Interactions/Weak Force/Leptons

- Quantum Picture of Interactions
- Yukawa Theory
- Boson Propagator
- Feynman Diagrams
- Electromagnetic Interactions
- Renormalization and Gauge Invariance
- Weak and Electroweak Interactions
- Lepton Flavors and Decays
- Lepton Universality
- Neutrinos
- Neutrino Oscillations
Quantum Picture of Interactions

• Quantum Theory views action at a distance through the exchange of quanta associated with the interaction.

• These exchanged quanta are virtual and can “violate” the conservation laws for a time defined by the Uncertainty Principle:
  \[ \Delta E \Delta t = \hbar \]
Yukawa Theory

• During the 1930’s, Yukawa was working on understanding the short range nature of the nuclear force ($R \approx 10^{-15} m$)

• He postulated that this was due to the exchange of massive quanta which obey the Klein-Gordon equation:

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 (\nabla^2 - \frac{m^2 c^2}{\hbar^2}) \psi$$
Yukawa Theory

\[ \frac{\partial^2 \psi}{\partial t^2} = c^2 (\nabla^2 - \frac{m^2 c^2}{\hbar^2}) \psi \]

for a static potential, this becomes:

\[ \nabla^2 \psi = (\frac{m^2 c^2}{\hbar^2}) \psi \]

We can interpret \( \psi \) as the potential \( U(r) \) and solve for \( U \):

\[ U(r) = g_0 e^{-r/R} / 4\pi r, \]

where \( R = \frac{\hbar}{mc} \),

and \( g_0 \) is a constant (the strength)
Yukawa Theory

The range of the nuclear force was known, 
\[ R \approx 10^{-15} \text{m} \]

Therefore, the mass of this new exchange particle could be predicted:
\[ R = \frac{\hbar}{mc}, \]
\[ mc^2 = \frac{\hbar c}{R} \approx 200 \text{ MeV-fm/1 fm} \approx 200 \text{ MeV} \]

- The pion with mass 140 MeV/c\(^2\) was discovered in 1947! (the muon was discovered in 1937 and mis-identified as Yukawa’s particle, the “mesotron”)
- We now realize that this interaction is actually a residual interaction, so Yukawa was a bit fortunate to find a particle with the predicted mass
Boson Propagator

The rate for a particular interaction mediated by boson exchange is proportional to the “propagator” squared, where the “propagator” is written as:

\[ f(q) = \frac{g_0 g}{q^2 + m^2}, \]

where \( q^2 = (\Delta p)^2 - (\Delta E)^2 \), is the 4-momentum transfer

\[ \Delta p = p_3 - p_1 = p_2 - p_4 \]
\[ \Delta E = E_3 - E_1 = E_2 - E_4 \]
Boson Propagator

- This “propagator” can be derived by taking the Fourier transform of the potential:

\[ f(q) = g \int U(r) e^{i q \cdot r} \, dV \]

- Therefore, the “propagator” describes the potential in momentum space

- Then, the boson “propagator” is:

\[ f(q) = g_0 g / (|q|^2 + m^2) \]

where \( q \) is the momentum of the boson, and \( m \) is its mass.
The “propagator” can be generalized to four-momentum transfer:

\[ f(q) = \frac{g_0 g}{(q^2 + m^2)}, \]

where now \( q^2 = (\Delta p)^2 - (\Delta E)^2 \),

is the 4-momentum transfer.

