QCD, Jets, & Gluons

- Quantum Chromodynamics (QCD)
- $e^+ e^- \rightarrow \mu^+ \mu^-$
- $e^+ e^-$ annihilation to hadrons ($e^+ e^- \rightarrow Q\overline{Q}$)
- Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$
- Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$
- Elastic lepton-nucleon scattering
- Deep inelastic scattering and partons
- Deep inelastic scattering and quarks
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon

Quantum Chromodynamics

- Another gauge theory, as QED
- Interactions described by exchange of massless, spin-1 bosons – so-called "gauge bosons"
- Force is long range, due to massless gluons – but long range force is cancelled by combinations of color in mesons and baryons

The color quantum number

- Color was invented to explain:
  - Δ^{++} = uuu
  - e+e- → hadrons
  - Also explains $\pi^0 \rightarrow \gamma\,\gamma$
- Color of a quark has three possible values
  - say red, blue, green
- Antiquarks carry anticolor
  - Anti-red, anti-blue, anti-green
- Bosons mediating the quark-quark interaction are called gluons
  - Gluons are to the strong force what the photon is to the EM force
  - Gluons carry a color and an anticolor
    - 9 possible combinations of color and anticolor
    - $r\bar{r}+g\bar{g}+b\bar{b}$ is color neutral, leaving 8 effective color combinations
- Results in potential $V = -\frac{4}{3}\,\alpha_s/r + kr$

Color

- 3 different color states
  - x_C = r, g, b
- Color hypercharge
  - Y^C
- Color isospin
  - $I_3{}^C$

Hadrons are combinations of quarks with $Y^C = 0$ and $I_3{}^C = 0$

so baryons = r g b

& mesons = r anti-r, etc.

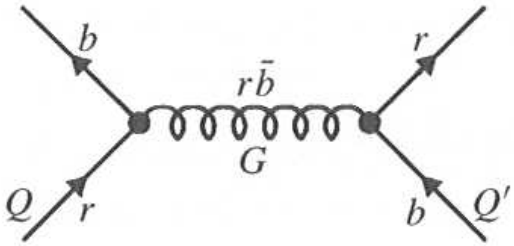
(a) Quarks			(b) Antiquarks		
	I_3^C	Y^C		I_3^C	Y^C
r	1/2	1/3	$\bar{r}$	−1/2	−1/3
g	−1/2	1/3	$\bar{g}$	1/2	−1/3
b	0	−2/3	$\bar{b}$	0	2/3

The color quantum number

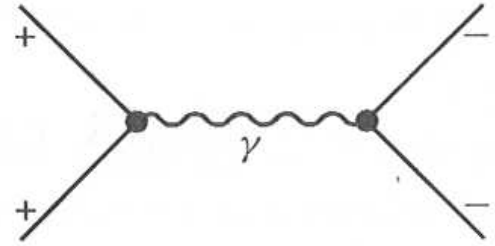
- A possible basis for the gluons:
  - $r\bar{b}$, $r\bar{g}$, $b\bar{g}$, $b\bar{r}$, $g\bar{r}$, $g\bar{b}$, $^1/_{\sqrt{2}}(r\bar{r}-b\bar{b})$, $^1/_{\sqrt{6}}(r\bar{r}+b\bar{b}-2g\bar{g})$

    (color singlet – no color – excluded: $^1/_{\sqrt{3}}(r\bar{r}+b\bar{b}+g\bar{g})$

- Gluon exhanges are analogous to photon exchanges, but different



gluons carry color charge

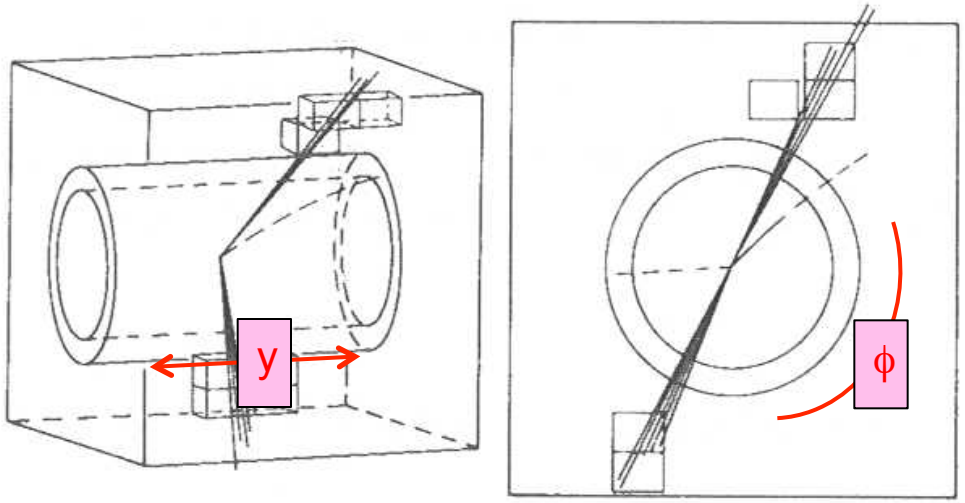


Photons **do not** carry electric charge

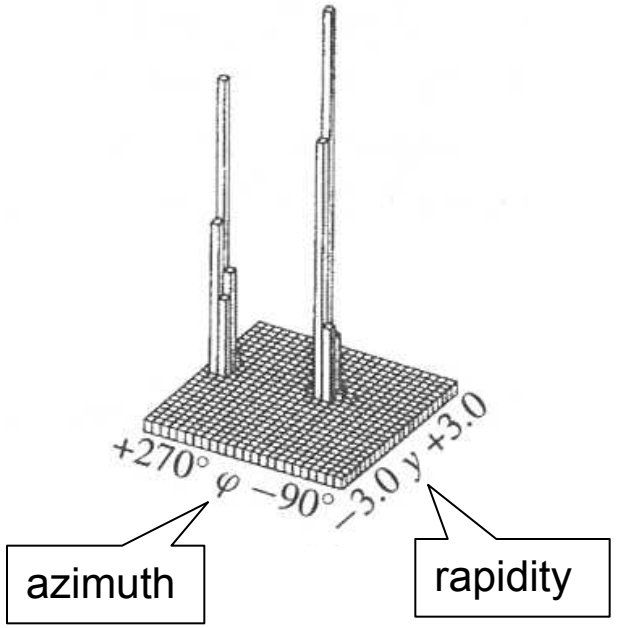
- Hadrons are always color neutral (or color-singlets)
  - eg. $Q_r\bar{Q}_r$, $Q_rQ_bQ_g$
  - Note – quark combinations such as QQ or $Q\bar{Q}\bar{Q}$ are not (normally) bound

The QCD Potential at Short Distance

- Jets produced in high energy collisions are direct support for the short range behavior

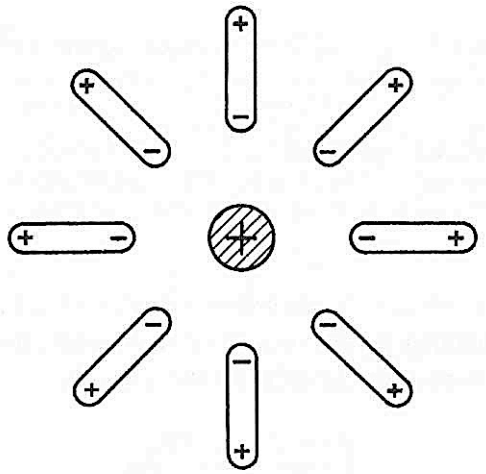


UA1 at CERN

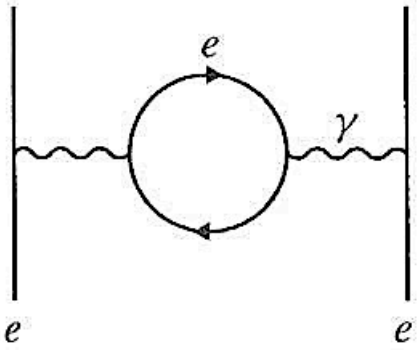


Running Couplings in QED

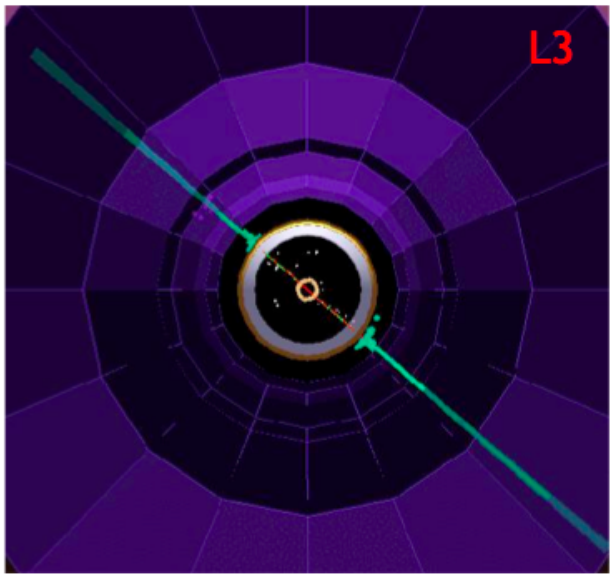
- Shielding effect is created around a charge, effectively changing the strength of coupling

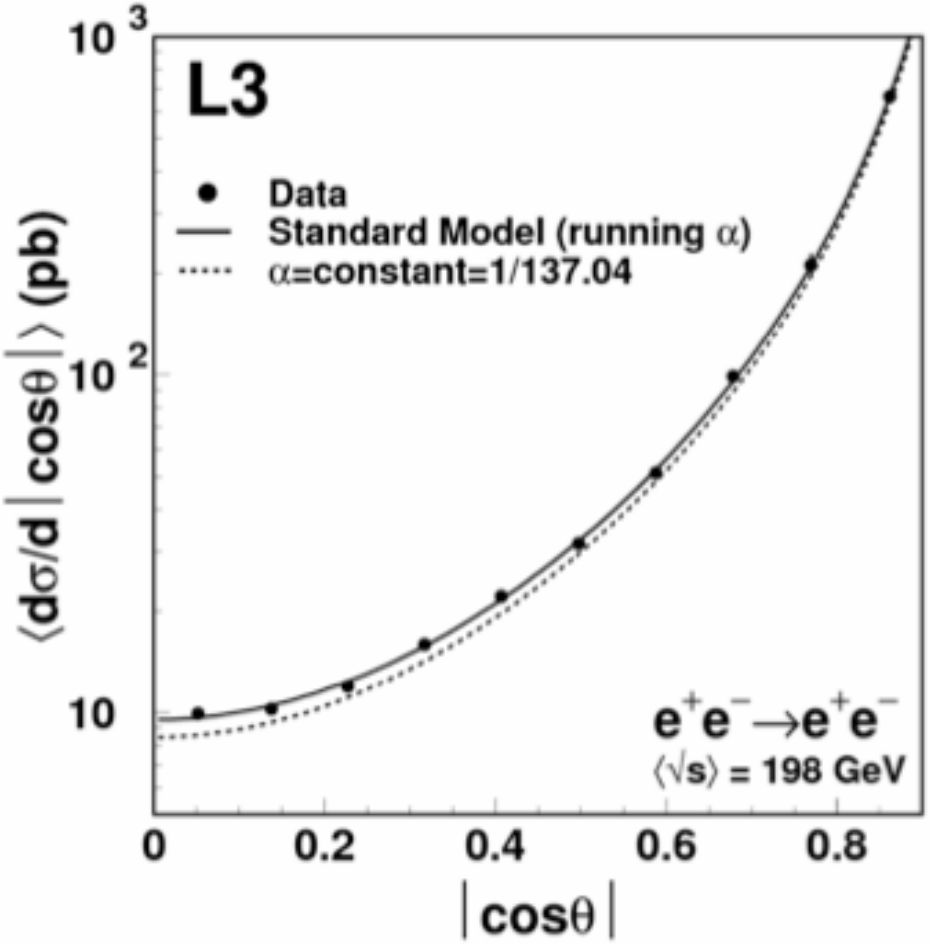


- Pairs of charge will be created and reabsorbed, producing shielding effect ("vacuum polarization")

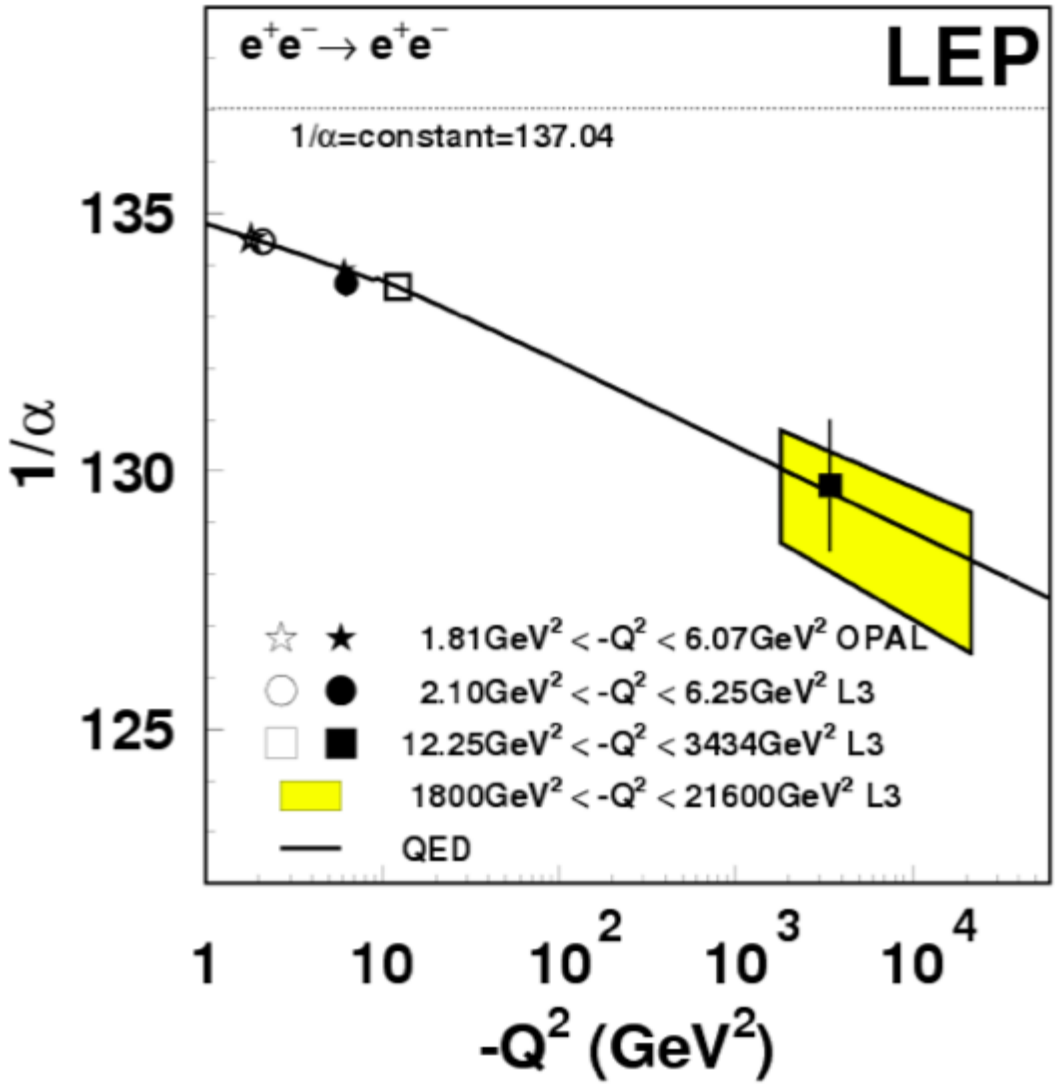


Bhabha measurement at large momentum-transfer (L3)





Running Couplings in QED



S.Mele

QED predictions from
Burkhardt&Pietrzyk
PLB **513(2001)46**

Running Couplings

- Dependence of coupling on momentum transfer (and therefore mass scale) follows the renormalization group equation:

$$\frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha(q^2)} + \beta_0 \ln\left(\frac{q^2}{\mu^2}\right) + \cdots$$

- β_0 depends on the number of degrees of freedom

$$\beta_0 = \frac{1}{12\pi}(4n_f - 11n_b)$$

- where n_b are bosons degrees of freedom and n_f are fermions degrees of freedom in the vacuum loops
- In QED there are no bosons in the loops, so at high energy

  $n_b = 0$, $n_f = 3$ (where all three families are active)

  consequently, $\alpha(M_Z = 91\ GeV) = 1/129$

Running Couplings in QCD

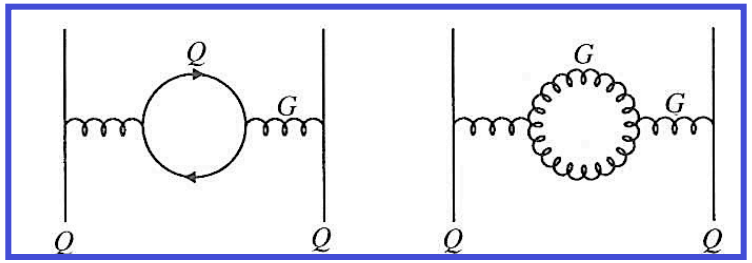
- In QCD there are three degrees of freedom (3 colors)

