Invariance Principles and Conservation Laws

- **Outline**
  - Translation and rotation
  - Parity
  - Charge Conjugation
  - Charge Conservation and Gauge Invariance
  - Baryon and lepton conservation
  - CPT Theorem
  - CP violation and T violation
  - Isospin symmetry
<table>
<thead>
<tr>
<th>Conserved quantity</th>
<th>strong</th>
<th>EM</th>
<th>weak</th>
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</thead>
<tbody>
<tr>
<td>energy-momentum</td>
<td></td>
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<tr>
<td>charge</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>baryon number</td>
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<td>lepton number</td>
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<td>CPT</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>P (parity)</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
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<tr>
<td>C (charge conjugation parity)</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
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<tr>
<td>CP (or T)</td>
<td>yes</td>
<td>yes</td>
<td>$10^{-3}$ violation</td>
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<td>I (isospin)</td>
<td>yes</td>
<td>no</td>
<td>no</td>
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</table>
Discrete and Continuous Symmetries

• Continuous Symmetries
  – Space-time symmetries
  – Lorentz transformations
  – Poincare transformations
    • combined space-time translation and Lorentz T.

• Discrete symmetries
  – cannot be built up from succession of infinitesimally small transformations

• Other types of symmetries
  – Dynamical symmetries - of the equations of motion
  – Internal symmetries - such as spin, charge, color, or isospin
Conservation Laws

- Continuous symmetries lead to additive conservation laws:
  - energy, momentum

- Discrete symmetries lead to multiplicative conservation laws
  - parity, charge conjugation
Translation and Rotation

• Invariance of the energy of an isolated physical system under space translations leads to conservation of linear momentum.

• Invariance of the energy of an isolated physical system under spatial rotations leads to conservation of angular momentum.

• Noether’s Theorem


Parity

- **Spatial inversion**
  - $(x,y,z) \rightarrow (-x,-y,-z)$
  - discrete symmetry
- $P \psi(r) = \psi(-r)$
  - $P$ is the parity operator
- $P^2 \psi(r) = P \psi(-r) = \psi(r)$,
  - therefore $P^2 = 1$ and the parity of an eigen-system is 1 or -1
• Example, the spherical harmonics:

\[ Y_{L}^{M} \]

\[ Y_{1}^{0} = \sqrt{\frac{3}{4\pi}} \cos \theta \]
\[ Y_{1}^{1} = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi} \]
\[ Y_{2}^{0} = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{2} \cos^{2} \theta - \frac{1}{2} \right) \]
\[ Y_{2}^{1} = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi} \]
\[ Y_{2}^{2} = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^{2} \theta e^{2i\phi} \]

• The spherical harmonics describe a state in a spherically symmetric potential with definite angular momentum

• \( p = (-1)^{L} \)
Parity

- Parity is a multiplicative quantum number

- Composite system:
  - parity is equal to product of the parts
  - eg. two pions in an angular momentum L state
    \[ P = P(\pi_1) \cdot P(\pi_2) \cdot (-1)^L \]

- Intrinsic parity of particles:
  - proton and neutron (+1 by definition, could be -1)
    - parity of fermion and anti-fermion are opposite, and assignment of one is arbitrary
  - pion (-1, measured)
Pion spin and parity

• 1. Spin of the charged pion
  - \( p + p \leftrightarrow \pi^+ + d \) \( (|M_{if}|^2 = |M_{fi}|^2) \)
  - then, “detailed balance” gives:
    \[ \sigma (pp \rightarrow \pi^+d) \sim (2s_\pi + 1)(2s_d+1)p_\pi^2 \]
    \[ \sigma (\pi^+d \rightarrow pp) \sim 1/2 (2s_p+1)^2 p_p^2 \]
    (1/2 because protons are identical)
  - Measurements of \( \sigma \)'s reveals \( s_\pi = 0 \)

• 2. Spin of the neutral pion
  - \( \pi^0 \rightarrow \gamma\gamma \) shows it must be 0 (see following)
Pion spin and parity

• 2. Spin of the neutral pion
  - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0
  - along the flight path of the $\gamma$s in the $\pi^0$ rest frame, the total photon spin ($S_z$) must be 0 or 2
  - If $S_{\pi} = 1$, then $S_z$ must be 0
  - If $S_{\pi} = 1$, $S_z = 0$, the two-photon amplitude must behave as $P_{l}^{m=0}(\cos \theta)$, which is antisymmetric under interchange ($=180^0$ rotation)
  - This corresponds to two right or left circularly polarized photons travelling in opposite directions, which must be symmetric by Bose statistics -> forbidden $S_{\pi} \neq 1$
  - Therefore, $S_{\pi} = 0$ or $S_{\pi} \geq 2$ (which is ruled out by pion production statistics - all three charges produced equally)
Parity

- Parity of the charged pion
  - the observation of the reaction
    \[ \pi^- + d \rightarrow n + n \]
    is evidence that the charged pion has \( p = -1 \)

- Capture takes place from an s-state, \( L_i=0 \) (\( S_d=1 \))
  - (X-ray emissions following capture)

- \( J=1 \), since \( S_d=1 \) and \( S_\pi=0 \)

- In the two neutron system, \( L+S \) must be even by the antisymmetric requirement on identical fermions
  - thus, \( (2S+1)J_L = 3P_1 \) state with \( p = (-1)^L = -1 \)
  - since initial state is \( p_d p_\pi (-1)^{L_i} = (+1) p_\pi \), \( p_\pi = -1 \)
Parity

