Particle Physics Phenomenology

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Electroweak Interactions and the Standard Model

- Divergences in the Weak Interactions
- Introduction of Neutral Currents
- The Weinberg-Salam Model
- Intermediate Boson Masses
- Electroweak Couplings of Leptons and Quarks
- Neutrino Scattering via Z Exchange
- Asymmetries in the Scattering of Polarized Electrons by Deuterons
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- Spontaneous Symmetry Breaking and the Higgs Mechanism
- Higgs Production and Detection

Spontaneous Symmetry Breaking and the Higgs Mechanism

- The Electroweak field Lagrangian terms:
  \[ L = g J_\mu \cdot W_\mu + g' J^Y_\mu B_\mu \]

- The Higgs field Lagrangian terms:
  \[ L = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda \phi^4 \]

- The covariant derivative:
  \[ D_\mu = \partial_\mu - ig \tau \cdot W_\mu - ig' Y B_\mu \]

- The Higgs Lagrangian becomes:
  \[ \mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \]
Spontaneous Symmetry Breaking and the Higgs Mechanism

\[ \mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \]

The Higgs field is now a doublet since \( D_\mu \) is.

\[ \Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \\ \phi^0 \\ \phi^0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{bmatrix} \]

The minimum in the potential occurs at:

\[ \frac{dV}{d(\Phi^\dagger \Phi)} = 0 \quad \Rightarrow \quad \mu^2 + 2\lambda (\Phi^\dagger \Phi) = 0 \]

\[ (\Phi^\dagger \Phi)_{\text{min}} = -\frac{\mu^2}{2\lambda} \]

\[ \Phi^\dagger \Phi = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) \]
Spontaneous Symmetry Breaking and the Higgs Mechanism

\[ \mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \]

\[ \Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \\ \phi_3 + i\phi_4 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{bmatrix} \]

The minimum: \textbf{we choose} \( \phi_1 = \phi_2 = \phi_4 = 0 \)

\[ \Phi_{\text{min}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \nu \end{bmatrix} \]

\[ (\Phi^\dagger \Phi)_{\text{min}} = -\frac{\mu^2}{2\lambda} \equiv \frac{\nu^2}{2} \]

Expand around the minimum:

\[ \Phi = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \nu + H(x) \end{bmatrix} \]

\[ D_\mu \Phi = \begin{bmatrix} \frac{i}{\sqrt{2}} W_\mu^+ (\nu + H) \\ \left( \partial_\mu - \frac{1}{2} (g \cos \theta_w \cos \theta_w + g' \sin \theta_w \sin \theta_w) Z_\mu \right) (\nu + H) \end{bmatrix} \]
Spontaneous Symmetry Breaking and the Higgs Mechanism

\[ \mathcal{L}_\Phi = \left( D_\mu \Phi \right)^\dagger \left( D_\mu \Phi \right) - \mu^2 \Phi^\dagger \Phi - \lambda \left( \Phi^\dagger \Phi \right)^2 \]

\[ = \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{1}{4} g^2 \left( v^2 + 2 v H + H^2 \right) W_\mu^+ W_-^\mu + \frac{1}{8} \left( g^2 + g'^2 \right) \left( v^2 + 2 v H + H^2 \right) Z_\mu Z^\mu \]

\[ - \frac{\mu^2 v^2}{4} + \mu^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \]

Identify the Z and W mass terms:

\[ \frac{1}{2} m_Z^2 Z_\mu Z^\mu \quad \Rightarrow \quad m_Z = \frac{1}{2} \left( g^2 + g'^2 \right)^{1/2} v \equiv \frac{1}{2} \frac{g}{\cos \theta_W} v \]

\[ m_W^2 W_\mu^+ W_-^\mu \quad \Rightarrow \quad m_W = \frac{1}{2} g v \quad \left( \Rightarrow m_Z = \frac{m_W}{\cos \theta_W} \right) \]

\[ - \frac{1}{2} m_H^2 H^2 \quad \Rightarrow \quad m_H = \sqrt{-2 \mu^2} \]
Spontaneous Symmetry Breaking and the Higgs Mechanism

\[ \frac{1}{2}(gv/2)^2 W_\mu^+W_\mu^- \implies M_W = (gv/2) \]
\[ \frac{1}{8}(v)^2 (g^2+g'^2) Z_\mu^2 \implies M_Z = \frac{1}{2} v \sqrt{g^2+g'^2} \]
\[ 0 \ A_\mu^2 \implies M_\gamma = 0 \]