  $n_b = 3$

  $n_f = 3$ (where all three families are active)

$$\beta_0 = \frac{1}{12\pi}(4n_f - 11n_b) = (12-33)/12\pi = -7/4\pi$$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{7}{4\pi}\alpha_s(\mu^2)\ln\left(\frac{q^2}{\mu^2}\right)}$$



or $$\alpha_s(q^2) = \frac{1}{B\ln(q^2/\Lambda^2)}$$

where $B = -\beta$ $\Lambda^2 = \mu^2 \exp\{-1/B\ \alpha_s(\mu^2)\}$

Running Couplings in QCD

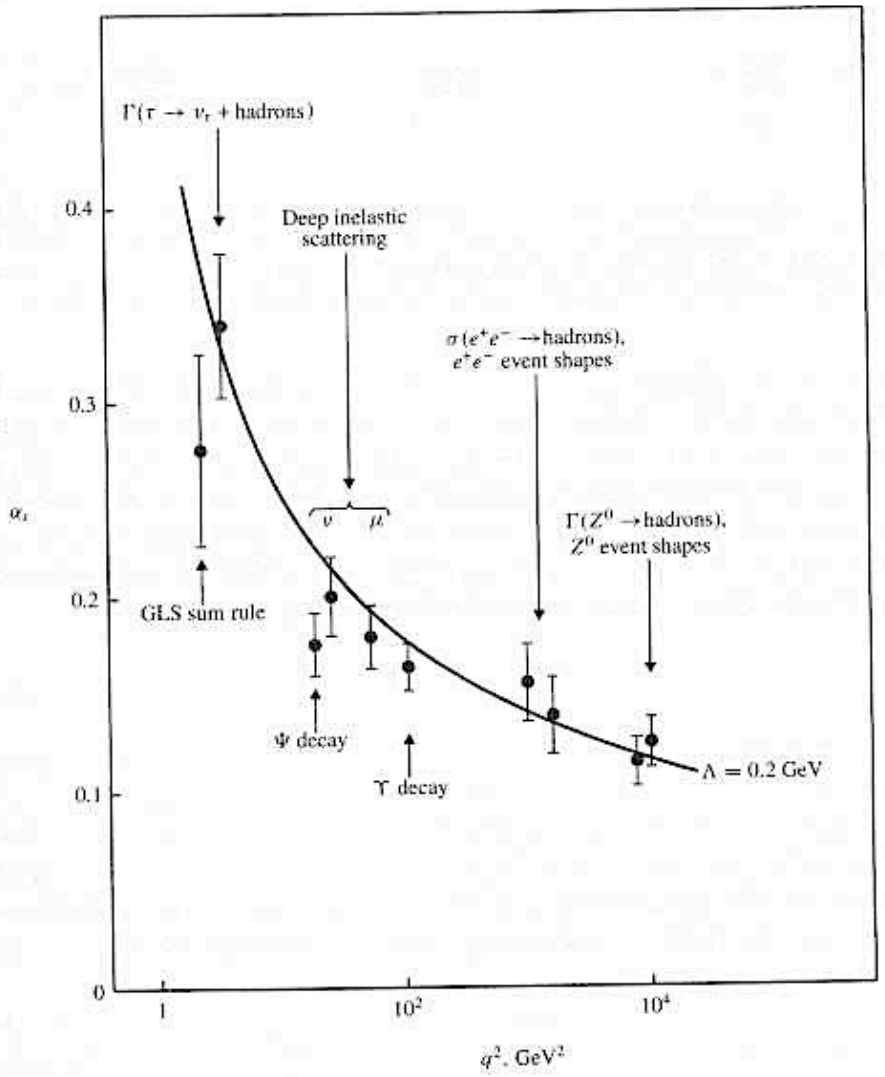
$$\alpha_s(q^2) = \frac{1}{B \ln(q^2/\Lambda^2)}$$

- α_s decreases with increasing q^2
  - this is the opposite dependence from QED
  - it is typical of non-Abelian field, where the field particles carry charge and are self-coupling
  - the longitudinal gluons have an antishielding effect, spread out the color charge, weakening the interaction

- Large q^2 $\alpha_s \rightarrow 0$
  - asymptotic freedom
- Small q^2 $\alpha_s \rightarrow$ oo
  - confinement

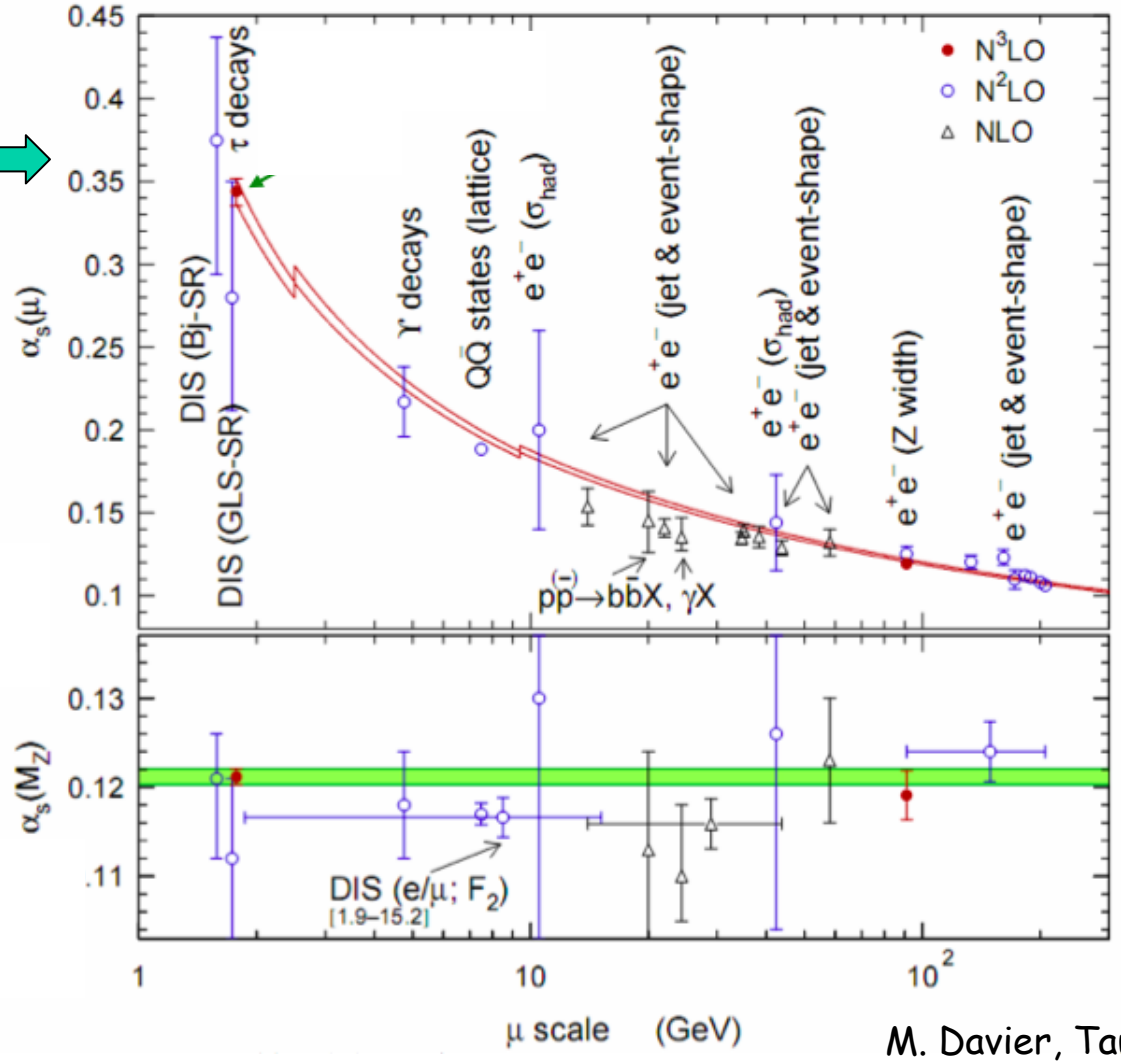
Perturbation theory applies for $q^2 >> L$

Running Couplings in QCD



Running of Strong Coupling

Confinement



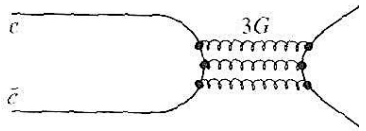
Asymptotic freedom

M. Davier, Tau2010

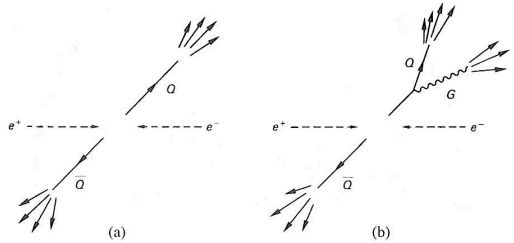
Running Couplings in QCD

Many observables are sensitive to the value of α_s

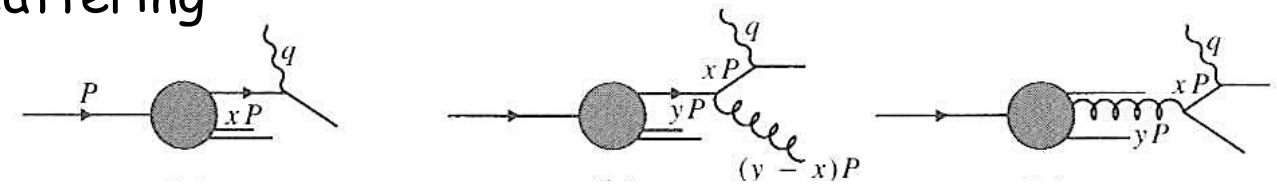
- Widths of bounds state of cc and bb



- R = $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(\text{point})$ requires factor $(1 + \alpha_s/\pi)$
- Event shapes depend on fraction of 3 jets



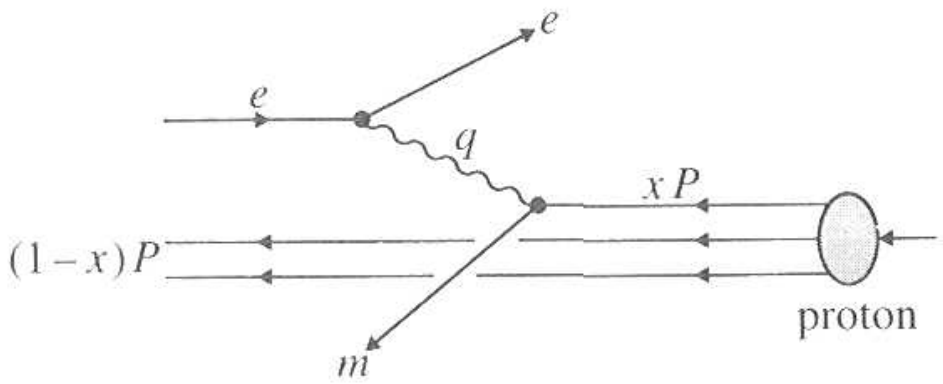
- Scaling deviations in deep inelastic lepton-nucleon scattering



- Hadronic width of the Z

The "discovery" of quarks

- deep inelastic lepton-nucleon scattering revealed dynamical understanding of quark substructure

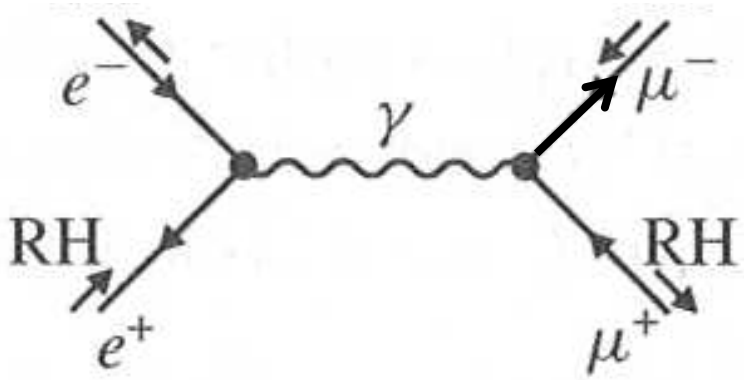


- leptoproduction of hadrons could be interpreted as elastic scattering of the lepton by a pointlike constituent of the nucleon, the quark
- theory of scattering of two spin-1/2, pointlike particles required

$e^+ e^- \rightarrow \mu^+ \mu^-$

- The electromagnetic process is dominated by single-photon exchange
- $M_{if} = e^2/q^2 = 4\pi\alpha/q^2$
- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} f(\theta)$$

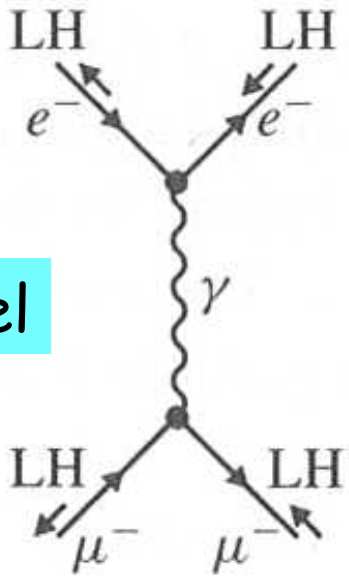
  (neglecting the muon mass)
- What about spins?
  - The conservation of helicity at high energy for the EM interaction means only LR and RL states will interact



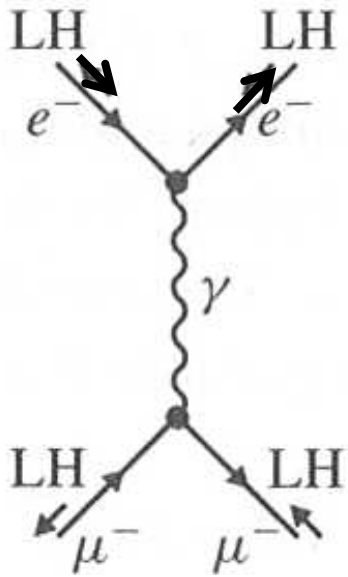
$e^+ e^- \rightarrow \mu^+ \mu^-$

- Conservation of helicity
  - consider conservation of helicity at high energy in scattering of electron

t channel



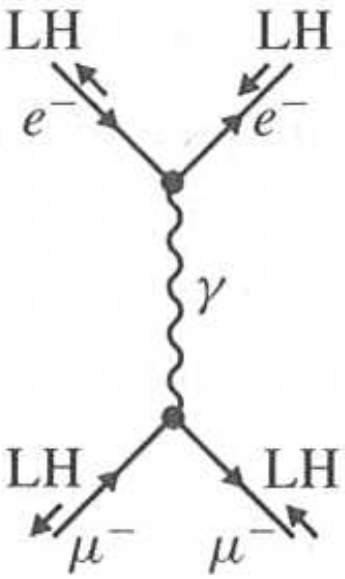
$e^-_R \rightarrow e^-_R$
and
$e^-_L \rightarrow e^-_L$

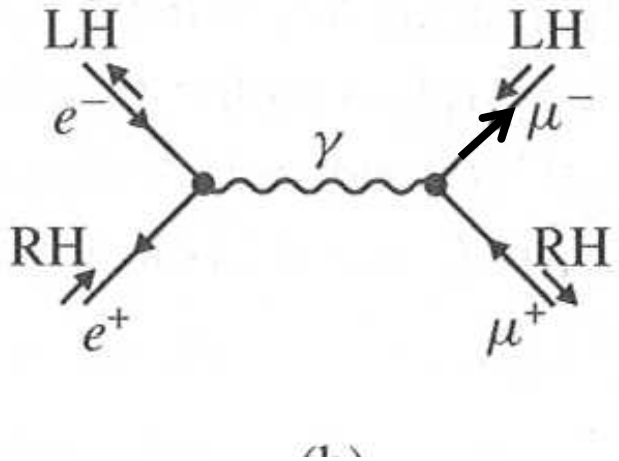


$e^+ e^- \rightarrow \mu^+ \mu^-$

- Conservation of helicity
  - consider crossed diagrams
  - $e^-_L \rightarrow e^-_L$ t-channel, and e^-_L only couples with e^+_R in s-channel (e^-_R only couples with e^+_L)