• Parity of the neutral pion
  \[ \pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-) \] (rare decay of the pion)
  - the planes of the pairs follow the E vectors of
    the internally converted photons (\( \pi^0 \rightarrow \gamma\gamma \))
  - even system of two photons (bosons)
    \[ A \sim (E_1 \cdot E_2) \] (P=+1)
    or \[ A \sim (E_1 \times E_2) \cdot k \] (P=-1)
  - Intensities or rates \( \sim |A|^2 \)
    \[ |A|^2 \sim \cos^2 \phi \]
    or \[ |A|^2 \sim \sin^2 \phi \]
Parity

• Parity of the neutral pion
  \[ \pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-) \]
  scalar\((0^+)\) would follow dashed line, while pseudoscalar \((0^-)\) would follow solid line
Parity of particles and antiparticles

• Dirac theory required fermions and antifermions to have opposite intrinsic parity.

• This was checked in decays of the spin singlet ground state of positronium

\[ e^+ e^- \left((2S+1)J_L = 1S_0\right) \rightarrow 2 \gamma \]

\[ P = (+1) \cdot (-1) \cdot (-1)^0 = -1 \]

• This is exactly the case of the \( \pi^0 \) decay
  - the photon polarizations must have a \( \sin^2\phi \) form.
Parity of particles and antiparticles

\[
\frac{\text{Rate}(90^0)}{\text{Rate}(0^0)} = 2.04 \pm 0.08
\]

expect 2.00

conclude \( P(e^+) = -P(e^-) \)
Parity of particles and antiparticles

- Two-particle systems
  - Fermion-antifermion
    - \( p = (-1)^{L+1} \)
    - eg, pion which is q-antiq with \( L=0 \) has \( p = -1 \)
  - Boson-antiboson
    - same parity
    - \( p = (-)^{L} \)
    - eg. \( \rho \to \pi^+ \pi^- \)
    - \( 1^- \to 0^- 0^- \), \( L=1 \)
Tests of parity conservation

- Strong and EM interactions conserve parity, but weak interactions do not

- LH neutrino observed, but RH neutrino is not
  - this is a maximally violated symmetry

\[ \sigma_z \quad p \]

LH neutrino (a)  \quad RH neutrino (b)
Tests of parity conservation

• In interactions dominated by the strong or the EM interaction, some parity violation may be observed due to the small contribution of the weak int. to the process

\[ H = H_{\text{strong}} + H_{\text{EM}} + H_{\text{weak}} \]

• In nuclear transitions, for example, the degree of parity violation will be of the order of the ratio of the weak to the strong couplings (\(\sim 10^{-7}\))
Tests of parity conservation

• Examples of nuclear transitions
  - fore-aft asymmetry in gamma emission
    $$^{19}\text{F}^* \rightarrow ^{19}\text{F} + \gamma \ (110 \text{ keV})$$
    $$\mathcal{J}^P: \ 1/2^- \rightarrow 1/2^+$$

    $$\Delta \sim 10^{-4}$$

  - very narrow decay
    $$^{16}\text{O}^* \rightarrow ^{12}\text{C} + \alpha$$
    $$\mathcal{J}^P: \ 2^- \rightarrow 2^+$$

    $$\Gamma = 10^{-10} \text{ eV} \quad \text{(note} \ ^{16}\text{O}^* \rightarrow ^{16}\text{O} + \gamma, \ \Gamma = 3 \times 10^{-3} \text{ eV})$$
Charge Conjugation Invariance

• Charge conjugation reverses sign of charge and magnetic moment, leaving other coord unchanged

• Good symmetry in strong and EM interactions:
  - eg. mesons in $p + \bar{p} \rightarrow \pi^+ + \pi^- + \ldots$.

• Only neutral bosons are eigenstates of $C$

• The charge conjugation eigenvalue of the $\gamma$ is -1:
  - EM fields change sign when charge source reverses sign

• Since $\pi^0 \rightarrow \gamma \gamma$, $C|\pi^0> = +1$

• Consequence of $C$ of $\pi^0$ and $\gamma$, and $C$-cons. in EM int,
  $\pi^0 \rightarrow \gamma \gamma$ forbidden
  - experiment: $\pi^0 \rightarrow \gamma \gamma \gamma / \pi^0 \rightarrow \gamma \gamma < 3 \times 10^{-8}$
Charge Conjugation Invariance