Rates are proportional to the propagator:

\[ W = |f|^2 \times \text{Phase Space} \ldots \]
Feynman Diagrams

• Interactions can be depicted with Feynman diagrams
  - electrons
  - photons
  - positrons
    • (equivalent to electron moving backward in time)
  
  - electron emits a photon
    \[ A \sim e \]
  - electron absorbs a photon
    \[ A \sim e \]
Feynman Diagrams

- Virtual particles
  - lines joining vertices represent virtual particles (undefined mass)

- Vertices are represented by coupling constants, and virtual particles by propagators
Electromagnetic Interactions

• The fine structure constant specifies the strength of the EM interaction between particle and photons:

\[ \alpha = \frac{e^2}{4\pi \hbar c} = 1 / 137.0360 \ldots \]

• Emission and absorption of a photon represents the basic EM interaction

vertex amplitude = \( \sqrt{\alpha} = e \)

• cannot occur for free particle
Electromagnetic Interactions

- **Coulomb scattering between two electrons:**

  ![Coulomb Scattering Diagram]

  - **Amplitude:** $\alpha / q^2$
  - **Cross Section** $= |\text{Amp}|^2$: $\alpha^2 / q^4$
    - the Rutherford scattering formula
Electromagnetic Interactions

• Bremstrahlung:
  - electron emits photon in field of the nucleus

  \[ \sim \alpha^3 Z^2 \]

\[ \begin{align*}
  &\text{Ze} \\
  &\text{Ze}
\end{align*} \]
Electromagnetic Interactions

- Pair production ($\gamma \rightarrow e^+e^-$)

  \[ \text{cross section: } \sim \alpha^3 Z^2 \]

- This process is closely related to bremsstrahlung ("crossed diagrams")
Electromagnetic Interactions

- Higher order processes
  - the diagrams we have seen so far are leading order diagrams, but the rate for a process will be the sum of all orders:
  - For example, Bhabha scattering: $e^+e^- \rightarrow e^+e^-$
    - leading order:
    - higher order:
Electromagnetic Interactions

• Example of higher order processes: the electron magnetic moment
  - lowest order:
  - higher order:

Correction $\sim e^2 \sim \alpha$

$\sim \alpha^2$ $\sim \alpha^2$ $\sim \alpha^2$
Electromagnetic Interactions

- Electron magnetic moment:
  - a Dirac electron has a magnetic moment of
  \[ \mu = g \mu_B s, \quad s = 1/2 \quad g = 2 \quad \mu_B = \frac{e\hbar}{2mc} \]

\( (g-2)/2 \) is the anomaly due to higher order terms

\[ (g-2)^{\text{th}}/2 = 0.5 \left( \frac{\alpha}{\pi} \right) - 0.32848 \left( \frac{\alpha}{\pi} \right)^2 + 1.19 \left( \frac{\alpha}{\pi} \right)^3 + \ldots \]

\[ = (115965230 \pm 10) \times 10^{-11} \]

experiment = \( (115965218.073 \pm 0.028) \times 10^{-11} \)

PRL 100, 120801 (2008)

this measurement provides very accurate value for the fine structure constant = \( 1/137.035\,999\,084(51) \)
Muon $g-2$

\[ \mu = g \mu_B s, \]
\[ s = 1/2 \quad g = 2 \]
\[ \mu_B = \frac{e\hbar}{mc} \]

\[ a = (g-2)/2 \]

M. Davier, Tau2010

\[ a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (29.6 \pm 8.1) \times 10^{-10} \]
\[ \Rightarrow 3.6 \text{ "standard deviations"} \]
g-2 could mean New Physics?

Supersymmetric particles (Marciano, Munich, 2011)

- Most likely (popular?)
- \( a_\mu(SUSY) = \text{sgn}(\mu) \times 10^{-11} \left( \frac{100\text{GeV}}{m_{SUSY}} \right)^2 \tan \beta \)
- \( \text{sgn}(\mu) = +, \tan \beta = 3 - 40, m_{SUSY} = 100 - 500 \text{ GeV} \)

If SUSY: \( \text{sgn}(\mu) += \), dark matter easier, SUSY at LHC likely, EDMS, ...