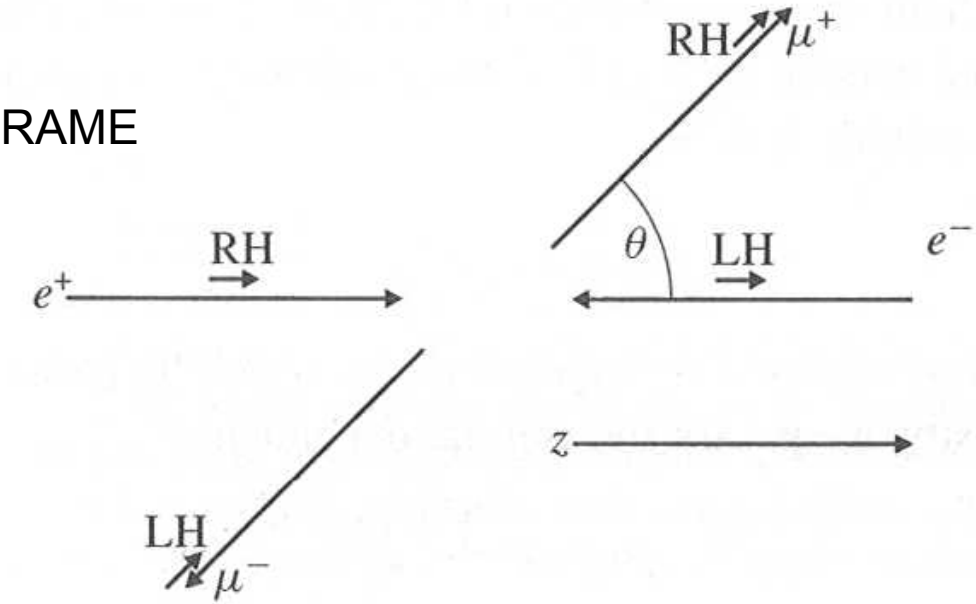
t chan





$e^+ e^- \rightarrow \mu^+ \mu^-$

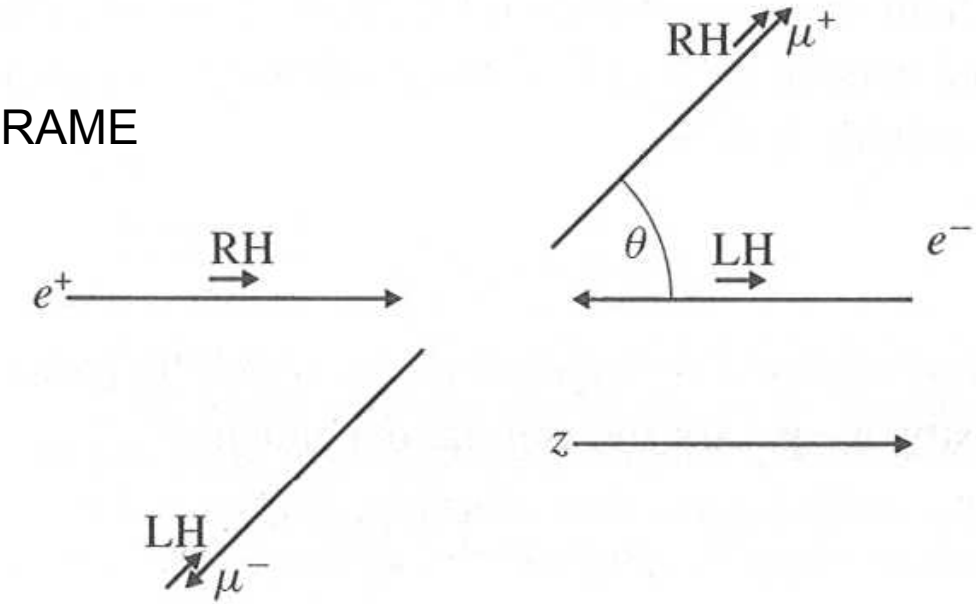
CMS FRAME



- Amplitude is $d^J_{mm'}(\theta) = d^1_{1,1}(\theta) = (1+\cos\theta)/2$
  - if RL → RL
- Amplitude is $d^J_{mm'}(\theta) = d^1_{1,-1}(\theta) = (1-\cos\theta)/2$
  - If LR → RL
- $M^2 = [(1+\cos\theta)/2]^2 + [(1-\cos\theta)/2]^2 = (1+\cos^2\theta)/2$

$e^+ e^- \rightarrow \mu^+ \mu^-$

CMS FRAME



- Now, for RL → RL and LR → RL we have

  $M^2 \sim [(1+\cos\theta)/2]^2 + [(1-\cos\theta)/2]^2 = (1+\cos^2\theta)/2$

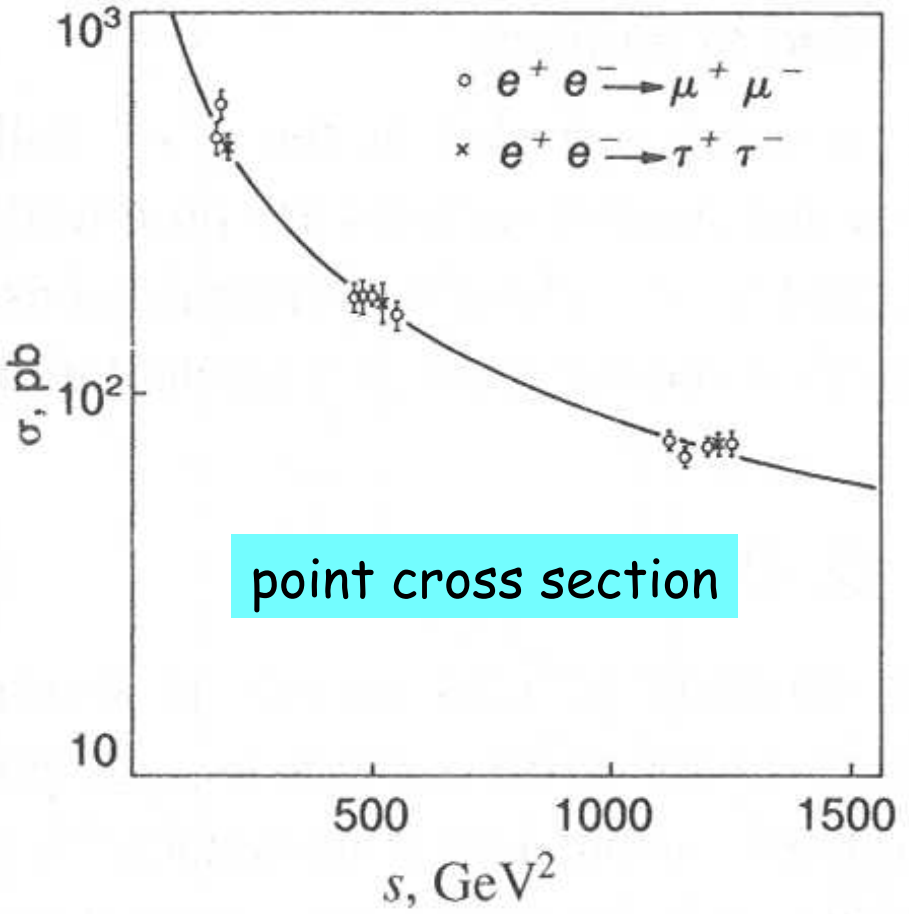
- Sum over final states, adding RL → LR and LR → LR, to double M^2

  $M^2 = (1+\cos^2\theta)$

$e^+ e^- \rightarrow \mu^+ \mu^-$

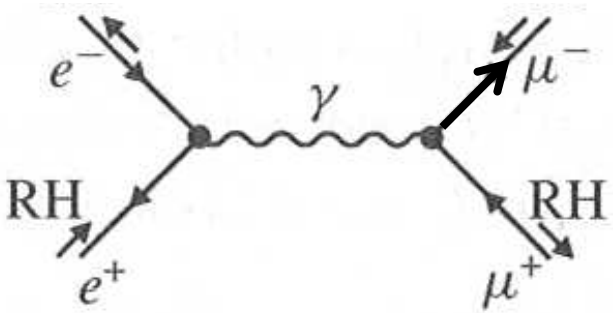
- $\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$

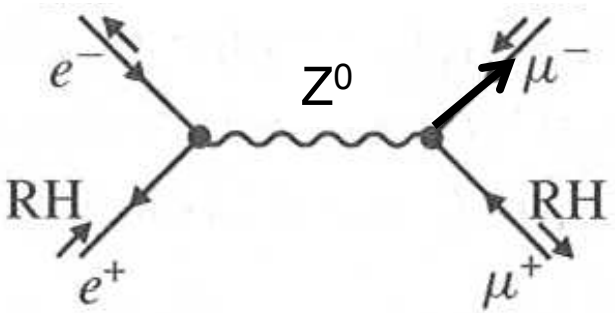
- $\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)$
  $= 4\pi\alpha^2 / 3s$
  $= 87\text{nb} / s(\text{GeV}^2)$
  (point cross section)



$e^+ e^- \rightarrow \mu^+ \mu^-$ at higher energy

- At higher energy, the Z^0 exchange diagram becomes important





$a_{em} \sim 4\pi\alpha/s$ $\qquad$ $a_{wk} \sim G$ (Fermi const.)

interference $\sim a_{wk}a_{em}/a_{em}^2 \sim Gs/(4\pi\alpha) \sim 10^{-4}s$

$e^+ e^- \rightarrow \mu^+ \mu^-$

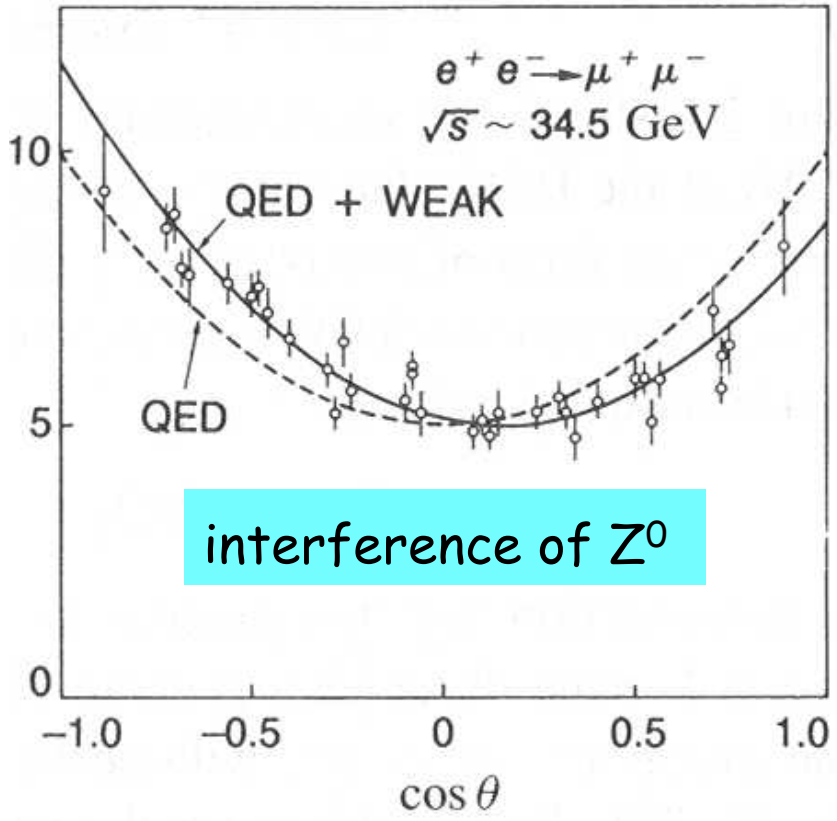
$f \sim a_{wk} a_{em} / a_{em}^2 \sim Gs/(4\pi\alpha)$

$\sim 10^{-4} s$ (interference)

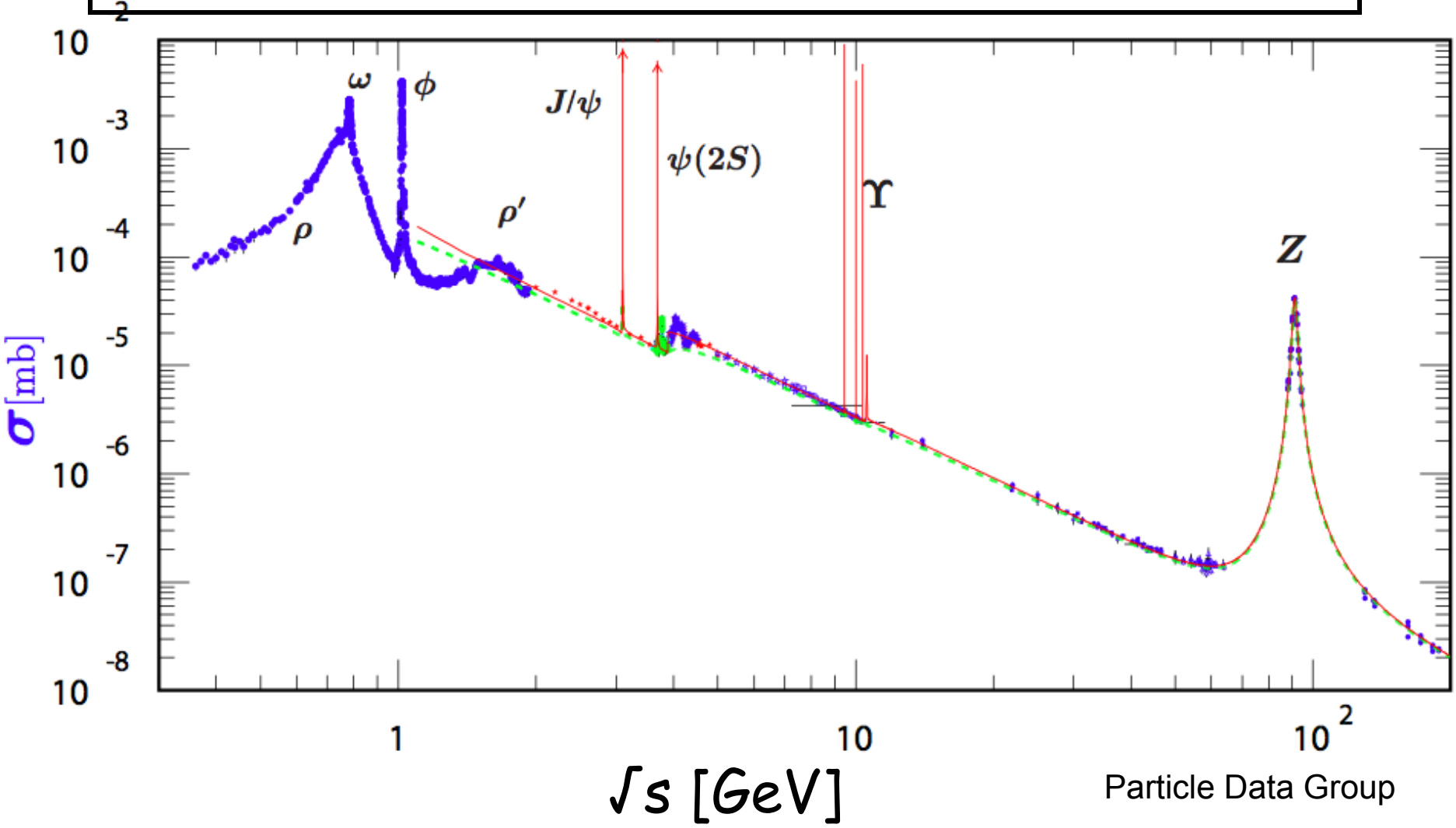
Asymmetry

$(B-F)/(F+B) = f$

$\simeq 10\%$ at $s=1000\ GeV^2$

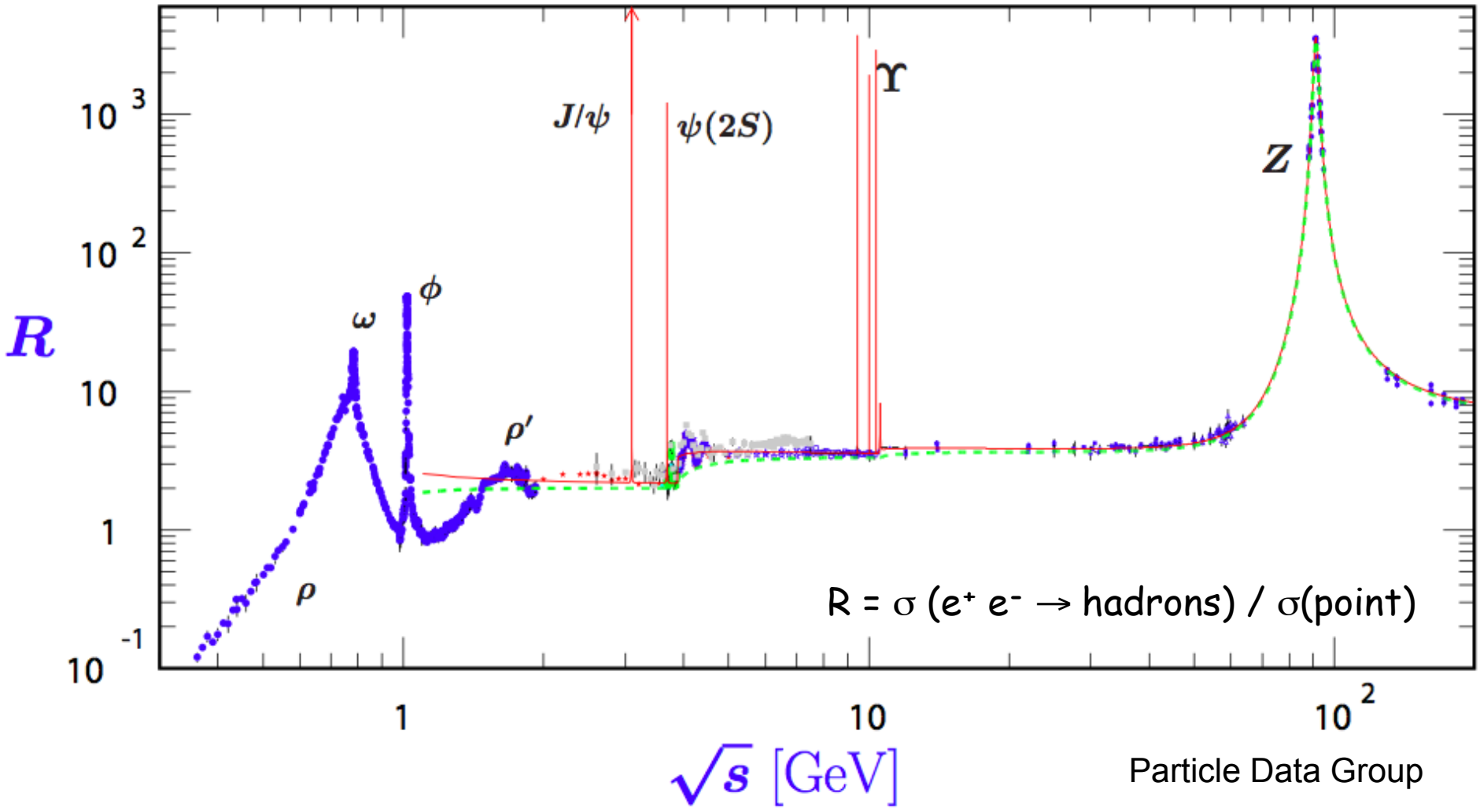


$e^+ e^-$ annihilation to hadrons



Particle Data Group

$e^+ e^-$ annihilation to hadrons



$e^+ e^-$ annihilation to hadrons

$e^+ e^-$ annihilation to hadrons

- $R = \sigma (e^+ e^- \rightarrow \text{hadrons}) / \sigma(\text{point})$
  - consider $e^+ e^- \rightarrow$ hadrons as $e^+ e^- \rightarrow Q\bar{Q}$,
    summed over all quarks