**Charge Conjugation (C) Invariance**

\[
\begin{align*}
\Gamma(\pi^0 \rightarrow 3\gamma)/\Gamma_{\text{total}} &< 3.1 \times 10^{-8}, \text{CL = 90}\% \\
\Gamma(\eta \rightarrow 3\gamma)/\Gamma_{\text{total}} &< 9 \times 10^{-5}, \text{CL = 90}\% \\
\Gamma(\eta \rightarrow 2\pi^0\gamma)/\Gamma_{\text{total}} &< 5 \times 10^{-4}, \text{CL = 90}\% \\
\Gamma(\eta \rightarrow \pi^0\gamma)/\Gamma_{\text{total}} &< 1.6 \times 10^{-5}, \text{CL = 90}\% \\
\Gamma(\eta \rightarrow \pi^0\mu^+\mu^-)/\Gamma_{\text{total}} &< 4 \times 10^{-5}, \text{CL = 90}\% \\
\Gamma(\eta \rightarrow \pi^0\mu^-\mu^+)/\Gamma_{\text{total}} &< 5 \times 10^{-6}, \text{CL = 90}\% \\
\Gamma(\omega(782) \rightarrow \eta\pi^0)/\Gamma_{\text{total}} &< 2.1 \times 10^{-4}, \text{CL = 90}\% \\
\Gamma(\omega(782) \rightarrow 2\pi^0)/\Gamma_{\text{total}} &< 2.1 \times 10^{-4}, \text{CL = 90}\% \\
\Gamma(\omega(782) \rightarrow 3\pi^0)/\Gamma_{\text{total}} &< 2.3 \times 10^{-4}, \text{CL = 90}\% \\
\text{asymmetry parameter for } \eta'(958) \rightarrow \pi^+\pi^-\gamma \text{ decay} &< 0.03 \pm 0.04 \\
\Gamma(\eta'(958) \rightarrow \pi^0 e^+ e^-)/\Gamma_{\text{total}} &< 1.4 \times 10^{-3}, \text{CL = 90}\% \\
\Gamma(\eta'(958) \rightarrow \eta e^+ e^-)/\Gamma_{\text{total}} &< 2.4 \times 10^{-3}, \text{CL = 90}\% \\
\Gamma(\eta'(958) \rightarrow 3\gamma)/\Gamma_{\text{total}} &< 1.0 \times 10^{-4}, \text{CL = 90}\% \\
\Gamma(\eta'(958) \rightarrow e^+ e^- \pi^0)/\Gamma_{\text{total}} &< 6.0 \times 10^{-5}, \text{CL = 90}\%
\end{align*}
\]
Charge Conjugation Invariance

- Weak interaction
  - respects neither $C$ nor $P$, but $CP$ is an approximate symmetry of the weak interaction
**Charge Conservation Invariance**

**ELECTRIC CHARGE (Q)**

\[ e \rightarrow \nu_e \gamma \text{ and astrophysical limits} \]
\[ \Gamma(n \rightarrow p\nu_e \bar{\nu}_e)/\Gamma_{\text{total}} \]

\[ [u] \quad > 4.6 \times 10^{26} \text{ yr, CL = 90\%} \]
\[ < 8 \times 10^{-27}, \text{ CL = 68\%} \]

\[ [u] \text{ This is the best limit for the mode } e^- \rightarrow \nu\gamma. \text{ The best limit for "electron disappearance" is } 6.4 \times 10^{24} \text{ yr.} \]
Charge Conservation and Gauge Invariance

• It is possible to described charged particles by wavefunctions with phases chosen arbitrarily at different times and places, provided:
  - the charges couple to a long-range field (the EM field) to which the same local gauge transformation is applied
  - charge is conserved
Charge Conservation and Gauge Invariance

- The pattern at C results from the interference of the electrons coming through slits A and B. If we allow an arbitrary phase for each slit, the pattern at C is altered.
Charge Conservation and Gauge Invariance

• After adding an arbitrary phase ($\theta$), the wave function of the electron will be
  \[ \psi = e^{i(p \cdot x - Et + e\theta)} = e^{i(px + e\theta)} \]
• Now the gradient of the phase is
  \[ \frac{\partial}{\partial x} i(px + e\theta) = i(p + e \frac{\partial \theta}{\partial x}) \]
• If the phase depends on location (it is local), the electron interaction with the EM field will also alter the pattern.
  \[ \psi = e^{i(px - eAx)} \quad A = (A, i\phi) \]
• Then, the space-time gradient of the phase becomes
  \[ \frac{\partial}{\partial x} i(px - eAx + e\theta) = i(p - eA + e \frac{\partial \theta}{\partial x}) \]
• If \( A \rightarrow A + \frac{\partial \theta}{\partial x} \) the pattern is unaltered
• This is an example of a local gauge transformation.
Charge Conservation and Gauge Invariance

• It is possible to describe charged particles by wavefunctions with phases chosen arbitrarily at different times and places (local gauge transformation), provided:
  - the charges couple to a long-range field (the EM field) to which the same local gauge transformation is applied
  - charge is conserved

• This property of local gauge symmetry is a critical ingredient in a field theory that will be renormalizable, such as QED, or the EW theory, leading to cross-sections and decay rates that are finite and calculable to all orders in the coupling constant.
Baryon Conservation

• Baryon number is conserved to a very high level:
  - If it didn’t, what would prevent \( p \rightarrow e^+ \pi^0 \) ?
• Does an underlying long range field lead to this?
  - Equivalency of gravitational and inertial mass established at the \( 10^{-12} \) level
  - Since the nuclear binding energy differs with nuclei, the ratio of mass to number of nucleons is not constant, providing a potential source for a difference between gravitational and inertial mass
Baryon Conservation

- Baryon conservation is established to a very low level of violation:
  - \( \tau(p \rightarrow e^+\pi^0) > 8 \times 10^{33} \text{ yr} \)

- This is a remarkably precise symmetry, yet we know of no long-range field that could cause it

- We have reason to suspect it is violated at some level:
  - baryon dominance in the Universe today
Baryon Conservation