Tom Blum (UConn and RIKEN BNL Research Center) | Muon g - 2 Theory
Renormalization and Gauge Invariance

- Electron line represents “bare” electron
- Observable particles are “dressed” by “infinite” number of virtual photons:
  - logarithmically divergent
- These divergences are swept away through renormalization:
  - “Bare” electron mass and charge is always multiplied by divergent integrals. We know this product must be the physical values of the mass and charge, so we set them to be, and the divergences are removed
Renormalization and Gauge Invariance

- In order for a theory to be “renormalizable” it must satisfy local gauge invariance
  - Examples of gauge invariance are familiar in EM and quantum
    - gauge transformations of scalar and vector potential in E&M do not change physical effects
    - wavefunction can change by an arbitrary phase without altering physics
Renormalization and Gauge Invariance

- The coupling constants that appear in the theory are actually not “constants”, but “run” with energy.
  - This is due again to virtual processes
  - For example, $\alpha = 1/137$ at very low energy, but $\alpha = 1/128$ at $\sqrt{s} = M_Z$
Leptons Flavors

3 pairs called generations

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e$ electron neutrino</td>
<td>$&lt;1\times10^{-8}$</td>
<td>0</td>
</tr>
<tr>
<td>$e_e$ electron</td>
<td>0.000511</td>
<td>-1</td>
</tr>
<tr>
<td>$\nu_\mu$ muon neutrino</td>
<td>$&lt;0.0002$</td>
<td>0</td>
</tr>
<tr>
<td>$\mu_e$ muon</td>
<td>0.106</td>
<td>-1</td>
</tr>
<tr>
<td>$\nu_\tau$ tau neutrino</td>
<td>$&lt;0.02$</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_e$ tau</td>
<td>1.7771</td>
<td>-1</td>
</tr>
</tbody>
</table>
# Leptons Flavor Interactions

<table>
<thead>
<tr>
<th></th>
<th>Strong</th>
<th>EM</th>
<th>Weak</th>
<th>Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\mu$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\tau$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Leptons Flavors

• The Standard Model includes massless neutrinos
  - left-handed neutrinos and RH antineutrinos
• Lepton flavors are conserved in interactions
  - $L_e, L_\mu, L_\tau$
  - $\text{mass}(\mu) = 106 \text{ MeV}/c^2$
  - $\text{mass}(e) = 0.5 \text{ MeV}/c^2$
  - However $\mu \rightarrow e \gamma$ is forbidden (exp: BR $< 1.2 \times 10^{-11}$)
• Neutrino oscillations are indications that the neutrinos have small masses and that flavor conservation will be violated at a small level
Lepton decays
Weak and Electroweak Interactions

- Nuclear $\beta$-decay
- Lepton conservation
- Weak due to very heavy mediating vector bosons:
  - $W^\pm$, 80 GeV/c$^2$
  - $Z^0$, 91 GeV/c$^2$
- charge-current
  - $\beta$-decay
- neutral-current
Weak Interactions

Neutrino Charged Current Interaction

Muon Decay

Σ⁺ Decay

Neutral Current
Weak and Electroweak Interactions

- Simplified picture of the Weak Interaction:
  - Propagator:
    \[ f(q) = \frac{g^2}{q^2 + M_{W,Z}^2} \]
  - for \( q^2 \ll M^2 \), \( f(q) = \frac{g^2}{M_{W,Z}^2} \)
  - Fermi’s early theory of \( \beta \)-decay postulated an interaction with strength \( G = 10^{-5} \), which we now recognize as \( G = \frac{g^2}{M_{W,Z}^2} = 10^{-5} \text{GeV}^{-2} \)

- Once we recognize the origin of the weakness, we can predict the masses of the \( W \) and \( Z \)
  \[ M_{W,Z} \sim \frac{e}{\sqrt{G}} \sim \sqrt{4\pi\alpha/G} \sim 90 \text{ GeV} \]

- Glashow, Weinberg, Salam (1961-8)
Lepton Decays

- **Muon decay rate**
  \[
  \Gamma(\mu^- \rightarrow e^- \nu_e \nu_\mu) = K G_F^2 m_\mu^5 = 2.2 \times 10^{-6} \text{ sec}
  \]

- **Tau**
  \[
  \Gamma(\tau^- \rightarrow e^- \nu_e \nu_\tau) = K G_F^2 m_\tau^5 = 3 \times 10^{-13} \text{ sec}
  \]

