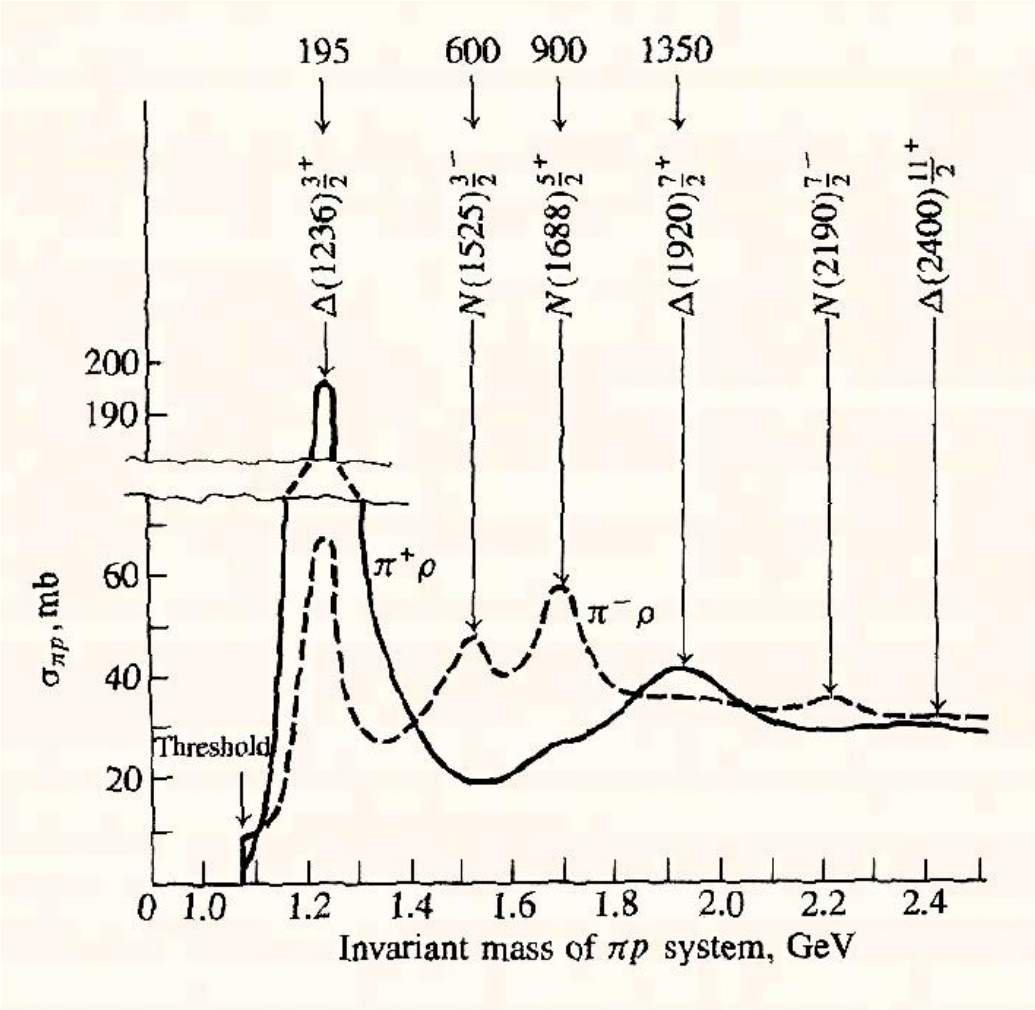
$M_3 >> M_1$

$\sigma_a : \sigma_b : \sigma_c = 9 : 1 : 2$

$M_1 >> M_3$

$\sigma_a : \sigma_b : \sigma_c = 0 : 2 : 1$

Isospin in Pion-nucleon System



Isospin Strangeness and Hypercharge

- $Q = [I_3 + (B+S)/2]\, e = (I_3 + Y/2)e$
  - B = baryon number
  - S = strangeness
  - Y = B+S = hypercharge
- Example, u and d quark

  u) $I_3 = 1/2$, $B = 1/3$, $S = 0$, $Q = [1/2 + 1/6]e = 2/3\, e$

  d) $I_3 = -1/2$, $B = 1/3$, $S = 0$, $Q = [-1/2 + 1/6]e = -1/3\, e$

Δ^+ Decays

- Baryon number conservation requires the Δ to decay to to p or n. (Higher mass baryons not accessible)
- Strong interaction favoured 10,000-fold over an EM decay.
- As the strong interaction dominates, use isospin to understand *relative* rates using:

$$|\Delta^+\rangle = |I = \tfrac{3}{2}, I_3 = \tfrac{1}{2}\rangle \quad |\mathrm{p}\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle \quad |\mathrm{n}\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$
$$|\pi^+\rangle = |1, 1\rangle \quad |\pi^0\rangle = |1, 0\rangle \quad |\pi^-\rangle = |1, -1\rangle$$

- Decompose $I = \frac{3}{2}$ state into possible combinations $I = \frac{1}{2}$ and $I = 1$

$$\Delta^+ = |\tfrac{3}{2}, \tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle|1, 1\rangle \;+\; \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle|1, 0\rangle$$
$$\tfrac{1}{\sqrt{3}}\;\; |\mathrm{n}\rangle \;\; |\pi^+\rangle \;+\; \sqrt{\tfrac{2}{3}}\;\; |\mathrm{p}\rangle \;\; |\pi^0\rangle$$

- And deducing branching ratios:

$$\frac{\mathcal{B}(\Delta^+ \to \pi^0\mathrm{p})}{\mathcal{B}(\Delta^+ \to \pi^+\mathrm{n})} = \frac{|\langle \pi^0\mathrm{p} \mid \Delta^+\rangle|^2}{|\langle \pi^+\mathrm{n} \mid \Delta^+\rangle|^2} = \frac{|\sqrt{2/3}|^2}{|\sqrt{1/3}|^2} = 2$$

From M. John, http://www-pnp.physics.ox.ac.uk/~mjohn/Lecture3.pdf

Mass Splittings

The mass splittings between members of isospin multiplets, as expected, are order of:

$$\frac{\Delta m}{m} \simeq \alpha \simeq 10^{-2}$$

	Δm (MeV/c^2)	m_{av} (MeV/c^2)	$10^3\ \Delta m/m$
$n - p$	1.3	939	1.4
$\Sigma^0 - \Sigma^+$	3.1	1190	2.6
$\Sigma^- - \Sigma^0$	4.9	1195	4.1
$\Xi^- - \Xi^0$	6.5	1318	4.9
$K^0 - K^\pm$	4.0	495	8.1
$\pi^\pm - \pi^0$	4.6	140	33

Note that particle and antiparticle must have identical mass (CTP Theorem). However Σ^+ and Σ^- have different masses, since they are both baryons, rather then baryon and antibaryon.

From L. Spogli, http://webusers.fis.uniroma3.it/~spogli/Isospin.pdf