BARYON NUMBER

\[ \frac{\Gamma(Z \to p e)}{\Gamma_{\text{total}}} < 1.8 \times 10^{-6}, \text{CL} = 95\% \]
\[ \frac{\Gamma(Z \to p \mu)}{\Gamma_{\text{total}}} < 1.8 \times 10^{-6}, \text{CL} = 95\% \]
\[ \frac{\Gamma(\tau^- \to \bar{p} \gamma)}{\Gamma_{\text{total}}} < 3.5 \times 10^{-6}, \text{CL} = 90\% \]
\[ \frac{\Gamma(\tau^- \to \bar{p} \pi^0)}{\Gamma_{\text{total}}} < 1.5 \times 10^{-5}, \text{CL} = 90\% \]
\[ \frac{\Gamma(\tau^- \to \bar{p} 2\pi^0)}{\Gamma_{\text{total}}} < 3.3 \times 10^{-5}, \text{CL} = 90\% \]
\[ \frac{\Gamma(\tau^- \to \bar{p} \eta)}{\Gamma_{\text{total}}} < 8.9 \times 10^{-6}, \text{CL} = 90\% \]
\[ \frac{\Gamma(\tau^- \to \bar{p} \pi^0 \eta)}{\Gamma_{\text{total}}} < 2.7 \times 10^{-5}, \text{CL} = 90\% \]
\[ \frac{\Gamma(\tau^- \to \Lambda \pi^-)}{\Gamma_{\text{total}}} < 7.2 \times 10^{-8}, \text{CL} = 90\% \]
\[ \frac{\Gamma(\tau^- \to \Lambda \pi^-)}{\Gamma_{\text{total}}} < 1.4 \times 10^{-7}, \text{CL} = 90\% \]
\[ \frac{\Gamma(D^0 \to p e^-)}{\Gamma_{\text{total}}} < 1.0 \times 10^{-5}, \text{CL} = 90\% \]
\[ \frac{\Gamma(D^0 \to \bar{p} e^+)}{\Gamma_{\text{total}}} < 1.1 \times 10^{-5}, \text{CL} = 90\% \]
\[ \rho \text{ mean life} > 2.1 \times 10^{29} \text{ years, CL} = 90\% \]

\( \tau(N \to e^+ \pi) \)
\[ > 158 \,(n), \ > 8200 \,(p) \times 10^{30} \text{ years, CL} = 90\% \]
\( \tau(N \to \mu^+ \pi) \)
\[ > 100 \,(n), \ > 6600 \,(p) \times 10^{30} \text{ years, CL} = 90\% \]
\( \tau(N \to e^+ K) \)
\[ > 17 \,(n), \ > 150 \,(p) \times 10^{30} \text{ years, CL} = 90\% \]
\( \tau(N \to \mu^+ K) \)
\[ > 26 \,(n), \ > 120 \,(p) \times 10^{30} \text{ years, CL} = 90\% \]

limit on \( n \bar{n} \) oscillations (free \( n \))
\[ > 0.86 \times 10^8 \text{ s, CL} = 90\% \]

A few examples of proton or bound neutron decay follow. For limits on many other nucleon decay channels, see the Baryon Summary Table.
Lepton Conservation

• Direct searches have not been fruitful:
  - eg. $\tau(^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 0\nu + e^- + e^-) > 10^{26}$ yr
    “neutrinoless double beta decay”

• Yet we do observe neutrino oscillations:
  - (eg. $\nu_e \rightarrow \nu_\mu$)
  - these are small transitions
Lepton Conservation

\[ \Gamma(Z \rightarrow e^{\pm} \nu^{\mp})/\Gamma_{\text{total}} \]
\[ \Gamma(Z \rightarrow e^{\pm} \tau^{\mp})/\Gamma_{\text{total}} \]
\[ \Gamma(Z \rightarrow \mu^{\pm} \tau^{\mp})/\Gamma_{\text{total}} \]
\[ \sigma(e^+ e^- \rightarrow e^{\pm} \tau^{\mp}) / \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) \]
\[ \sigma(e^+ e^- \rightarrow \mu^{\pm} \tau^{\mp}) / \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) \]

limit on \( \mu^- \rightarrow e^- \) conversion
\[ \sigma(\mu^- 32S \rightarrow e^- 32S) / \sigma(\mu^- 32S \rightarrow \nu_{\mu} 32P^*) \]
\[ \sigma(\mu^- Ti \rightarrow e^- Ti) / \sigma(\mu^- Ti \rightarrow \text{capture}) \]
\[ \sigma(\mu^- Pb \rightarrow e^- Pb) / \sigma(\mu^- Pb \rightarrow \text{capture}) \]

limit on muonium \( \rightarrow \) antimuonium conversion \( R_g = \frac{G_C}{G_F} \)
\[ \Gamma(\mu^- \rightarrow e^- \nu e \bar{\nu}_{\mu})/\Gamma_{\text{total}} \]
\[ \Gamma(\mu^- \rightarrow e^- \gamma)/\Gamma_{\text{total}} \]
\[ \Gamma(\mu^- \rightarrow e^- e^+ e^-)/\Gamma_{\text{total}} \]
\[ \Gamma(\mu^- \rightarrow e^- 2\gamma)/\Gamma_{\text{total}} \]
\[ \Gamma(\tau^- \rightarrow e^- \gamma)/\Gamma_{\text{total}} \]
\[ \Gamma(\tau^- \rightarrow \mu^- \gamma)/\Gamma_{\text{total}} \]

\[ m \]

\[ \ll 1.7 \times 10^{-6}, \text{ CL = 95\%} \]
\[ \ll 9.8 \times 10^{-6}, \text{ CL = 95\%} \]
\[ \ll 1.2 \times 10^{-5}, \text{ CL = 95\%} \]
\[ <8.9 \times 10^{-6}, \text{ CL = 95\%} \]
\[ <4.0 \times 10^{-6}, \text{ CL = 95\%} \]
\[ <7 \times 10^{-11}, \text{ CL = 90\%} \]
\[ <4.3 \times 10^{-12}, \text{ CL = 90\%} \]
\[ <4.6 \times 10^{-11}, \text{ CL = 90\%} \]
\[ <0.0030, \text{ CL = 90\%} \]
\[ <1.2 \times 10^{-2}, \text{ CL = 90\%} \]
\[ <1.2 \times 10^{-11}, \text{ CL = 90\%} \]
\[ <1.0 \times 10^{-12}, \text{ CL = 90\%} \]
\[ <7.2 \times 10^{-11}, \text{ CL = 90\%} \]
\[ <3.3 \times 10^{-8}, \text{ CL = 90\%} \]
\[ <4.4 \times 10^{-8}, \text{ CL = 90\%} \]
CPT Invariance