- **Lepton lifetime**
  \[
  \tau_l = 1/\Gamma_{tot} = \frac{B(l^- \rightarrow e^- \nu_e \nu_l)}{\Gamma(l^- \rightarrow e^- \nu_e \nu_l)}
  \]
  - Since \( B(l^- \rightarrow e^- \nu_e \nu_l) = \Gamma(l^- \rightarrow e^- \nu_e \nu_l) / \Gamma_{tot} \)

- **Ratio**
  \[
  \frac{\tau_\tau}{\tau_\mu} = \frac{B(\tau^- \rightarrow e^- \nu_e \nu_\tau)}{B(\mu^- \rightarrow e^- \nu_e \nu_\mu)} \left(\frac{m_\mu}{m_\tau}\right)^5 = 1.3 \times 10^{-7}
  \]
Classification of Weak Interactions

- Weak interactions are mediated by the “intermediate bosons” $W^\pm$ and $Z^0$
- Just as the EM force between two current carrying wires depends on the EM current, the weak interaction is between two weak currents, describing the flow of conserved weak charge, $g$
  \[ j \propto \psi^* \psi \]
- Two types of interactions:
  - CC (charged current)
  - NC (neutral current)
Classification of Weak Interactions

- Weak interactions occur between all types of leptons and quarks, but are often hidden by the stronger EM and strong interactions.

- Semi-leptonic

- Leptonic

- Non-leptonic
Lepton universality

- Unit of weak charge
  - all the leptons carry the same weak charge and therefore couple to the $W^\pm$ with the same strength
  - The quarks DO NOT carry the same unit of weak charge

- Muon decay

$$
\Gamma (\mu \rightarrow e\nu_e\bar{\nu}_\mu) = \frac{1}{\tau} \propto G^2 m_\mu^5
$$

$$
= \frac{G^2 m_\mu^5}{192\pi^3}
$$

- experimental: $\tau_\mu = 2.197 \times 10^{-6} \text{ sec}$
Lepton universality

- Tau decay

\[ \Gamma(\tau \rightarrow e\nu_e\bar{\nu}_\tau) = B(\tau \rightarrow e\nu\nu) \frac{1}{\tau} \propto G^2 m_\tau^5 \]

\[ = \frac{G^2 m_\tau^5}{192\pi^3} \]

- \( B(\tau \rightarrow e\nu\nu) = 17.80 \pm 0.06\% \)

- Test universality: since \( \Gamma \sim G^2 \sim g^4 \)

\[ g_\tau^4 \propto \frac{B(\tau \rightarrow e\nu\nu)}{(m_\tau^5 \tau_\tau)} \]

\[ \left( \frac{g_\tau}{g_\mu} \right)^4 = B(\tau \rightarrow e\nu_e\bar{\nu}_\tau) \left( \frac{m_\mu}{m_\tau} \right)^5 \left( \frac{\tau_\mu}{\tau_\tau} \right) \]
Lepton universality

- Test universality:

\[
\left( \frac{g_\tau}{g_\mu} \right)^4 - B(\tau \rightarrow e\nu\bar{\nu}_\tau) \left( \frac{m_\mu}{m_\tau} \right)^5 \left( \frac{\tau_\mu}{\tau_\tau} \right)
\]

With \( \tau_\mu = 2.197 \times 10^{-6} \text{ s} \), \( \tau_\tau = (291.0 \pm 1.5) \times 10^{-15} \text{ s} \), \( m_\mu = 105.658 \text{ MeV} \), \( m_\tau = 1777.0 \text{ MeV} \) and \( B(\tau \rightarrow e\nu\nu) = 17.80 \pm 0.06\% \)

\[
\frac{g_\tau}{g_\mu} = 0.999 \pm 0.003
\]

\[
\frac{g_\mu}{g_e} = 1.001 \pm 0.004
\]
Lepton universality

- Lepton universality also holds for the $Z$ couplings:

$$Z^0 \rightarrow e^+ e^- : \mu^+ \mu^- : \tau^+ \tau^- = 1 : 1.000 \pm 0.004 : 0.999 \pm 0.005$$

- From the muon lifetime we can compute the Fermi constant, $G$:

$$\frac{G}{(\hbar c)^3} = 1.1664 \times 10^{-5} \text{ GeV}^{-2}$$
Neutrinos could contribute significantly to the total energy density of the Universe if they have a mass in the eV range.