- $R = \Sigma e_i^2 / e^2$

  $= N_c ((1/3)^2 + (2/3)^2 + (1/3)^2 + (2/3)^2 + (1/3)^2 + \ldots.)$

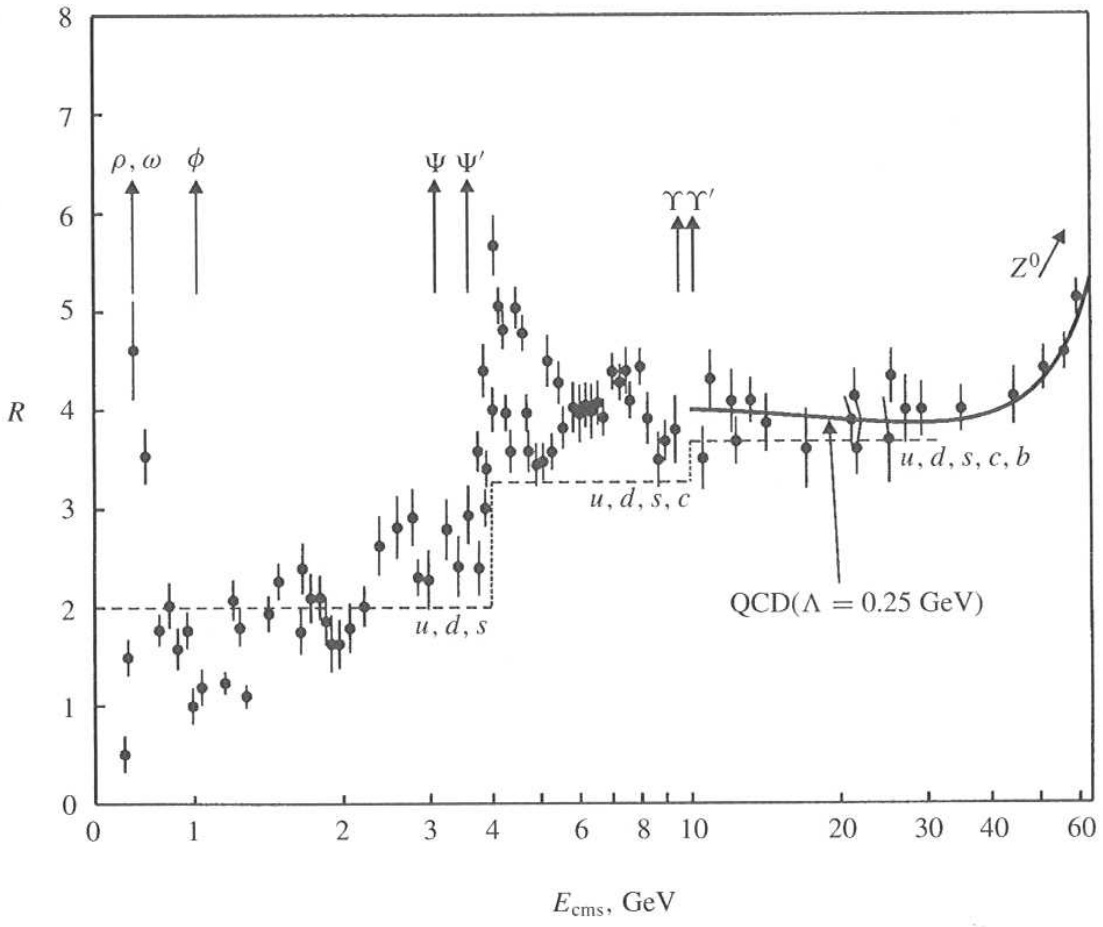
  d u s c b

  - $N_c=3$

- Therefore, R should increase by a well defined value as each flavor threshold is crossed

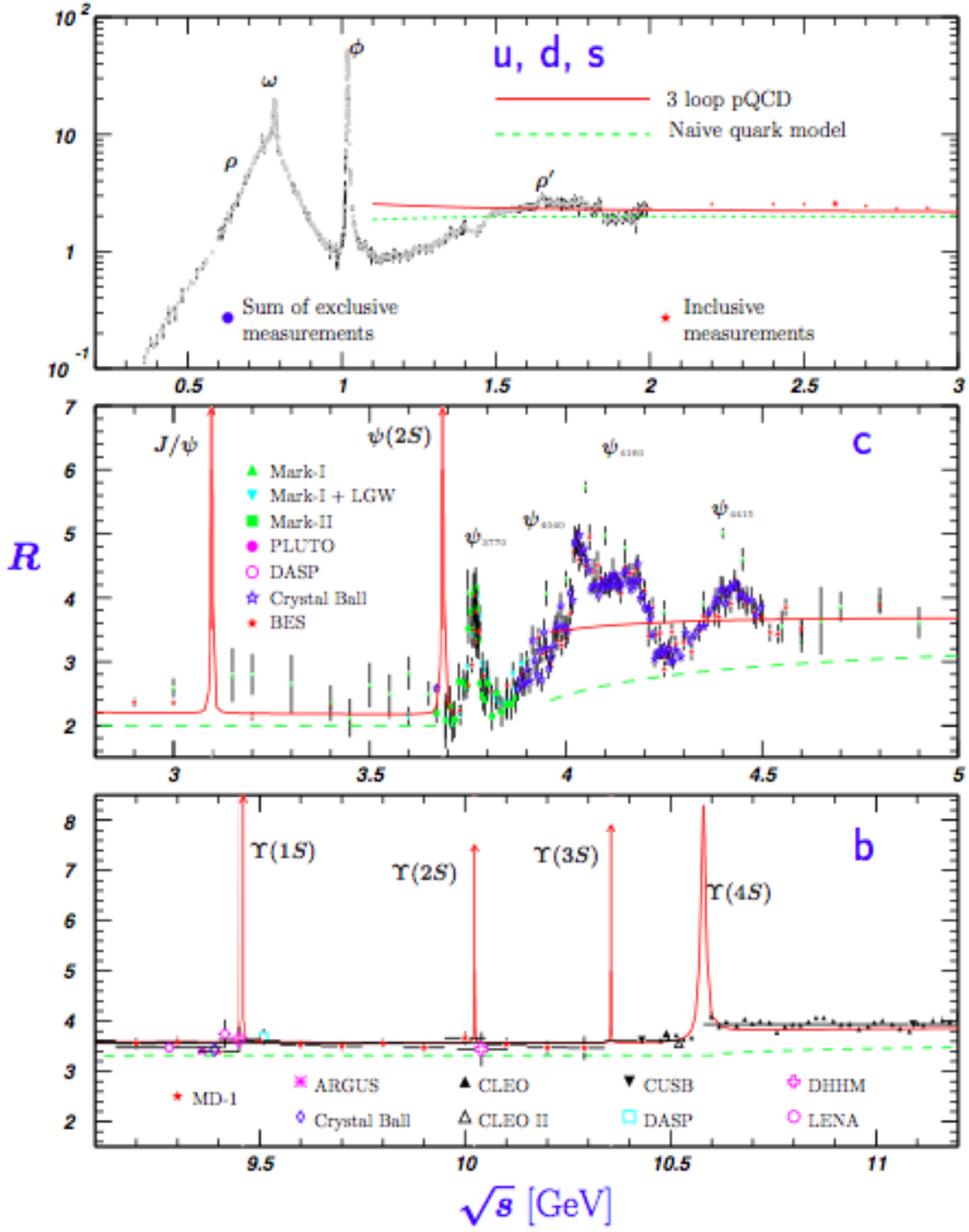
$e^+ e^-$ annihilation to hadrons

Threshold	R
below c	2
charm	3 1/3
bottom	3 2/3



$e^+ e^-$ annihilation to hadrons

Threshold	R
below c	2
charm	3 1/3
bottom	3 2/3

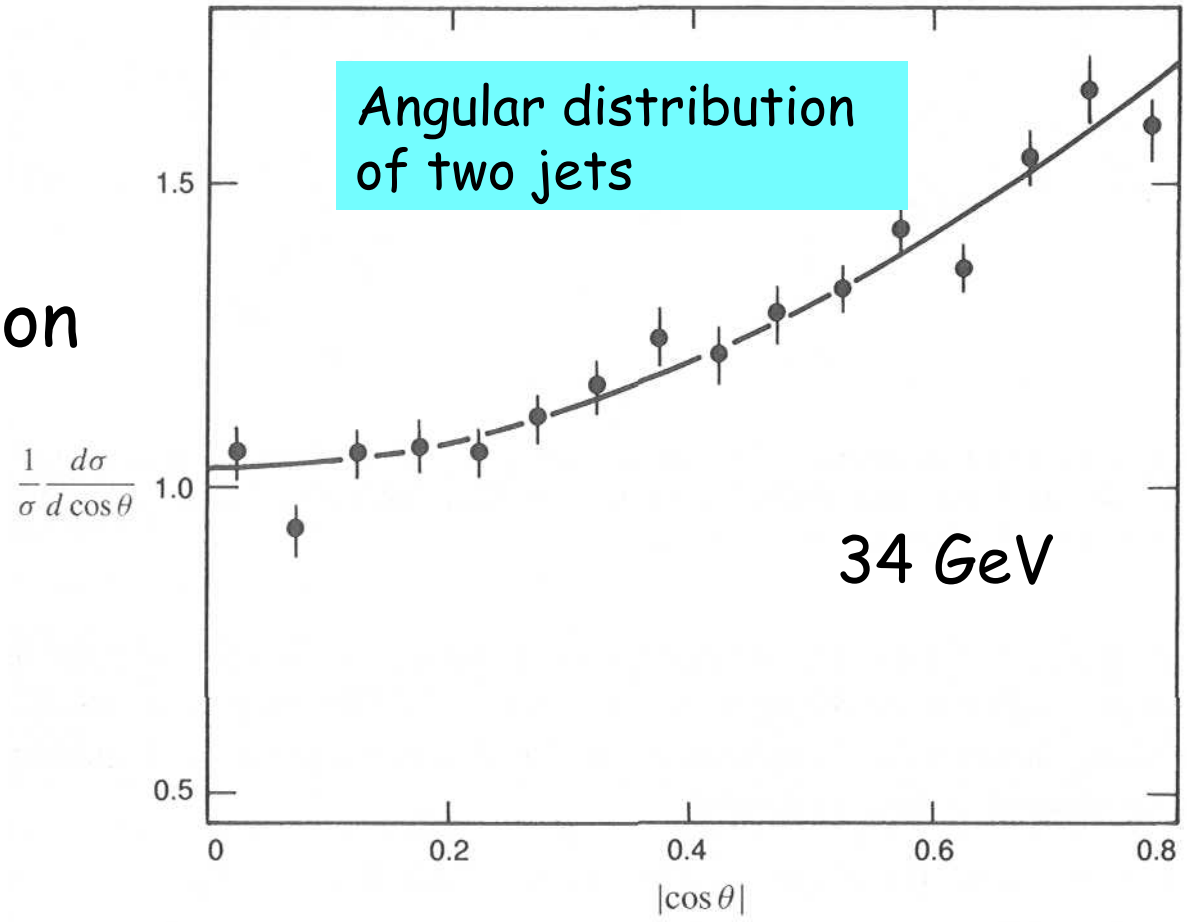


$e^+ e^-$ annihilation to hadrons

$e^+ e^- \rightarrow Q\overline{Q}$

If Q is spin $\frac{1}{2}$, this distribution should be

$(1+\cos^2\theta)$

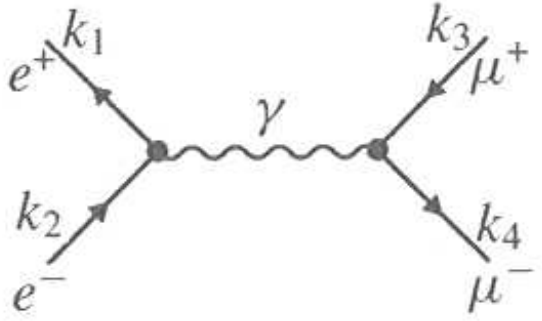


$e^+ e^-$ annihilation to hadrons

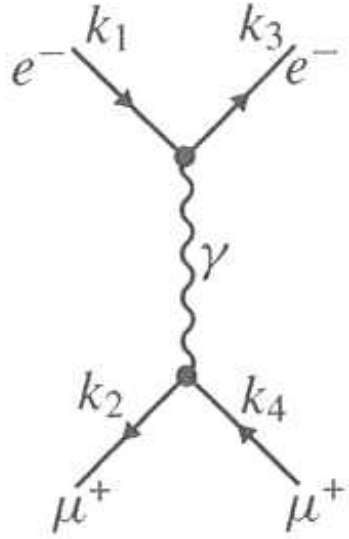
- Constancy of R indicates pointlike constituents
- Angular distributions of hadron jets prove spin 1/2 partons
- Values of R consistent with partons of quarks with expected charge and color quantum number

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

s channel



t channel



- Mandelstam variables:

$s = -(k_1 + k_2)^2 = -(k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4$

$t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4$

$u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4$

for m = 0

$k = (p_x, p_y, p_z, iE)$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Mandelstam variables:

$s = -(k_1 + k_2)^2 = -(k_3 + k_4)^2 = -2k_1k_2 = -2k_3k_4$

suppose $k_1 = (p, iE)$ $k_2 = (-p, iE)$

$s = -(p-p)^2 - (2iE)^2 = 4E^2$ for m = 0

$t = q^2 = (k_1 - k_3)^2 = (k_2 - k_4)^2 = -2k_1k_3 = -2k_2k_4$

suppose $k_3 = (p \cos\theta, p \sin\theta, 0, iE)$

$t = (p - p\cos\theta)^2 + (-p\sin\theta)^{2+} (E - E)^2 = 2p^2(1-\cos\theta)$

$= 4p^2\sin^2\theta/2$

$u = (k_2 - k_3)^2 = (k_1 - k_4)^2 = -2k_2k_3 = -2k_1k_4 = 4p^2\cos^2\theta/2$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Annihilation cross-section ($e^+e^- \rightarrow \mu^+\mu^-$)
- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2}\left(\frac{t^2+u^2}{s^2}\right) = \frac{\alpha^2}{8p^2}\,[\sin^4(\theta/2) + \cos^4(\theta/2)]$$

$$= \frac{\alpha^2}{4s}\,[1 + \cos^2\theta]$$

$2\sin^2(\theta/2) = 1-\cos\theta$
$2\cos^2(\theta/2) = 1+\cos\theta$

$s=4p^2$

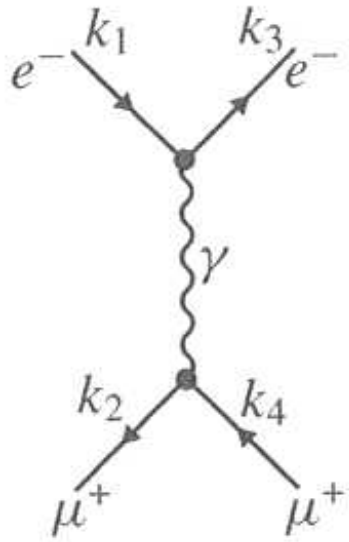
- Now consider the <u>crossed channel</u>, $e^- \mu^+ \rightarrow e^- \mu^+$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

crossed channel ($s \leftrightarrow -t$)

- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2}\left(\frac{s^2 + u^2}{t^2}\right)$$

$$= \frac{\alpha^2}{8p^2 \sin^4(\theta/2)}\left[1 + \cos^4(\theta/2)\right]$$



So far, all of these expressions are for the center-of-momentum system

What about the laboratory frame?