- Very general assumptions in quantum field theory:
  - spin-statistics relation: integer and half-integer spin particles obey Bose and Fermi statistics
  - Lorentz invariance

lead to the CPT Theorem:
  - all interactions are invariant under the successive operation of C (=charge conjugation), P (=parity operation), and T (= time reversal)

- Masses, lifetimes, moments, etc. of particles and antiparticles must be identical
CPT Invariance

- Tests of the CPT Theorem

\[
\begin{align*}
\left( m_{W^+} - m_{W^-} \right) / m_{\text{average}} &= -0.002 \pm 0.007 \\
\left( m_{e^+} - m_{e^-} \right) / m_{\text{average}} &= <8 \times 10^{-9}, \text{ CL } = 90\% \\
|q_{e^+} + q_{e^-}|/e &= <4 \times 10^{-8} \\
\left( g_{e^+} - g_{e^-} \right) / g_{\text{average}} &= (-0.5+2.1) \times 10^{-12} \\
\left( \tau_{\mu^+} - \tau_{\mu^-} \right) / \tau_{\text{average}} &= (2 \pm 8) \times 10^{-5} \\
\left( g_{\mu^+} - g_{\mu^-} \right) / g_{\text{average}} &= (-0.11 \pm 0.12) \times 10^{-8} \\
\left( m_{\tau^+} - m_{\tau^-} \right)/m_{\text{average}} &= <2.8 \times 10^{-4}, \text{ CL } = 90\% \\
2 \left( m_t - m_{\bar{t}} \right) / (m_t + m_{\bar{t}}) &= 0.022 \pm 0.022 \\
\left( m_{\pi^+} - m_{\pi^-} \right) / m_{\text{average}} &= (2+5) \times 10^{-4} \\
\left( \tau_{\pi^+} - \tau_{\pi^-} \right) / \tau_{\text{average}} &= (6+7) \times 10^{-4} \\
\left( m_{K^+} - m_{K^-} \right) / m_{\text{average}} &= (-0.6 \pm 1.8) \times 10^{-4} \\
\left( \tau_{K^+} - \tau_{K^-} \right) / \tau_{\text{average}} &= (0.10 \pm 0.09)\% (S = 1.2) \\
K^{\pm} \rightarrow \mu^{\pm} \nu_{\mu} \text{ rate difference/average} &= (-0.5 \pm 0.4)\% \\
\nu_{\pm} \rightarrow \nu_{\pm} \pm 0 \text{ rate difference/average} &= (0.8 \pm 1.2)\% 
\end{align*}
\]
CPT Invariance (cont.)

\[
\begin{align*}
\text{phase difference } & \phi_{00} - \phi_{+-} \\
\text{Re}\left( \frac{2}{3} \eta_{+-} + \frac{1}{3} \eta_{00} \right) & - \frac{A_L}{2} \\
A_{CPT}(D^0 \to K^- \pi^+) & \\
|m_p - m_{\bar{p}}|/m_p & \\
(|q_{\bar{p}}| - |q_p|)/q_p & \\
|q_p + q_{\bar{p}}|/e & \\
(\mu_p + \mu_{\bar{p}})/\mu_p & \\
(m_n - m_{\bar{n}})/m_n & \\
(m_{\Lambda} - m_{\bar{\Lambda}})/m_{\Lambda} & \\
(\tau_{\Lambda} - \tau_{\bar{\Lambda}})/\tau_{\Lambda} & \\
(\tau \Sigma^+ - \tau \Sigma^-)/\tau \Sigma^+ & \\
(\mu \Sigma^+ + \mu \Sigma^-)/\mu \Sigma^+ & \\
(m_{\Xi^-} - m_{\Xi^+})/m_{\Xi^-} & \\
(\tau_{\Xi^-} - \tau_{\Xi^+})/\tau_{\Xi^-} & \\
(\mu_{\Xi^-} + \mu_{\Xi^+})/|\mu_{\Xi^-}| & \\
(m_{\Omega^-} - m_{\Omega^+})/m_{\Omega^-} & \\
(\tau_{\Omega^-} - \tau_{\Omega^+})/\tau_{\Omega^-} & \\
\end{align*}
\]

(0.2 ± 0.4)°

(−3 ± 35) × 10^{-6}

0.008 ± 0.008

[k] <2 × 10^{-9}, CL = 90%

(−9 ± 9) × 10^{-11}

[k] <2 × 10^{-9}, CL = 90%

(−0.1 ± 2.1) × 10^{-3}

(9 ± 6) × 10^{-5}

(−0.1±1.1) × 10^{-5} (S = 1.6)