<table>
<thead>
<tr>
<th>$m_{\nu_e}$</th>
<th>$m_{\nu_{\mu}}$</th>
<th>$m_{\nu_{\tau}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 2 eV</td>
<td>&lt; 190 keV</td>
<td>&lt; 18 MeV</td>
</tr>
</tbody>
</table>

\[ ^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e \quad \text{(electron energy spectrum - endpoint)} \]

\[ p^+ \rightarrow m^+ + n_m \quad (m_{\nu_\mu}^2 = m_{\pi}^2 + m_{\mu}^2 - 2m_{\pi}\sqrt{m_{\mu}^2 + p^2_{\mu}}) \]

\[ \tau \rightarrow 5\pi^{\pm} + \nu_\tau \quad \tau \rightarrow 5\pi^{\pm} + \pi^0 + \nu_\tau \quad \text{(missing E and p)} \]

Analysis of the WMAP cosmic microwave background radiation and large scale structure measurements has put a limit on the sum of the neutrino masses (2013):

\[ \Sigma m_\nu < 0.44 \text{ eV} \]
Solar Neutrinos

Solar Neutrinos: The “Standard Solar Model”

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Neutrino energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p + p \rightarrow ^{2}\text{H} + e^+ + \nu_e$</td>
<td>$\leq 0.42$ MeV</td>
</tr>
<tr>
<td>or $p + e^- + p \rightarrow ^{2}\text{H} + \nu_e$</td>
<td>1.442 MeV</td>
</tr>
<tr>
<td>$^{2}\text{H} + p \rightarrow ^{3}\text{He} + \gamma$</td>
<td></td>
</tr>
<tr>
<td>$^{3}\text{He} + ^{3}\text{He} \rightarrow ^{4}\text{He} + p + p$</td>
<td></td>
</tr>
<tr>
<td>or $^{3}\text{He} + ^{4}\text{He} \rightarrow ^{7}\text{Be} + \gamma$</td>
<td></td>
</tr>
<tr>
<td>or $^{3}\text{He} + p \rightarrow ^{4}\text{He} + e^+ + \nu_e$</td>
<td>$\leq 18.8$ MeV</td>
</tr>
<tr>
<td>$^{7}\text{Be} + e^- \rightarrow ^{7}\text{Li} + \nu_e$</td>
<td>0.86 MeV</td>
</tr>
<tr>
<td>$^{7}\text{Li} + p \rightarrow ^{4}\text{He} + ^{4}\text{He}$</td>
<td></td>
</tr>
<tr>
<td>or $^{7}\text{Be} + p \rightarrow ^{8}\text{B} + \gamma$</td>
<td></td>
</tr>
<tr>
<td>$^{8}\text{B} \rightarrow ^{8}\text{Be}^* + e^+ + \nu_e$</td>
<td>$&lt; 15$ MeV</td>
</tr>
<tr>
<td>$^{8}\text{Be}^* \rightarrow ^{4}\text{He} + ^{4}\text{He}$</td>
<td></td>
</tr>
</tbody>
</table>
Homestake Mine

beginning in the 1960’s, Ray Davis et al pioneered detection of solar neutrinos

615 tons of cleaning fluid, $C_2Cl_4$

$$\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e^-$$

Argon is chemically extracted and single atoms are counted in subsequent decay
Homestake Mine
about 15 atoms are counted each month
average rate over 20 years: $2.6 \pm 0.2$ SNU
(Solar Neutrino Unit: $1 \text{ SNU} = 10^{-36} \text{ s}^{-1}$)
while Standard Solar Model predicts $7.9 \pm 2.6$ SNU