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

- Suppose the muon is a target at rest in the laboratory
  (Note: this is not practical since the lifetime of the muon is 2.2 microseconds)
- γ is the boost from cms to lab
- p and θ are the projectile parameters in the cms

- In the lab (neglecting lepton masses):

$$E_\mu = \gamma (p - \beta p \cos\theta) \quad \text{(scattered muon)}$$
$$= \gamma p (1 - \cos\theta)$$
$$E_e = \gamma (p + \beta p) = 2\gamma p \quad \text{(incident electron)}$$
$$y \equiv E_\mu / E_e = (1 - \cos\theta)/2$$

$$\text{so } 1-y = 1 - (1 - \cos\theta)/2 = (1+\cos\theta)/2 = \cos^2\theta/2$$
$$d\Omega = 2\pi\, d(\cos\theta) = 4\pi\, dy$$

Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2 \sin^4(\theta/2)} [1 + \cos^4(\theta/2)]$$

$$\frac{d\sigma}{4\pi dy} = \frac{\alpha^2}{q^4/2p^2} [1 + (1-y)^2]$$

$$\frac{d\sigma}{dy} = \frac{2\pi \alpha^2 s}{q^4} [1 + (1-y)^2]$$

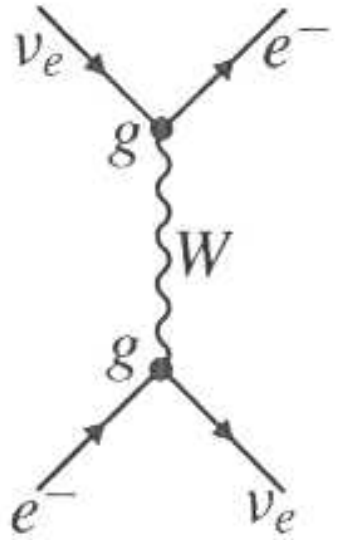
$d\Omega = 4\pi\, dy$
$\cos^2 \theta/2 = 1-y$
$q^2 = t = 4p^2 \sin^2 \theta/2$

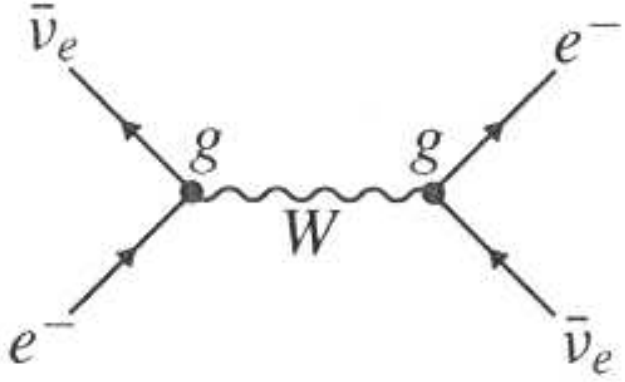
same helicity scattering

opposite helicity scattering

As y -> 0, we recover the Rutherford formula

Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$





Consider only the charged-current reaction

$$M(\nu_e e^- \rightarrow \nu_e e^-) = (g/\sqrt{2})^2 / (q^2 + M_w^2)$$

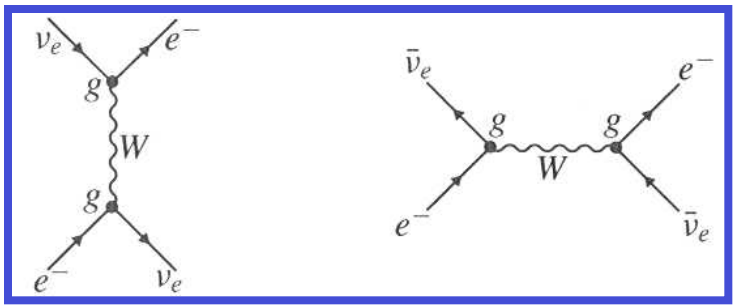
$$= g^2 / 2M_w^2 \quad \text{for } q^2 << M_w^2$$

Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$

$M(\nu_e e^- \rightarrow \nu_e e^-) = g^2/2M_W^2$

$G \equiv \sqrt{2}\, g^2/8M_W^2$

$M(\nu_e e^- \rightarrow \nu_e e^-) = 2\sqrt{2}\, G$



$$\frac{d\sigma}{d\Omega} = \frac{W}{\phi_i} = \frac{W}{v_i} = \frac{2\pi}{\hbar}\frac{|M_{if}|^2}{v_i}\frac{1}{(2\pi\hbar)^3}p_f^2\frac{dp_f}{dE_0}$$

$d\sigma/dq^2 = G^2/\pi$

$\sigma = G^2 s/\pi$ since $q^2_{max} = s$

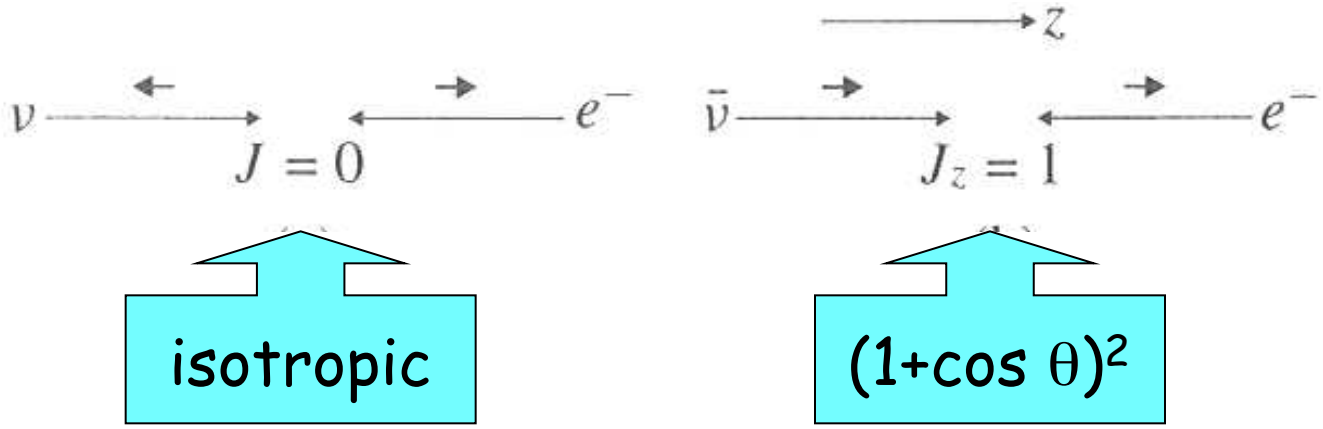
Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$

- Leptons participate in charged-current weak interactions as longitudinally polarized particles

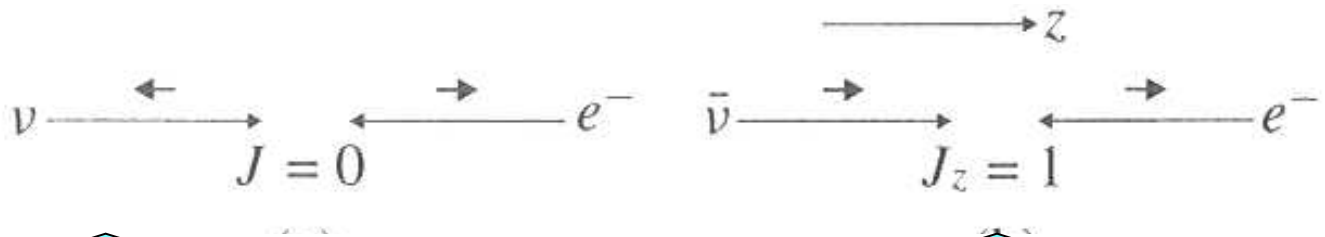
$P = (I_+ - I_-)/(I_+ + I_-) = \alpha\, p/E = \alpha\, v/c$

$\alpha = -1$ for leptons

$\alpha = +1$ for antileptons



Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$



isotropic

$(1+\cos\theta)^2$

$\sigma = G^2 s/\pi$

$d\sigma/d\cos\theta = (G^2/8\pi)s\,(1+\cos\theta)^2$

$\sigma = (G^2/3\pi)s$

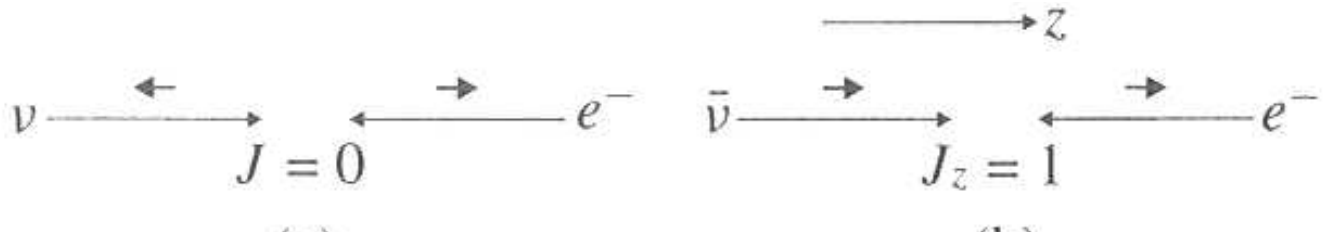
$\sigma(\nu_e e^- \rightarrow \nu_e e^-) = 3\ \sigma(\overline{\nu}_e e^- \rightarrow \overline{\nu}_e e^-)$

Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$

Consider the cross section in terms of $y = E_e/E_\nu$ in the lab:

as before, $(1+\cos\theta)^2 = 4(1-y)^2$

and $d\cos\theta = 2\,dy$



$\nu_e e^- \rightarrow \nu_e e^-$

$d\sigma/dy = G^2 s/\pi$

$\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$

$d\sigma/dy = (G^2/\pi)\, s\, (1-y)^2$

Elastic Lepton-Nucleon Scattering

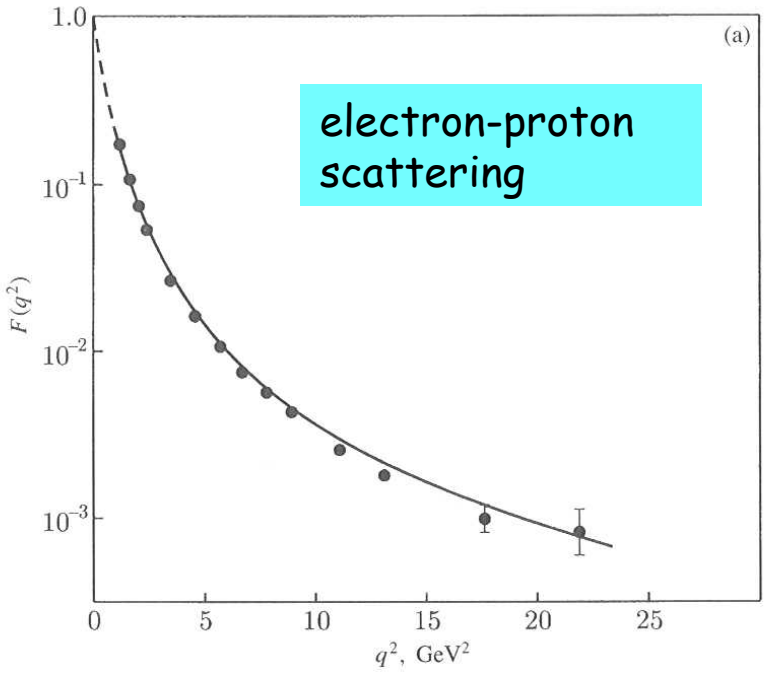
If the nucleon were a point like particle the cross section would follow the $e\mu \rightarrow e\mu$ calc.

$$\frac{d\sigma}{dq^2} = \frac{2\pi\,\alpha^2}{q^4}\,[1 + (1-q^2/s)^2]$$

But this equation must be modified by the addition of a form factor

$$\frac{d\sigma}{dq^2} \rightarrow \frac{d\sigma}{dq^2}\,F(q^2)$$

therefore, the nucleon has structure



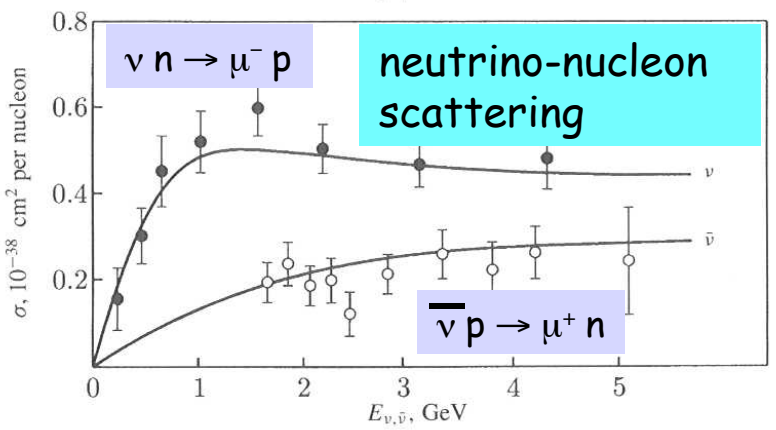
Elastic Lepton-Nucleon Scattering

Neutrino cross-section is expected to rise with s:

$$d\sigma/dy = G^2 s/\pi$$

However, elastic and quasi-elastic cross section saturate due to strong q^2 dependence of form factor

This appears to represent an exponential charge distribution



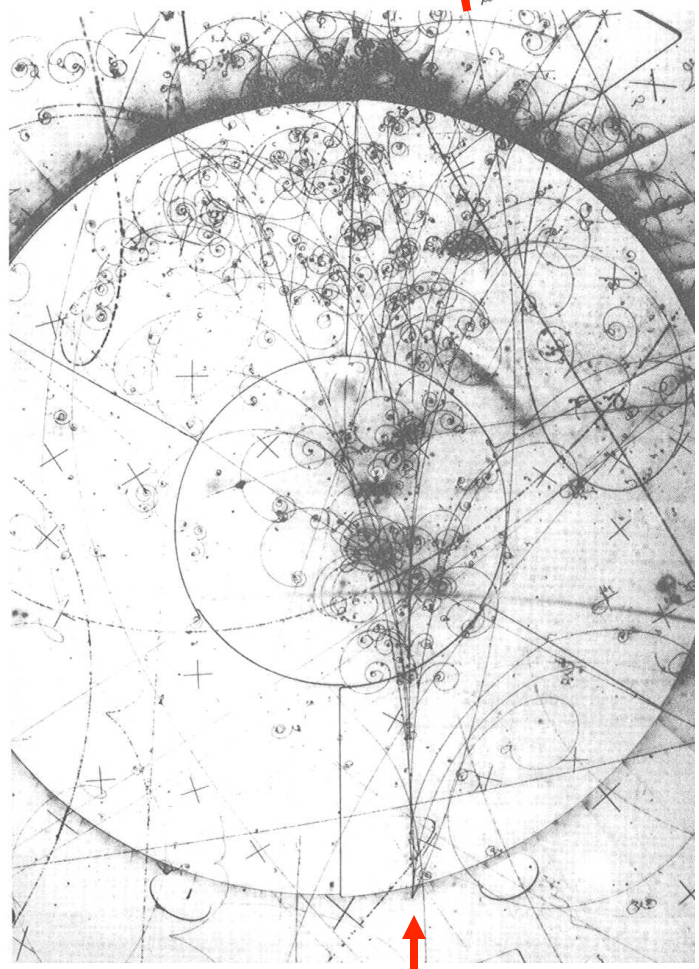
Deep Inelastic Scattering and Partons

Consider <u>inclusive</u> processes

lepton + nucleon

→ lepton' + anything

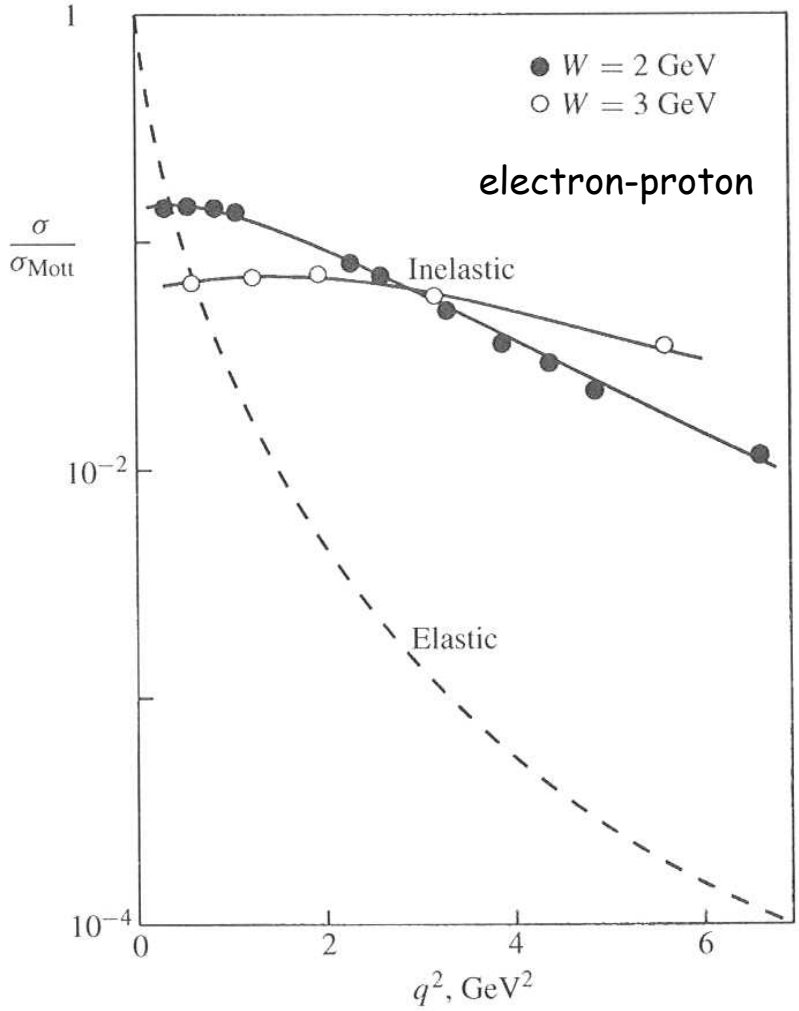
Example of such an interaction with neutrinos