−0.001 ± 0.009

(−0.6 ± 1.2) × 10^{-3}

0.014 ± 0.015

(−3 ± 9) × 10^{-5}

−0.01 ± 0.07

+0.01 ± 0.05

(−1 ± 8) × 10^{-5}

0.00 ± 0.05
Until 1964 it was believed that CP symmetry was respected in the weak interaction
- P and C were known to be separately violated
A small team led by Cronin and Fitch studying the long-lived neutral neutral kaon discovered that it violated CP with a rate of $2 \times 10^{-3}$

$$K_L^0 = \sqrt{\frac{1}{2}} (|K^0> - \bar{|K^0>})$$

- $K_L^0 \rightarrow \pi \pi \pi$ ($> 99 \%$)
- $CP = -1$ → $CP = -1$
- $K_L^0 \rightarrow \pi \pi$ (0. 2 %)
  $CP = -1$ → $CP = +1$
CP Violation

- The Standard Model of CP violation explains its origin from the CP-violating phase in the mixing matrix of the six quark flavors.

- Experiments at PEPIII (BaBar) at SLAC and KEKB (Belle) at KEK have confirmed the Standard Model explanation with measurements of CP violation in B decays.
T Violation

- The **CPT Theorem** dictates that if CP is violated, T must also be violated so that CPT can be invariant

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<th>Effect of T</th>
<th>Effect of P</th>
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<tbody>
<tr>
<td>position</td>
<td>r</td>
<td>r</td>
</tr>
<tr>
<td>momentum</td>
<td>p</td>
<td>-p</td>
</tr>
<tr>
<td>spin</td>
<td>(\sigma)</td>
<td>-(\sigma)</td>
</tr>
<tr>
<td></td>
<td>= ((r \times p))</td>
<td></td>
</tr>
<tr>
<td>electric field</td>
<td>E ((= -\nabla V))</td>
<td>E</td>
</tr>
<tr>
<td>magnetic field</td>
<td>B</td>
<td>-B</td>
</tr>
<tr>
<td>mag. dip. mom.</td>
<td>(\sigma \cdot B)</td>
<td>(\sigma \cdot B)</td>
</tr>
<tr>
<td>el. dipl. mom.</td>
<td>(\sigma \cdot E)</td>
<td>-(\sigma \cdot E)</td>
</tr>
<tr>
<td>long. pol.</td>
<td>(\sigma \cdot p)</td>
<td>(\sigma \cdot p)</td>
</tr>
<tr>
<td>trans. pol.</td>
<td>(\sigma \cdot (p_1 \times p_2))</td>
<td>-(\sigma \cdot (p_1 \times p_2))</td>
</tr>
</tbody>
</table>
T Violation

- Transverse polarization

<table>
<thead>
<tr>
<th>Effect of T</th>
<th>Effect of P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p_1 \times p_2)$</td>
<td>$-\sigma(p_1 \times p_2)$</td>
</tr>
</tbody>
</table>

- trans. pol.

- $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$

- $n \rightarrow p + e^- + \overline{\nu}_e$

- T-violating contributions below $10^{-3}$
T Violation

- Detailed Balance

\[ p + ^{27}\text{Al} \leftrightarrow \alpha + ^{24}\text{Mg} \]

Curve and points

$T$ violation $< 5 \times 10^{-4}$
Neutron Electric Dipole Moment

- Most sensitive tests of T violation come from searches for the neutron and electron dipole moments

  - electron \( 0.07 \pm 0.07 \times 10^{-26} \text{ e-cm} \)
  - neutron \( < 0.29 \times 10^{-25} \text{ e-cm} \)

- Note, the existence of these electric dipole moments would also violate parity
Neutron Electric Dipole Moment

• How big might we expect the EDM to be?
  - \( \text{EDM} = (\text{charge} \times \text{length}) \times \text{T-violating parameter} \)
    length \( \sim 1/\text{Mass} \)
    assume weak interaction \( \sim \frac{g^2}{M_W^2} \)
    \( \sim \frac{e^2 M_N}{M_W^2} \sim \frac{4\pi}{137} \frac{(0.94 \text{ GeV})}{(80\text{GeV})^2} \)
    \( \sim 10^{-5} \text{ GeV}^{-1} \) [200 MeV-fm]
    \( \sim 10^{-6} \text{ fm} \sim 10^{-19} \text{ cm} \)
    T-violating parameter \( \sim 10^{-3} \) (like \( K^0 \))

Then:
\[ \text{EDM} \sim e \left(10^{-19} \text{ cm}\right)\left(10^{-3}\right) \]
\[ \sim 10^{-22} \text{ e-cm} \]

Experiment: Neutron EDM \( < 0.29 \times 10^{-25} \text{ e-cm} \)
Neutron Electric Dipole Moment
Neutron Electric Dipole Moment

- Neutron EDM $< 0.63 \times 10^{-25}$ e-cm
T Violation

TIME REVERSAL ($T$) INVARIANCE

$e \text{ electric dipole moment}$
$\mu \text{ electric dipole moment}$
$\mu \text{ decay parameters}$
$\text{transverse } e^+ \text{ polarization normal to plane of } \mu$
$\text{spin, } e^+ \text{ momentum}$

$\rho \text{ electric dipole moment}$
$n \text{ electric dipole moment}$

$(0.07 \pm 0.07) \times 10^{-26} \text{ e cm}$
$(-0.1 \pm 0.9) \times 10^{-19} \text{ e cm}$

$(-2 \pm 8) \times 10^{-3}$

$<0.54 \times 10^{-23} \text{ e cm}$
$<0.29 \times 10^{-25} \text{ e cm, CL = 90\%}$
Isospin Symmetry

- The nuclear force of the neutron is nearly the same as the nuclear force of the proton
- 1932 - Heisenberg - think of the proton and the neutron as two charge states of the nucleon
- In analogy to spin, the nucleon has isospin $1/2$
  \[ I_3 = \frac{1}{2} \quad \text{proton} \]
  \[ I_3 = -\frac{1}{2} \quad \text{neutron} \]

  \[ Q = (I_3 + 1/2) e \]

- Isospin turns out to be a conserved quantity of the strong interaction
The notion that the neutron and the proton might be two different states of the same particle (the nucleon) came from the near equality of the n-p, n-n, and p-p nuclear forces (once Coulomb effects were subtracted).