This was the origin of the long-standing “Solar Neutrino Problem”
Solar Neutrinos

SAGE and GALLEX

\[ \nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^- \]

lower energy threshold than for Chlorine
0.233 MeV vs. 0.814 MeV
therefore, sensitive to larger fraction of neutrino flux,
and, in particular, the pp reaction

SAGE (Baksan, Russia)
GALLEX (Gran Sasso, Italy)

Standard Solar Model: 130 SNU
experiment: 70.3 ± 7 SNU
Solar Neutrinos

Super Kamiokande

Japanese mine Kamioka
Large water Cherenkov
originally 2.1 ktons
Super K ~ 20 ktons
Solar Neutrinos

Super K

Large threshold (~8 MeV)

fewer events

-> larger target needed

Direction measurement
Solar Neutrinos

SNO (Sudbury Neutrino Observatory) interactions in heavy water sensitive to neutral current interactions
Solar Neutrinos

SNO (Sudbury Neutrino Observatory)
interactions in heavy water
sensitive to neutral current interactions

Flux of $^8$B neutrinos (non-e vs. e)
### Solar Neutrinos

#### Solar Neutrino Experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>measured flux</th>
<th>ratio exp/BP98</th>
<th>threshold energy</th>
<th>Years of running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homestake</td>
<td>$2.56 \pm 0.16 \pm 0.16$</td>
<td>$0.33 \pm 0.03 \pm 0.05$</td>
<td>0.814 MeV</td>
<td>1970-1995</td>
</tr>
<tr>
<td>Kamiokande</td>
<td>$2.80 \pm 0.19 \pm 0.33$</td>
<td>$0.54 \pm 0.08 \pm 0.10 \pm 0.07$</td>
<td>7.5 MeV</td>
<td>1986-1995</td>
</tr>
<tr>
<td>SAGE</td>
<td>$75 \pm 7 \pm 3$</td>
<td>$0.58 \pm 0.06 \pm 0.03$</td>
<td>0.233 MeV</td>
<td>1990-2006</td>
</tr>
<tr>
<td>Gallex</td>
<td>$78 \pm 6 \pm 5$</td>
<td>$0.60 \pm 0.06 \pm 0.04$</td>
<td>0.233 MeV</td>
<td>1991-1996</td>
</tr>
<tr>
<td>Super-Kamiokande</td>
<td>$2.40 \pm 0.03 \pm 0.08$</td>
<td>$0.465 \pm 0.005 \pm 0.015$</td>
<td>5.5 (6.5) MeV</td>
<td>1996-</td>
</tr>
<tr>
<td>GNO</td>
<td>$66 \pm 10 \pm 3$</td>
<td>$0.51 \pm 0.08 \pm 0.03$</td>
<td>0.233 MeV</td>
<td>1998-</td>
</tr>
<tr>
<td>SNO</td>
<td>$1.75 \pm 0.07 \pm 0.12 \pm 0.05$ (CC)</td>
<td>$2.39 \pm 0.34 \pm 0.16$ (ES)</td>
<td>$0.347 \pm 0.029$ (CC)</td>
<td>6.75 MeV</td>
</tr>
</tbody>
</table>

- The values for Chlorine and Gallium experiments are given in SNU.
- The values for Cerenkov experiments are given in units of $10^{10}$ counts/m²·s.
- The errors for the relative values correspond to experimental and theoretical errors, respectively, with the statistical and systematic errors added quadratically. Some of the relative values are based on my own calculation from the published results.

from “The Ultimate Neutrino Page”

http://cupp.oulu.fi/neutrino/nd-sol2.html

J. Brau  Physics 661, Interactions/Weak Force/Leptons
Neutrino Oscillations

Two possible solutions to the Solar Neutrino Problem:
1. The Standard Solar Model is wrong
cross sections, temperature, whatever
2. Neutrinos behave differently
decay, transform, whatever