Deep Inelastic Scattering and Partons

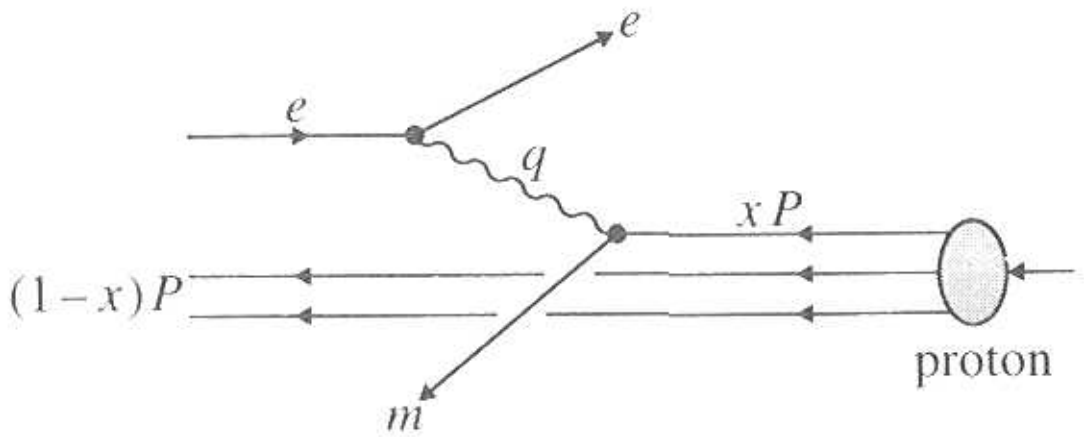
These inclusive reactions are observed to be only weakly q^2 dependent, unlike the case for exclusive reactions

This is a sign of elastic scattering from point-like constituents within the nucleon



Deep Inelastic Scattering and Partons

Infinite momentum frame (Feynman)

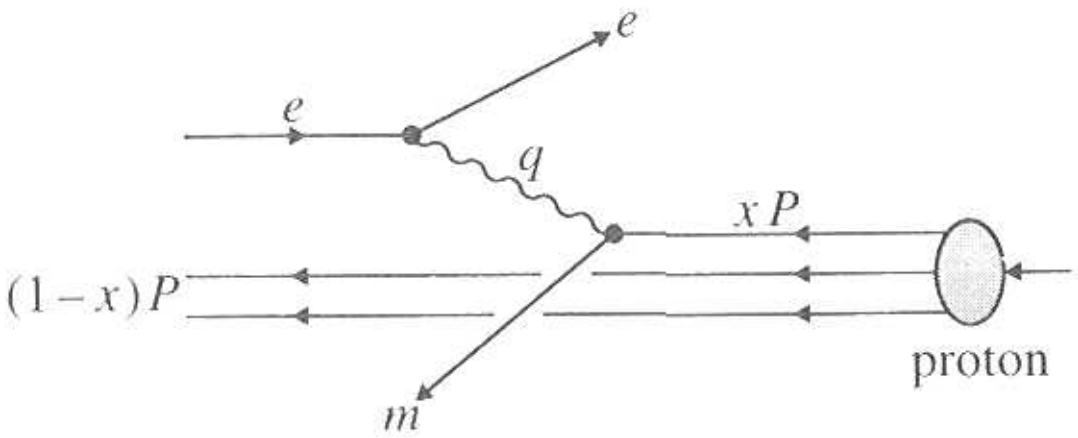


Very large proton momentum

proton P = (p,0,0,ip) as mass can be neglected
each parton has 4-momentum xp
now scatter by absorbing q

$(xP + q)^2 = -m^2 \approx 0$

Deep Inelastic Scattering and Partons



$$(xP + q)^2 = -m^2 \approx 0$$

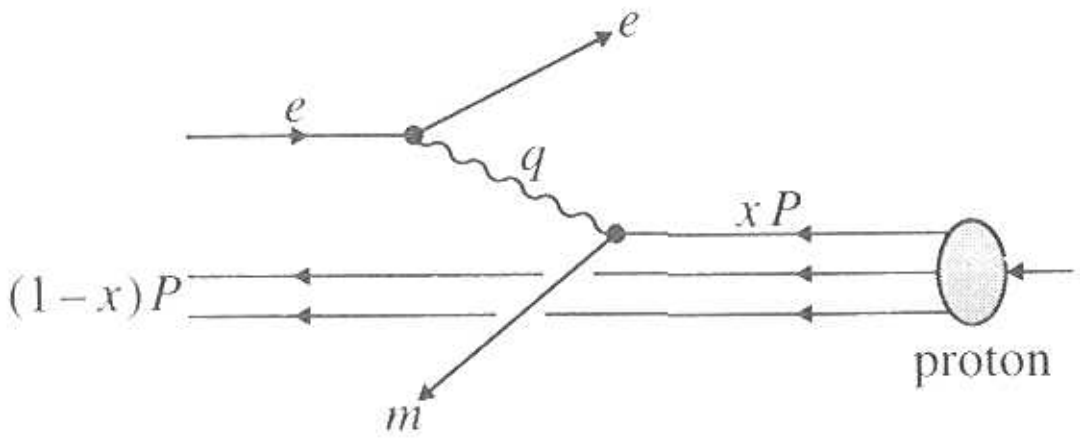
$$x^2 P^2 + q^2 + 2xPq \approx 0$$

If $|x^2 P^2| = x^2 M^2 \ll q^2$,

$$x = -q^2/2Pq = q^2/2M\nu$$

(ν is the energy transfer in the lab,)

Deep Inelastic Scattering and Partons



This model assumes the process occurs in two stages:

1. One parton is scattered

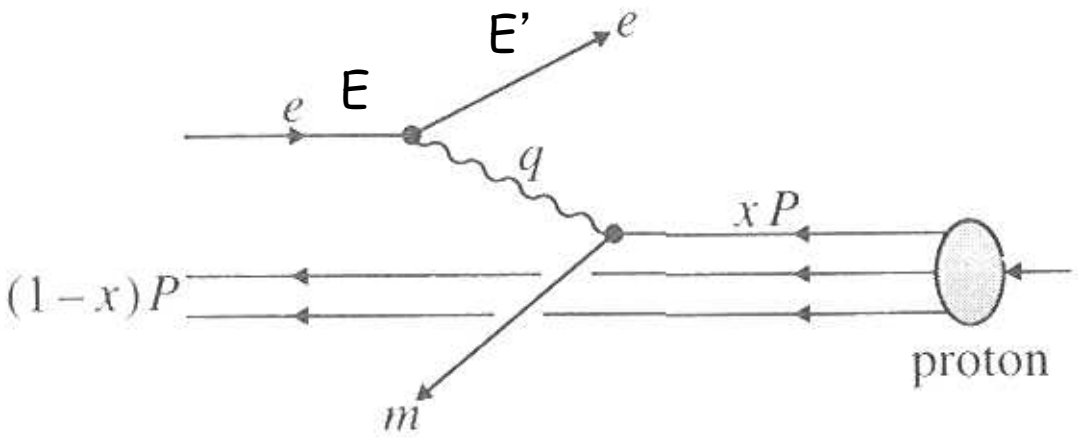
   collision time $t_1 \sim \hbar/\nu$

2. Partons recombine to form final state (M=W)

   time $t_2 \sim \hbar/W$

   $t_2 \gg t_1$

Deep Inelastic Scattering and Partons



$x = q^2/2M\nu$

(ν is the energy transfer in the lab)

$\nu = E\text{-}E'$

$q^2 = (\vec{p}\text{-}\vec{p}\,')^2 - (E\text{-}E')^2$

Deep Inelastic Electron-Nucleon Scattering and Quarks

Evidence that partons are quarks:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8p^2\sin^4(\theta/2)}\left[1 + \cos^4(\theta/2)\right]$$

$e^- \mu^+ \rightarrow e^- \mu^+$

1. ang. dist. characterized by two terms:
   a. $\cos^2(\theta/2)$ - electric (Mott)
   b. $\sin^4(\theta/2)$ - magnetic (spin-flip)

$$[1 + \cos^4(\theta/2)] = 2\cos^2(\theta/2) + \sin^4(\theta/2)$$

Data reveals just this dependence

Deep Inelastic Electron-Nucleon Scattering and Quarks

Consider a model of the partons

$u(x)\,dx$ = number of u quarks carrying momentum fraction $x \rightarrow x+dx$

$d(x)\,dx$ = number of d quarks carrying momentum fraction $x \rightarrow x+dx$

Cross section depends on charge squared:

$$\frac{d^2\sigma(ep)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\,\frac{F_2^{ep}(x)}{x}\left[\frac{1+(1-y)^2}{2}\right]$$

$$F_2^{ep}(x) = x\{4/9[u(x)+\bar{u}(x)]+1/9[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$$

Deep Inelastic Electron-Nucleon Scattering and Quarks

What about electron-neutron scattering?

$$\frac{d^2\sigma(en)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\,\frac{F_2^{en}(x)}{x}\,\left[\frac{1+(1-y)^2}{2}\right]$$

$$F_2^{en}(x) = x\{4/9[d(x)+\bar{d}(x)]+1/9[u(x)+\bar{u}(x)+s(x)+\bar{s}(x)]\}$$

by isospin invariance

for a target with equal number of protons and neutrons:

$$F_2^{eN}(x) = x\{5/18[d(x)+\bar{d}(x)+u(x)+\bar{u}(x)]+1/9[s(x)+\bar{s}(x)]\}$$

Deep Inelastic Electron-Nucleon Scattering and Quarks

$$\frac{d^2\sigma(en)}{dydx} = \frac{4\pi\,\alpha^2\,xs}{q^4}\;\frac{F_2^{en}(x)}{x}\;\left[\frac{1+(1-y)^2}{2}\right]$$

$$F_2^{ep}(x) = x\{4/9[u(x)+\bar{u}(x)]+1/9[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)]\}$$

$q(x)\,dx$ = number of q quarks carrying momentum fraction $x \rightarrow x+dx$

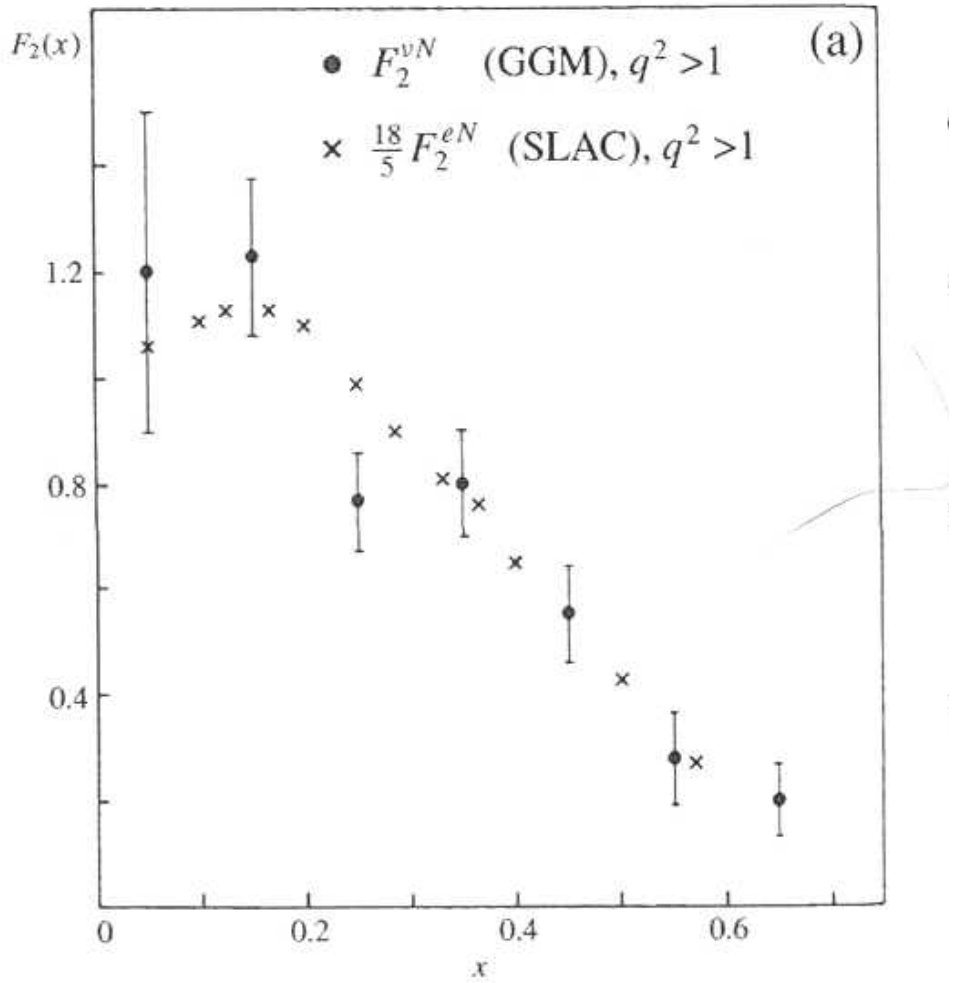
$$F_2^{en}(x) = x\{4/9[d(x)+\bar{d}(x)]+1/9[u(x)+\bar{u}(x)+s(x)+\bar{s}(x)]\}$$

by isospin invariance

for a target with equal number of protons and neutrons:

$$F_2^{eN}(x) = x\{5/18[d(x)+\bar{d}(x)+u(x)+\bar{u}(x)]+1/9[s(x)+\bar{s}(x)]\}$$

Deep Inelastic Scattering and Quarks



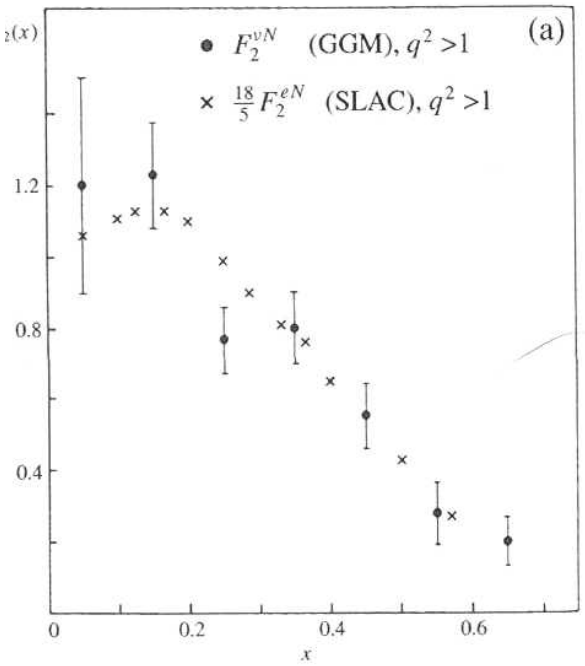
Deep Inelastic Neutrino-Nucleon Scattering and Quarks

Electron-nucleon scattering is sensitive to the parton charge, showing we have +2/3 and -1/3 quarks.

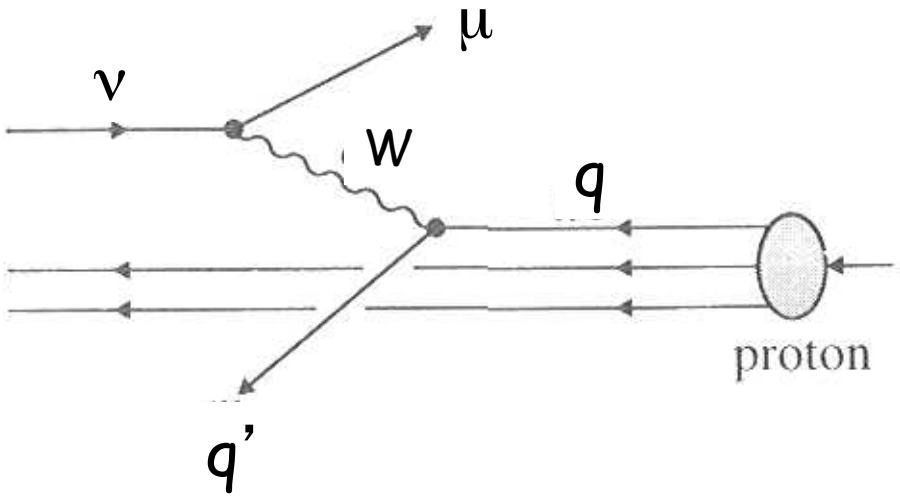
Neutrino nucleon scattering is not sensitive to charge.