Within the quark model, we can think of this symmetry as being a symmetry between the u and d quarks:

\[
\begin{align*}
p &= d \ u \ u \quad & I_3 (u) &= \frac{1}{2} \\
n &= d \ d \ u \quad & I_3 (d) &= -1/2
\end{align*}
\]
**Isospin Symmetry**

- **Example from the meson sector:**
  - the pion (an isospin triplet, \( I=1 \))

\[
\begin{align*}
\pi^+ & = u\bar{d} \quad (I_3 = +1) \\
\pi^- & = d\bar{u} \quad (I_3 = -1) \\
\pi^0 & = (d\bar{d} - u\bar{u})/\sqrt{2} \quad (I_3 = 0)
\end{align*}
\]

The masses of the pions are similar,
\[
M (\pi^\pm) = 140 \text{ MeV} \quad M (\pi^0) = 135 \text{ MeV},
\]
reflecting the similar masses of the \( u \) and \( d \) quark.
Isospin in Two-nucleon System

- Consider the possible two nucleon systems
  - pp $I_3 = 1 \Rightarrow I = 1$
  - pn $I_3 = 0 \Rightarrow I = 0$ or 1
  - nn $I_3 = -1 \Rightarrow I = 1$

- This is completely analogous to the combination of two spin 1/2 states
  - p is $I_3 = 1/2$
  - n is $I_3 = -1/2$
• Combining these doublets yields a triplet plus a singlet
\[ 2 \otimes 2 = 3 \oplus 1 \]

• Total wavefunction for the two-nucleon state:
\[ \psi(\text{total}) = \phi(\text{space}) \alpha(\text{spin}) \chi(\text{isospin}) \]

• Deuteron = pn
  \[ \text{spin} = 1 \quad \Rightarrow \quad \alpha \text{ is symmetric} \]
  \[ L = 0 \quad \Rightarrow \quad \phi \text{ is symmetric} \quad [(-1)^L] \]
   Therefore, Fermi statistics required \( \chi \) be antisymmetric
   \[ I = 0, \text{ singlet state without related pp or nn states} \]
Isospin in Two-nucleon System

Consider an application of isospin conservation

- (i) \( p + p \rightarrow d + \pi^+ \) (isospin of the final state is 1)
- (ii) \( p + n \rightarrow d + \pi^0 \) (isospin of the final state is 1)

initial state:

- (i) \( I = 1 \)
- (ii) \( I = 0 \) or \( 1 \) (CG coeff say 50%, 50%)

Therefore \( \sigma(\text{ii})/\sigma(\text{i}) = 1/2 \)

which is experimentally confirmed
Isospin in Pion-nucleon System

- Consider pion-nucleon scattering:
  - three reactions are of interest for the contributions of the I=1/2 and I=3/2 amplitudes
    (a) \( \pi^+ p \rightarrow \pi^+ p \) (elastic scattering)
    (b) \( \pi^- p \rightarrow \pi^- p \) (elastic scattering)
    (c) \( \pi^- p \rightarrow \pi^0 n \) (charge exchange)

- \( \sigma \propto \langle \psi_f | H | \psi_i \rangle^2 = M_{if}^2 \)

\[ M_1 = \langle \psi_f (1/2) | H_1 | \psi_i (1/2) \rangle \]
\[ M_3 = \langle \psi_f (3/2) | H_3 | \psi_i (3/2) \rangle \]
Clebsch-Gordon Coefficients

\[
\begin{array}{cccc}
1 \times 1/2 & 3/2 & 3/2 & 3/2 \\
+1 & +1/2 & 1 & +1/2 \\
+1 & -1/2 & 1/3 & 2/3 \\
0 & +1/2 & 2/3 & -1/3 \\
-1 & +1/2 & 0 & -1/2 \\
-1 & -1/2 & -1 & -1/2 \\
\end{array}
\]
Isospin in Pion-nucleon System

- (a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)
  - (b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)
  - (c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

- (a) is purely $I=3/2$
  - $\sigma_a = K |M_3|^2$

- (b) is a mixture of $I = 1/2$ and $3/2$
  - $|\psi_i\rangle = |\psi_f\rangle = \sqrt{1/3} \chi(3/2,-1/2) - \sqrt{2/3} \chi(1/2,-1/2)$
  - $\sigma_b = K |\langle\psi_f|H_1+H_3|\psi_i\rangle|^2$
    - $= K |(1/3)M_3 + (2/3)M_1|^2$
Isospin in Pion-nucleon System

- (a) $\pi^+ p \rightarrow \pi^+ p$ (elastic scattering)
  (b) $\pi^- p \rightarrow \pi^- p$ (elastic scattering)
  (c) $\pi^- p \rightarrow \pi^0 n$ (charge exchange)