If the neutrinos are massless, they will not decay
But if the neutrinos have mass they may decay:
\[ \nu_\alpha \rightarrow \nu_\beta + \gamma \]
but the estimate of this rate is very small in the SM
Another possibility, if they have mass, the different flavors may mix
weak-interaction and mass eigenstates may be different
Neutrino Oscillations

\[ |\nu_f > = \sum_m c_{fm} |\nu_m > \]

Consider two flavors

\[
\begin{pmatrix}
\nu_\mu \\
\nu_e
\end{pmatrix}
= 
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\]

\[ |\nu_e(t) > = -\sin \theta e^{-iE_1 t} |\nu_1 > + \cos \theta e^{-iE_2 t} |\nu_2 > \]

\[ E_i = p + \frac{m_i^2}{2p} \]

since \( m_i \ll p \)

\[ P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 (2\theta) \sin^2 \left[ \frac{1}{2}(E_2 - E_1)t \right] \]

\[ P(\nu_e \rightarrow \nu_\mu) = \sin^2 (2\theta) \sin^2 \left[ \frac{\Delta m^2}{4E} t \right] \]
Neutrino Oscillations

$$A = \sin^2 (2\theta)$$

$$L_\nu = \frac{4\pi E \hbar}{\Delta m^2 c^3}$$

$$L_\nu = 2.48 \left( \frac{E}{1 \text{ MeV}} \right) \left( \frac{1 \text{ eV}^2}{\Delta m^2} \right) \text{ metres.}$$
Mixing Matrix

\[ \nu_{lL}(x) = \sum_j U_{lj} \nu_{jL}(x), \quad l = e, \mu, \tau, \]

where \( \nu_{jL}(x) \) is the LH component of the field of \( \nu_j \) possessing a mass \( m_j \) and \( U \) is a unitary matrix - the neutrino mixing matrix \([1,17,18]\). The matrix \( U \) is often called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) or Maki-Nakagawa-Sakata (MNS) mixing matrix. Obviously, Eq. (13.1) implies that the individual lepton charges \( L_l, l = e, \mu, \tau, \) are not conserved.

\[
U = \begin{pmatrix}
\nu_e & \nu_1 & \nu_2 & \nu_3 \\
\nu_\mu & c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
\nu_\tau & -s_{12}c_{23} - c_{12}s_{23}c_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}c_{13}e^{i\delta} & -s_{12}s_{23}c_{13}e^{i\delta} \\
& s_{12}c_{23} - c_{12}s_{23}c_{13}e^{i\delta} & c_{12}s_{23}c_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \times \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)
\]
Mixing Matrix

\[
\begin{pmatrix}
    \nu_e \\
    \nu_\mu \\
    \nu_\tau
\end{pmatrix}
= \begin{pmatrix}
    c_{12} & s_{12} & 0 \\
    -s_{12} & c_{12} & 0 \\
    0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    1 & 0 & 0 \\
    0 & c_{23} & s_{23} \\
    0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
    0 & s_{13}e^{-i\delta} & e^{i\alpha_1/2} & 0 & 0 \\
    0 & 1 & 0 & e^{i\alpha_2/2} & 0 \\
    -s_{13}e^{i\delta} & 0 & c_{13} & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    \nu_1 \\
    \nu_2 \\
    \nu_3
\end{pmatrix}
\]

\[c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}, \quad \{\delta, \alpha_1, \alpha_2\} \equiv \text{CP - Violating Phases}\]
Neutrino Oscillations

Experiments:

Reactor

disappearance of electron neutrino
since muon cannot be produced by MeV neutrinos

Accelerator

appearance of electron neutrino,
from muon neutrino beam

Solar

disappearance of electron neutrino

Atmospheric

electron and muon neutrinos produced in atmosphere by cosmic rays
## Neutrino Oscillations