\( v = 246 \text{ GeV} \)

Homework assignment: show \( M_W \) and \( M_Z \) yield this
Spontaneous Symmetry Breaking and the Higgs Mechanism

- The Higgs mechanism also endows the fermions with mass
- The full Lagrangian has terms coupling all the fermions to the Higgs field

\[ L = m_f \bar{e} e - \frac{m_f \bar{e} e H}{v} \]

Yukawa coupling

- electron: \[ \frac{m_e}{v} = 2 \times 10^{-6} \]
- top quark: \[ \frac{m_t}{v} = 0.7 \]

The size of the top quark mass seems much more natural than the mass of the lighter fermions given the value of \( v = 246 \, \text{GeV} \)
Higgs Couplings
Higgs Production and Detection

• The isospin doublet of scalar Higgs particles in the minimal Standard Model yields one real particle to be found.

• Four real components of the new fields are reduced to one when three are “eaten” by the massless W and Z to produce W and Z mass.

\[
\begin{pmatrix}
\phi^+ \\
\phi^0
\end{pmatrix} = \begin{pmatrix}
\frac{(\phi_1 + i\phi_2)}{\sqrt{2}} \\
\frac{(\phi_3 + i\phi_4)}{\sqrt{2}}
\end{pmatrix}
\]

\[
\begin{array}{ccc}
I & Y = Q - I_3 \\
+\frac{1}{2} & +\frac{1}{2} \\
-\frac{1}{2} & +\frac{1}{2}
\end{array}
\]

• The mass of the remaining physical neutral boson is unknown.

• Limits on the mass can be determined.
Higgs Production and Detection

- Upper limit on the Higgs mass

  consider \( \Gamma_H \sim G M_H^3 \)

- The Higgs must be weakly coupled \( \Rightarrow \Gamma_H < M_H \)

  \[ M_H < G^{-1/2} < (10^5 \text{ GeV})^{1/2} \approx 300 \text{ GeV} \]

- We also have a unitarity limit on WW scattering

  \[ M_H < (8\pi \sqrt{2/3})^{1/2} G^{-1/2} \approx 1 \text{ TeV} \]
Higgs Production and Detection

- The Higgs boson might be produced in an electron-positron collider

\[ e^+ e^- \rightarrow H^0 Z^0 \]
\[ H^0 \rightarrow b \bar{b}, \tau \bar{\tau}, \ldots \]
\[ Z^0 \rightarrow Q \bar{Q}, l \bar{l}, \nu \bar{\nu} \]

- The Higgs couples proportionally to mass, so it should decay preferentially to the heaviest possible quark or lepton

- In an electron-positron collider, the Higgs signal would show up dramatically recoiling from a Z decay
Higgs Production and Detection

LEP results

$M_H > 114 \text{ GeV}/c^2$ (95% CL)
Higgs Production and Detection

- With LEP now off, the Higgs search proceeds
  - Fermilab TeVatron Collider
  - Large Hadron Collider at CERN
  - future Linear Collider

- Hadron colliders
Hadron Collider Cross sections

\[ \sigma(pp \rightarrow H + X) \ [pb] \]
\[ \sqrt{s} = 14 \text{ TeV} \]
\[ M_t = 175 \text{ GeV} \]
CTEQ4M

\[ gg \rightarrow H \]
\[ qq \rightarrow HW \]
\[ qq \rightarrow Hqq \]
\[ gg, qq \rightarrow Htt \]
\[ gg, qq \rightarrow Hbb \]
\[ qq \rightarrow HZ \]
Light Scalars Are Unnatural

- Higgs mass grows with cut-off, $\Lambda$

\[
\delta M_h^2 = \frac{G_F}{4\sqrt{2}\pi^2} \Lambda^2 \left( 6M_W^2 + 3M_Z^2 + M_h^2 - 12M_t^2 \right)
\]

\[
= -\left( \frac{\Lambda}{0.7\text{TeV}} \right)^2 \left( 200\text{GeV} \right)
\]

\[M_h \leq 200 \text{ GeV} \text{ requires large cancellations}\]
SUSY...Our favorite model*

- Quadratic divergences cancelled automatically if SUSY particles at TeV scale
- Cancellation result of *supersymmetry*, so happens at every order

\[ \delta M^2_h \approx (\ldots) G_F \Lambda^2 (M_t^2 - M_{\tilde{t}}^2) \]

*Spires: 7421 papers after 1990 with title supersymmetry or supersymmetric!
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