- (c) is a mixture of $I = 1/2$ and $3/2$

\[
|\psi_i\rangle = \sqrt{\frac{1}{3}} |\chi(3/2, -1/2)\rangle - \sqrt{\frac{2}{3}} |\chi(1/2, -1/2)\rangle
\]
\[
|\psi_f\rangle = \sqrt{\frac{2}{3}} |\chi(3/2, -1/2)\rangle + \sqrt{\frac{1}{3}} |\chi(1/2, -1/2)\rangle
\]

\[
\sigma_c = K |\langle \psi_f | H_1 + H_3 | \psi_i \rangle|^2
\]
\[
= K |\sqrt{(2/9)}M_3 - \sqrt{(2/9)}M_1|^2
\]

Therefore:

\[
\sigma_a : \sigma_b : \sigma_c = |M_3|^2 : (1/9)|M_3 + 2M_1|^2 : (2/9)|M_3 - M_1|^2
\]
Isospin in Pion-nucleon System

\[ \sigma_a : \sigma_b : \sigma_c = \frac{1}{9} |M_3 + 2M_1|^2 : \frac{2}{9} |M_3 - M_1|^2 \]

Limiting situations:

- \(M_3 \gg M_1\)
  \[\sigma_a : \sigma_b : \sigma_c = 9 : 1 : 2\]

- \(M_1 \gg M_3\)
  \[\sigma_a : \sigma_b : \sigma_c = 0 : 2 : 1\]
Isospin in Pion-nucleon System
Isospin Strangeness and Hypercharge

- \( Q = [I_3 + (B+S)/2]e = (I_3 + Y/2)e \)
  - \( B = \) baryon number
  - \( S = \) strangeness
  - \( Y = B+S = \) hypercharge

- Example, \( u \) and \( d \) quark
  - \( u \)  \( I_3 = 1/2, B = 1/3, S = 0, \)  \( Q = [1/2 + 1/6]e = 2/3 \ e \)
  - \( d \)  \( I_3 = -1/2, B = 1/3, S = 0, \)  \( Q = [-1/2 + 1/6]e = -1/3 \ e \)
\[\Delta^+ \quad \text{Decays}\]

- Baryon number conservation requires the \(\Delta\) to decay to \(p\) or \(n\). (Higher mass baryons not accessible)
- Strong interaction favoured 10,000-fold over an EM decay.
- As the strong interaction dominates, use isospin to understand relative rates using:

\[
|\Delta^+\rangle = |I = \frac{3}{2}, I_3 = \frac{1}{2}\rangle \quad |p\rangle = |I = \frac{1}{2}, \frac{1}{2}\rangle \quad |n\rangle = |I = \frac{1}{2}, -\frac{1}{2}\rangle \\
|\pi^+\rangle = |1, 1\rangle \quad |\pi^0\rangle = |1, 0\rangle \quad |\pi^-\rangle = |1, -1\rangle
\]

- Decompose \(I = \frac{3}{2}\) state into possible combinations \(I = \frac{1}{2}\) and \(I = 1\)

\[
\Delta^+ = |\frac{3}{2}, \frac{1}{2}\rangle = \frac{1}{\sqrt{3}} |\frac{1}{2}, -\frac{1}{2}\rangle |1, 1\rangle + \sqrt{\frac{2}{3}} |\frac{1}{2}, \frac{1}{2}\rangle |1, 0\rangle
\]

\[
\frac{1}{\sqrt{3}} |n\rangle \quad |\pi^+\rangle + \sqrt{\frac{2}{3}} |p\rangle \quad |\pi^0\rangle
\]

- And deducing branching ratios:

\[
\frac{\mathcal{B}(\Delta^+ \to \pi^0p)}{\mathcal{B}(\Delta^+ \to \pi^+n)} = \frac{|\langle \pi^0p | \Delta^+ \rangle|^2}{|\langle \pi^+n | \Delta^+ \rangle|^2} = \frac{|\sqrt{2/3}|^2}{|\sqrt{1/3}|^2} = 2
\]

The mass splittings between members of isospin multiplets, as expected, are of order of:

\[ \frac{\Delta m}{m} \sim \alpha \sim 10^{-2} \]

<table>
<thead>
<tr>
<th></th>
<th>( \Delta m ) (MeV/c²)</th>
<th>( m_n ) (MeV/c²)</th>
<th>( 10^3 \Delta m/m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n - p )</td>
<td>1.3</td>
<td>939</td>
<td>1.4</td>
</tr>
<tr>
<td>( \Sigma^0 - \Sigma^+ )</td>
<td>3.1</td>
<td>1190</td>
<td>2.6</td>
</tr>
<tr>
<td>( \Sigma^- - \Sigma^0 )</td>
<td>4.9</td>
<td>1195</td>
<td>4.1</td>
</tr>
<tr>
<td>( \Xi^- - \Xi^0 )</td>
<td>6.5</td>
<td>1318</td>
<td>4.9</td>
</tr>
<tr>
<td>( K^0 - K^\pm )</td>
<td>4.0</td>
<td>495</td>
<td>8.1</td>
</tr>
<tr>
<td>( \pi^\pm - \pi^0 )</td>
<td>4.6</td>
<td>140</td>
<td>33</td>
</tr>
</tbody>
</table>

Note that particle and antiparticle must have identical mass (CTP Theorem). However, \( \Sigma^- \) and \( \Sigma^+ \) have different masses, since they are both baryons, rather then baryon and antibaryon.