Theory predicts:

$$F_2^{eN}(x) = 5/18\ F_2^{\nu N}(x)$$



Deep Inelastic Neutrino-Nucleon Scattering and Quarks



Dominant charged current interactions

$\nu_\mu + d \rightarrow \mu^- + u$ (W^+)

$\nu_\mu + \bar{u} \rightarrow \mu^- + \bar{d}$ (W^+)

$\bar{\nu}_\mu + u \rightarrow \mu^+ + d$ (W^-)

$\bar{\nu}_\mu + \bar{d} \rightarrow \mu^+ + \bar{u}$ (W^-)

Deep Inelastic Neutrino-Nucleon Scattering and Quarks

left-handed right-handed

$$d\sigma(\nu p)/dydx = G^2xs/\pi\,[d(x) + \overline{u}(x)(1-y)^2]$$

- through isospin invariance:

$$d\sigma(\nu n)/dydx = G^2xs/\pi\,[u(x) + \overline{d}(x)(1-y)^2]$$

left-handed right-handed

- for isoscalar targets (# p = # n)

$$d\sigma(\nu N)/dydx = G^2xs/2\pi\,\{[u(x) + d(x)] + [\overline{u}(x) + \overline{d}(x)](1-y)^2]\}$$

- and

$$d\sigma(\bar{\nu}N)/dydx = G^2xs/2\pi\,\{[u(x) + d(x)](1-y)^2 + [\overline{u}(x) + \overline{d}(x)]]\}$$

Deep Inelastic Neutrino-Nucleon Scattering and Quarks

- Define the neutrino structure functions:

$F_2^{\nu N}(x) = x\,[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]$

$F_3^{\nu N}(x) = x\,[u(x)-\bar{u}(x)+d(x)-\bar{d}(x)]$

Then

$$d\sigma^{\nu N,\,\bar{\nu}N}/dydx = G^2ME/\pi\left\{[F_2 \pm F_3]/2 + [F_2 \mp F_3](1-y)^2/2\right\}$$

Defining:

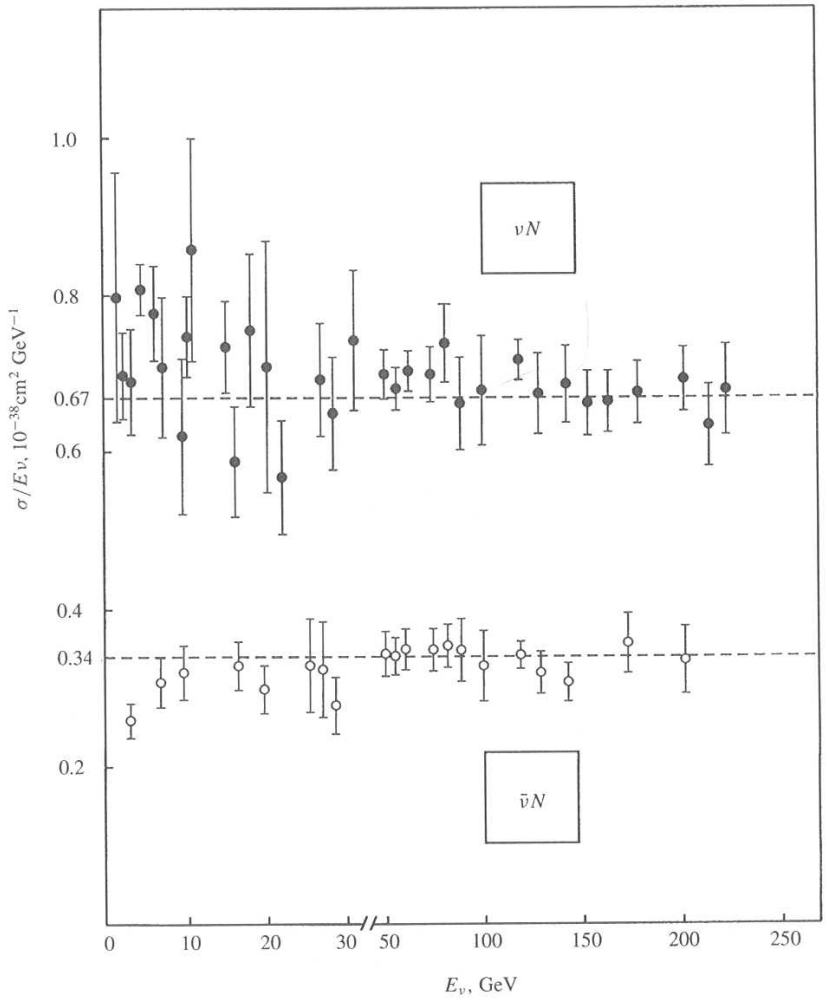
$Q = \int x[u(x)+d(x)]dx$ $\qquad$ $\bar{Q} = \int x[\bar{u}(x)+\bar{d}(x)]dx$

$$R = \sigma(\bar{\nu}N)/\sigma(\nu N) = (1+3\bar{Q}/Q)/(3+\bar{Q}/Q)$$

Quark distributions within the nucleon

- Several predictions of the nucleon model are validated
  - 1.) Total neutrino-nucleon cross section are proportional to energy

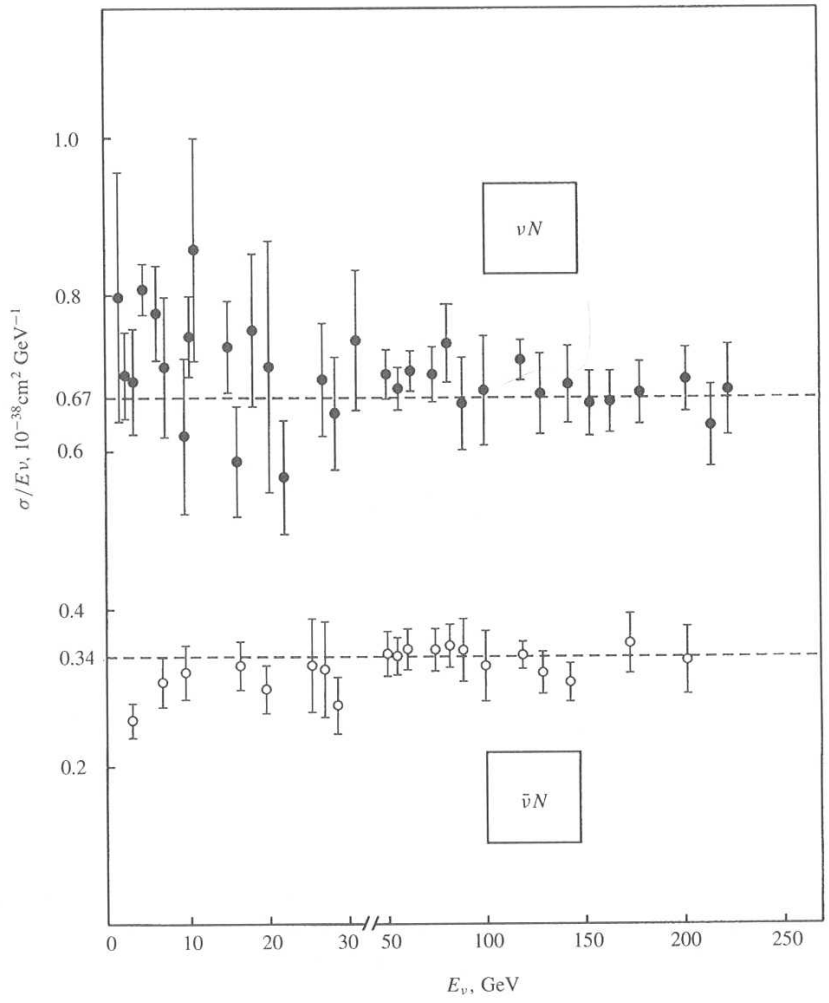
$$s = (E_\nu + m_p)^2 - E_\nu^2 = 2 E_\nu m_p$$



Quark distributions within the nucleon

- 2.) Ratio of antineutrino to neutrino cross sections indicates nucleon contains antiquarks with $\overline{Q}/Q \approx 0.15$

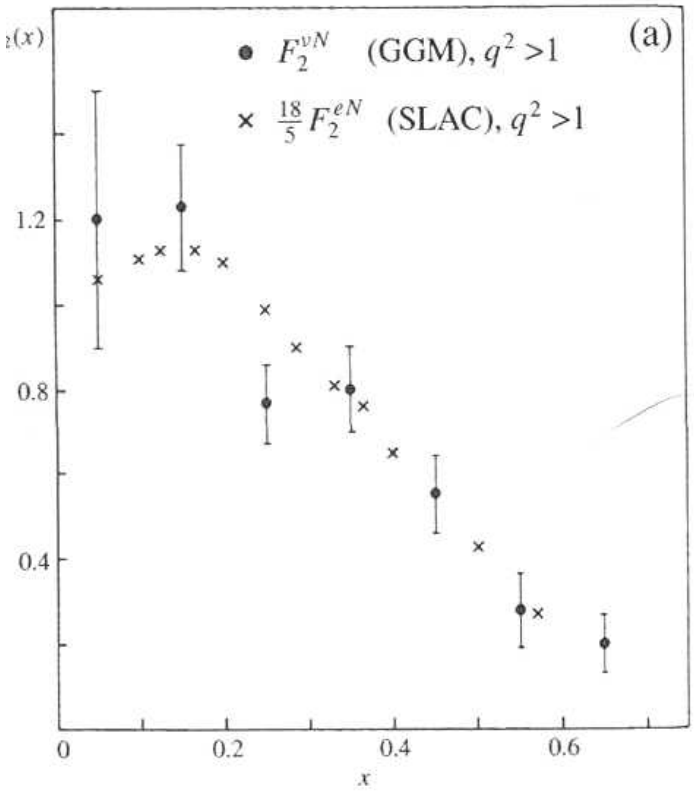
$$R = \frac{1 + 3\,\overline{Q}/Q}{3 + \overline{Q}/Q} \approx 0.45$$



Quark distributions within the nucleon

- 3.) Comparison of neutrino-nucleon cross sections with electron-nucleon cross sections establish fractionally charged quarks are real dynamical entities

$$F_2^{eN}(x) = 5/18\ F_2^{\nu N}(x)$$



Quark distributions within the nucleon

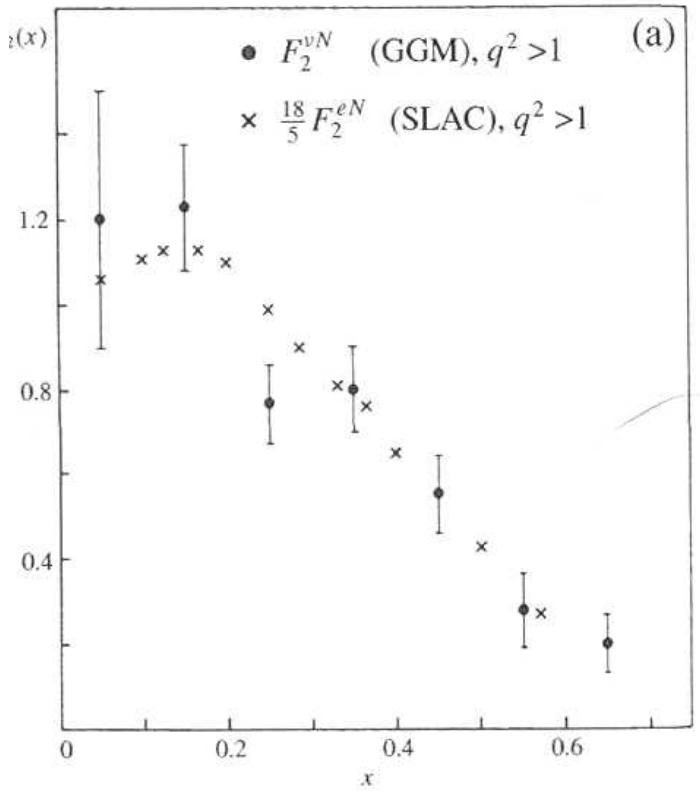
- 4.) Total momentum fraction carried by quarks and anti-quarks is ≈ 0.5

$$\int F_2^{\nu N}(x)\,dx = \tfrac{18}{5}\int F_2^{eN}(x)\,dx$$

$$\approx 0.5$$

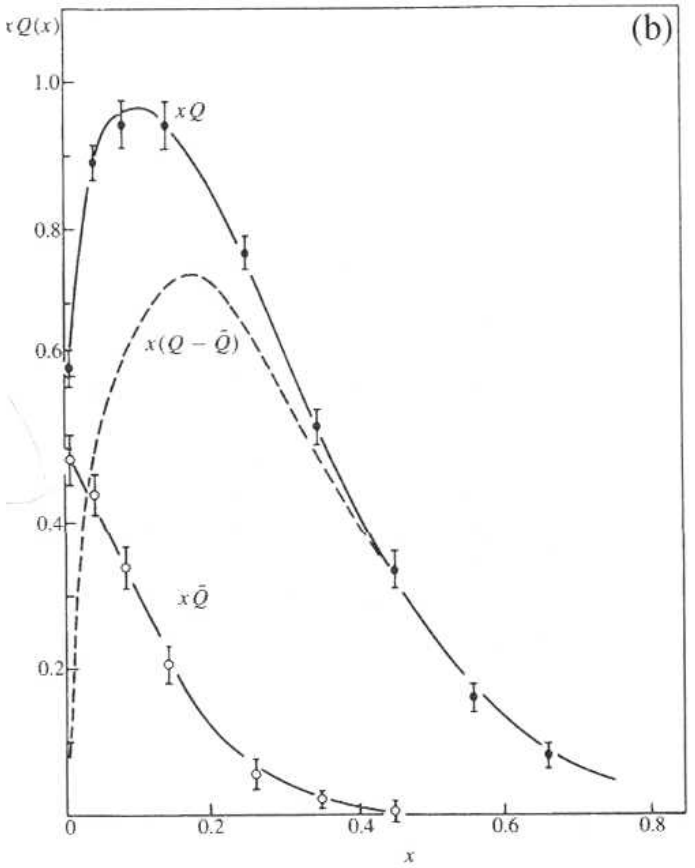
$$= \int x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]\,dx$$

Therefore 50% is carried by partons which do not couple to weak or EM forces (gluons)