### Table 13.1: Sensitivity of different oscillation experiments.

<table>
<thead>
<tr>
<th>Source</th>
<th>Type of $\nu$</th>
<th>$E$ [MeV]</th>
<th>$L$ [km]</th>
<th>min($\Delta m^2$) [eV$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor</td>
<td>$\overline{\nu}_e$</td>
<td>$\sim 1$</td>
<td>1</td>
<td>$\sim 10^{-3}$</td>
</tr>
<tr>
<td>Reactor</td>
<td>$\overline{\nu}_e$</td>
<td>$\sim 1$</td>
<td>100</td>
<td>$\sim 10^{-5}$</td>
</tr>
<tr>
<td>Accelerator</td>
<td>$\nu_\mu, \overline{\nu}_\mu$</td>
<td>$\sim 10^3$</td>
<td>1</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>Accelerator</td>
<td>$\nu_\mu, \overline{\nu}_\mu$</td>
<td>$\sim 10^3$</td>
<td>1000</td>
<td>$\sim 10^{-3}$</td>
</tr>
<tr>
<td>Atmospheric $\nu$'s</td>
<td>$\nu_\mu, e, \overline{\nu}_\mu, \overline{\nu}_e$</td>
<td>$\sim 10^3$</td>
<td>$10^4$</td>
<td>$\sim 10^{-4}$</td>
</tr>
<tr>
<td>Sun</td>
<td>$\nu_e$</td>
<td>$\sim 1$</td>
<td>$1.5 \times 10^8$</td>
<td>$\sim 10^{-11}$</td>
</tr>
</tbody>
</table>
Atmospheric Neutrinos

\[
p/n + N \rightarrow \pi^+/K^+ + \ldots
\]

\[
\pi^+/K^+ \rightarrow \mu^+ + \nu_\mu
\]

\[
\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e,
\]

\[
p/n + N \rightarrow \pi^-/K^- + \ldots
\]

\[
\pi^-/K^- \rightarrow \mu^- + \bar{\nu}_\mu
\]

\[
\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e
\]

\[
\frac{\varphi_{\nu_\mu} + \varphi_{\bar{\nu}_\mu}}{\varphi_{\nu_e} + \varphi_{\bar{\nu}_e}} = 2
\]

\[E_n \text{ has broad peak } \sim 0.1 \text{ GeV}\]

oscillation will depend on azimuth due to L dependence

\[L_{\text{max}} \approx 10^4 \text{ km} \Rightarrow Dm^2 \sim 10^{-5} \text{ eV}^2\]

\[L_\nu = 2.48 \left(\frac{E}{1 \text{ MeV}}\right) \left(\frac{1 \text{ eV}^2}{\Delta m^2}\right) \text{ metres.}\]
Atmospheric Neutrinos

\[ P(\mu_m \rightarrow \mu_m) = 1 - \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2}{4E} t \right) \]

\[ \sin^2 \theta \sim 1, \quad \Delta m^2 \sim 2.2 \times 10^{-3} \text{eV}^2 \]

\[ A = \sin^2(2\theta) \]

\[ L_\nu = \frac{4\pi E \hbar}{\Delta m^2 c^3} \]
Neutrino Oscillations

\[ \Delta m^2 \text{ [eV}^2] \]

\[ \tan^2 \theta \]

http://hitoshi.berkeley.edu/neutrino

http://hitoshi.berkeley.edu/neutrino
The diagram illustrates the neutrino mass spectrum with three different mass states, labeled $m_1^2$, $m_2^2$, and $m_3^2$. The mass states are associated with different types of neutrinos: $\nu_e$, $\nu_\mu$, and $\nu_\tau$.

- $m_1^2$ and $m_3^2$ are indicated as atmospheric neutrinos, with masses approximately $2.5 \times 10^{-3} eV^2$.
- $m_2^2$ is indicated as solar neutrinos, with masses approximately $7.6 \times 10^{-5} eV^2$.

The diagram also shows the uncertainties or excessively unknown values indicated by question marks (?) for the mass differences.