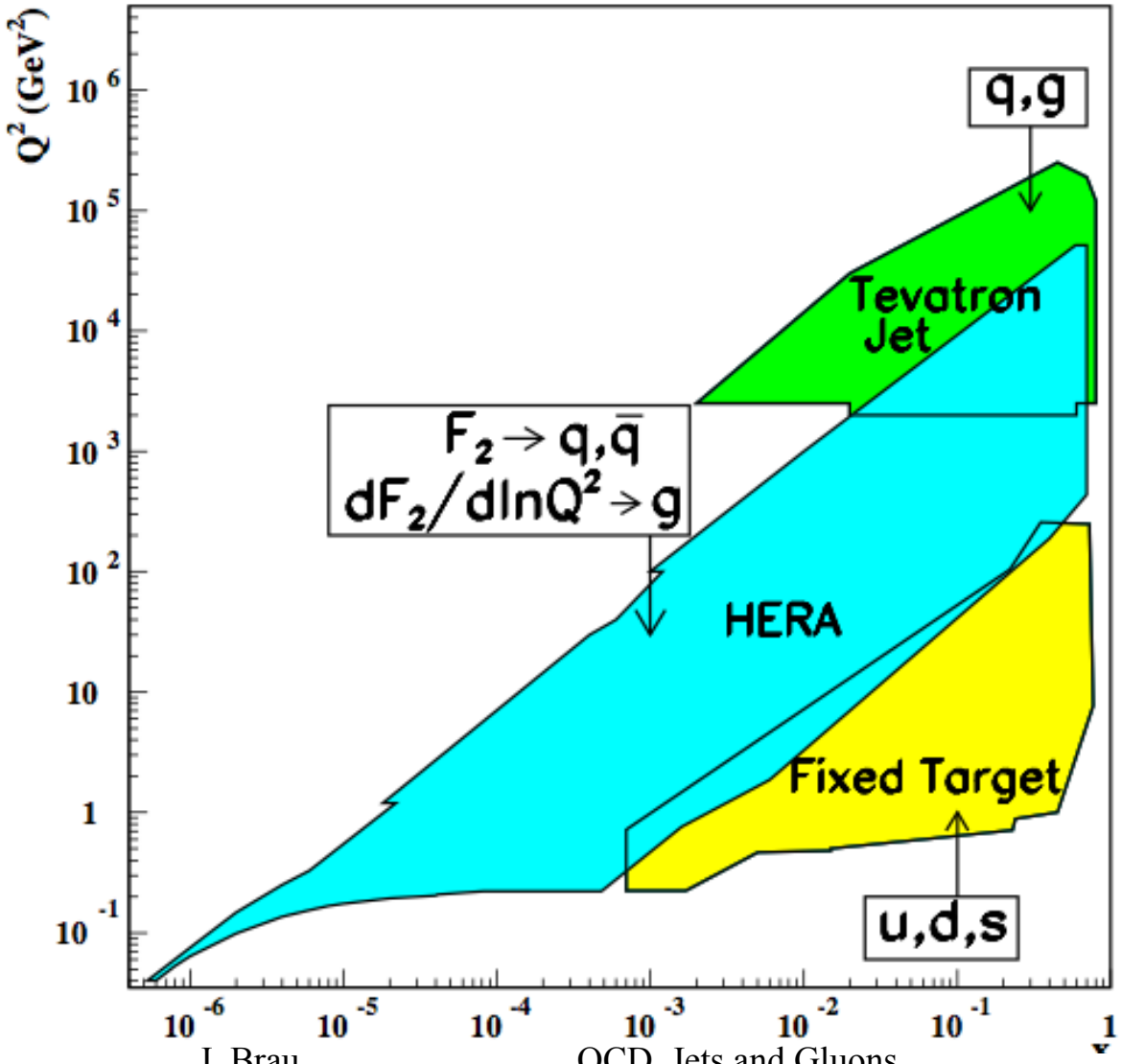


Quark distributions within the nucleon

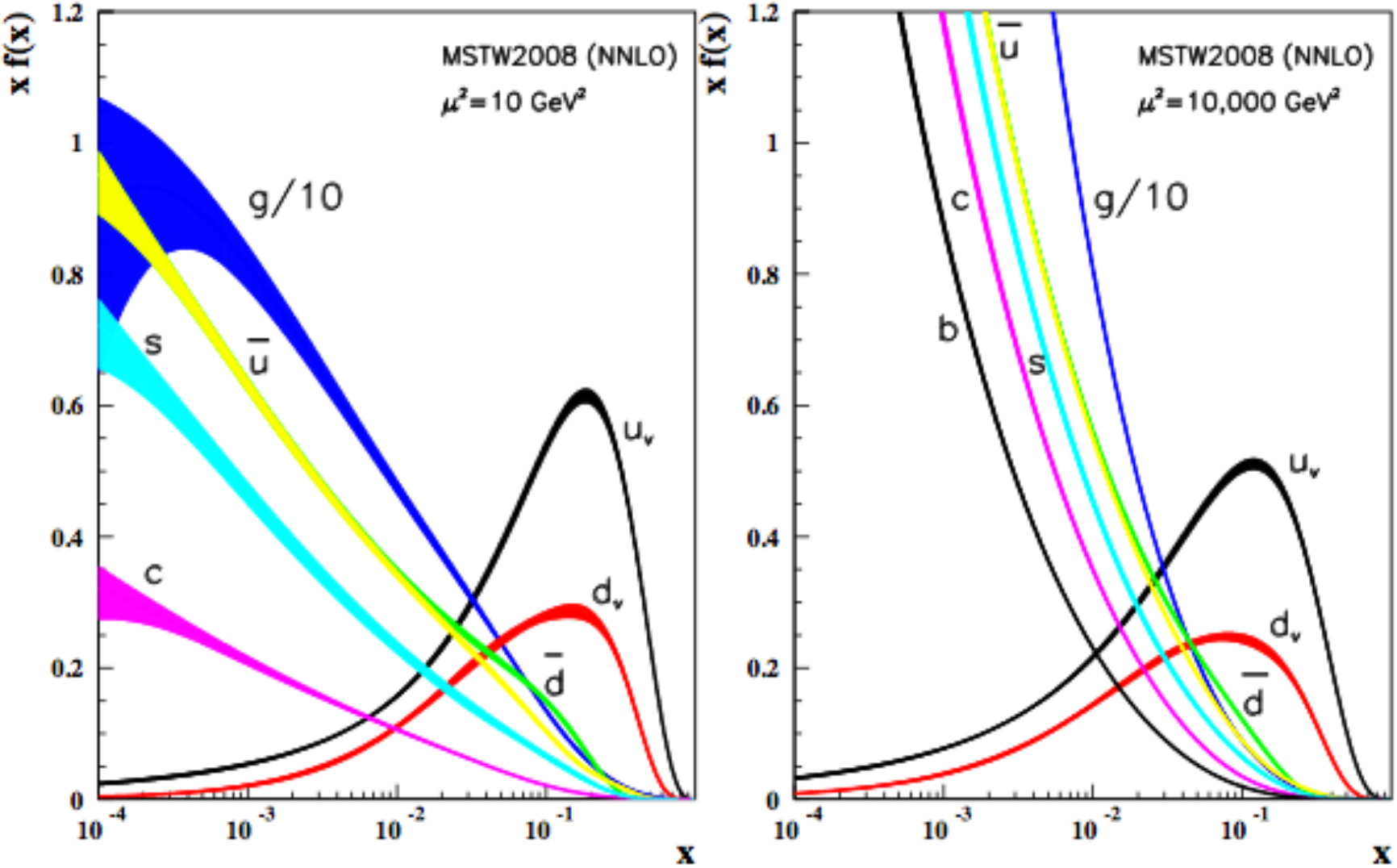
Quark and antiquark distributions can be determined from F_2 and F_3



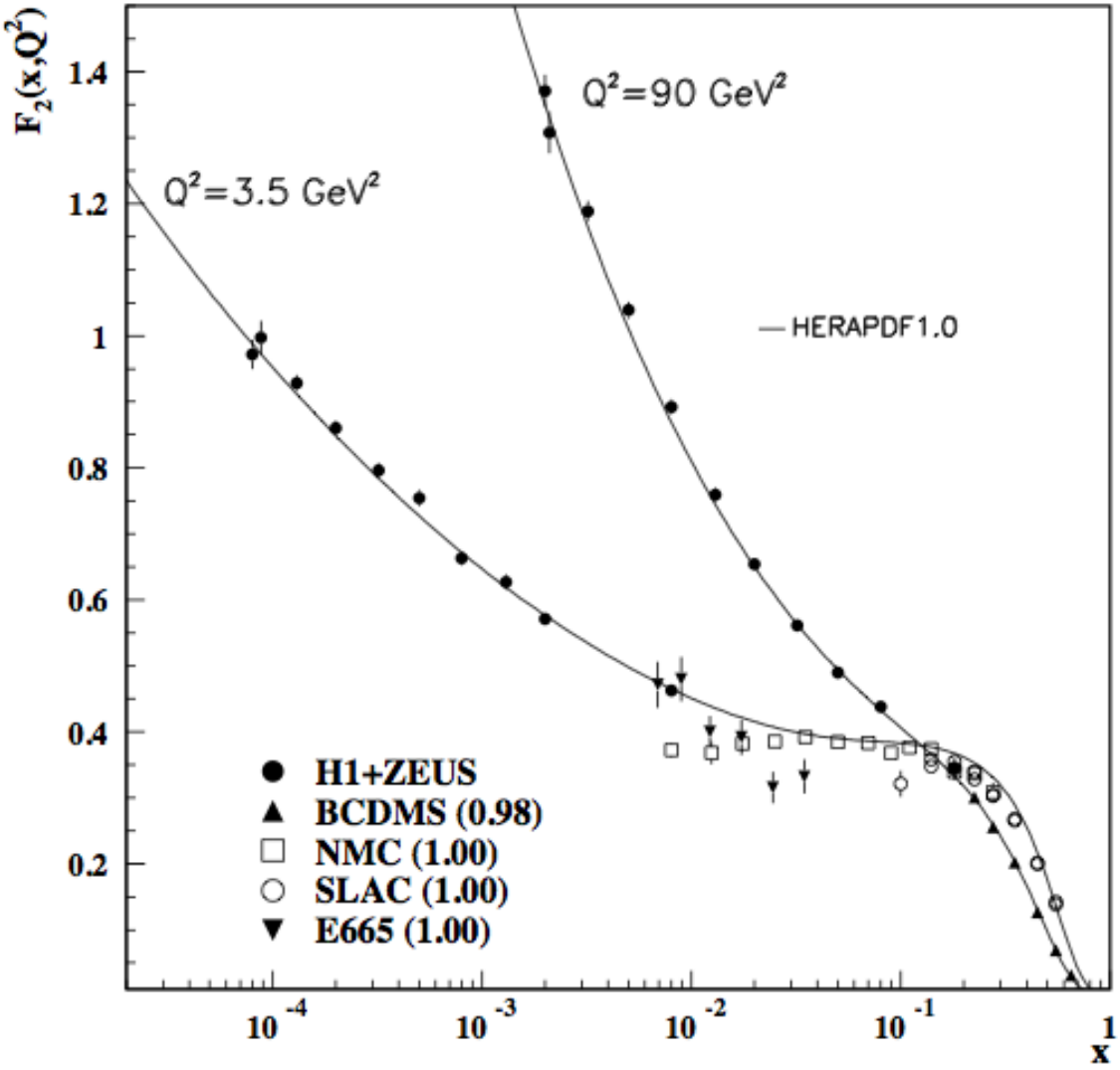
Quark distributions within the nucleon



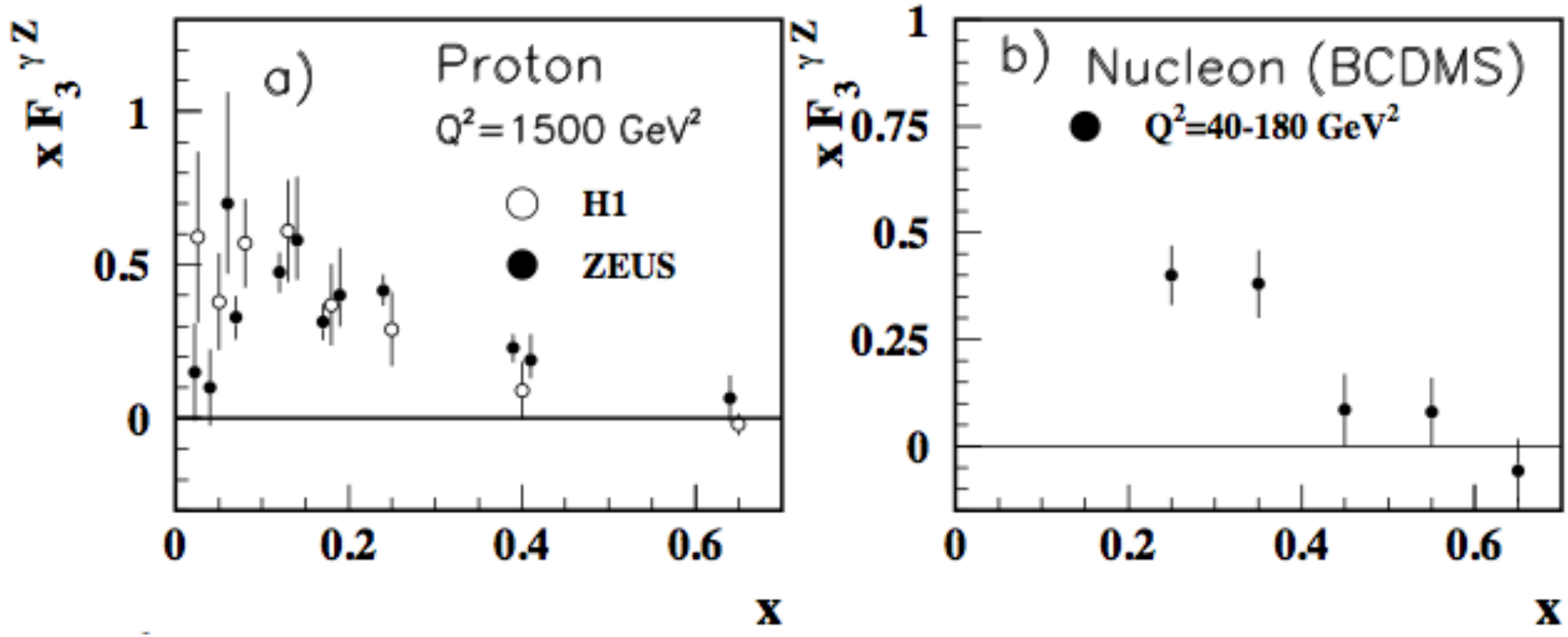
Quark distributions within the nucleon



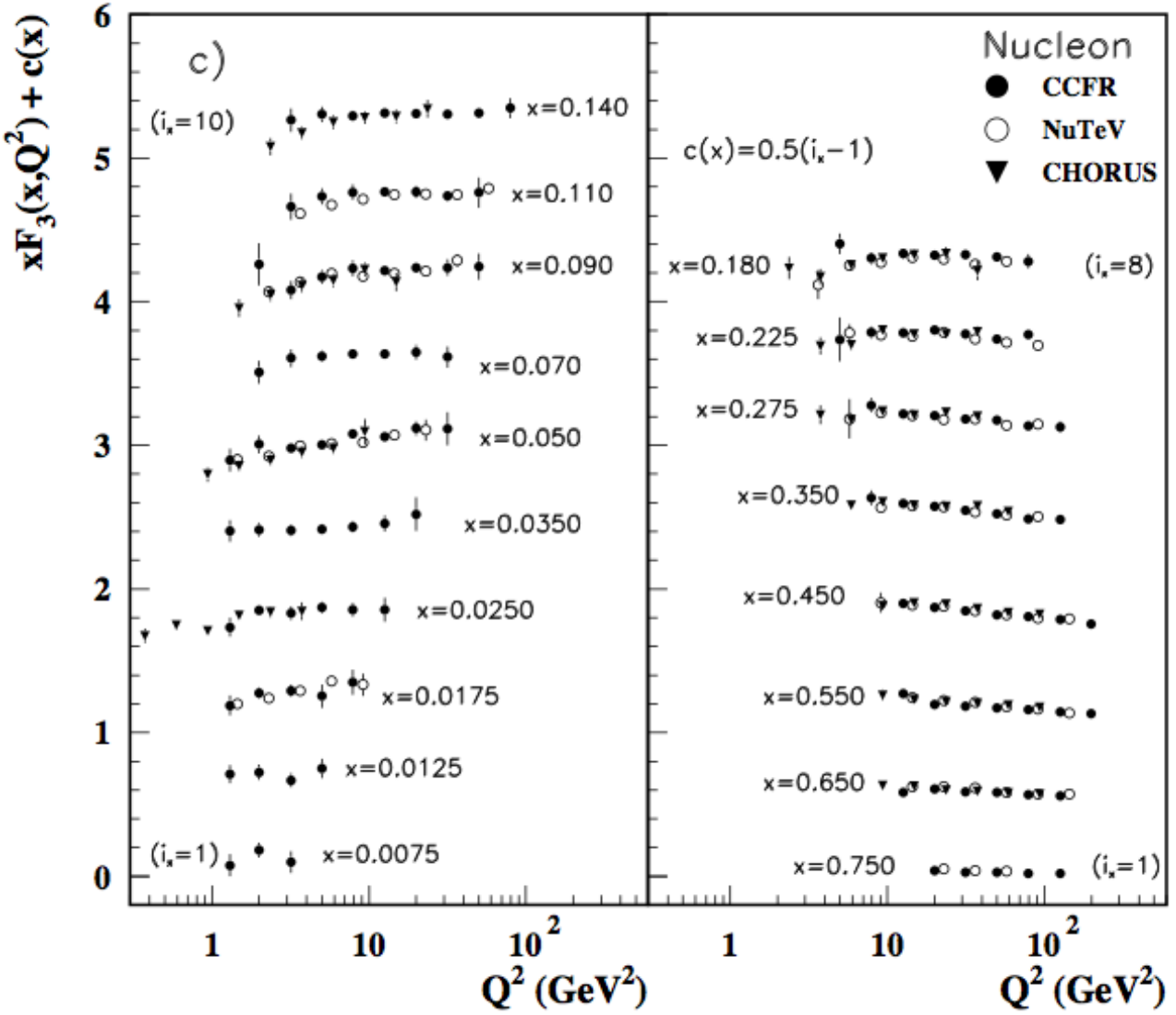
Quark distributions within the nucleon



Quark distributions within the nucleon



Quark distributions within the nucleon



Lepton and Quark Scattering

- $e^+ e^- \rightarrow \mu^+ \mu^-$
- $e^+ e^-$ annihilation to hadrons ($e^+ e^- \rightarrow Q\overline{Q}$)
- Electron-muon scattering, $e^- \mu^+ \rightarrow e^- \mu^+$
- Neutrino-electron scattering, $\nu_e e^- \rightarrow \nu_e e^-$
- Elastic lepton-nucleon scattering
- Deep inelastic scattering and partons
- Deep inelastic scattering and quarks
  - Electron-nucleon scattering
  - Neutrino-nucleon scattering
- Quark distributions within the nucleon

Data broadly in agreement with parton model

hadrons built of pointlike, quasi-free constituents with QNs of quarks and anti-quarks & gluons accounting for 50%

Deviations associated with interactions between quarks