Particle Physics

NOTE - I am having trouble with making PDF files, so some images on this file are corrupted. - JEB

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Particle Physics and Cosmology

- Hubble’s law and the expanding universe ✓
- Cosmological Models ✓
- Friedmann equation ✓
- Cosmic microwave radiation: the hot Big Bang
- Radiation and matter eras
- Nucleosynthesis in the Big Bang
- Baryon-antibaryon asymmetry
- Dark matter
- Inflation

  Bergstrom and Goobar, Cosmology and Particle Astrophysics
Particle Physics and Cosmology

- The ‘Standard Model’ of the early universe is supported by the following understood pieces of observational evidence
  - Hubble’s Law
  - the cosmic microwave background radiation
  - the cosmic abundances of the light elements
  - anisotropies in the background radiation
Four Evidences for the Big Bang

Hubble’s Law

Anisotropies in CMB

Cosmic Microwave Background (T=2.7K)

Light Element Abundances

1 Mpc = 3.1 \times 10^{19} \text{ km} = 3.26 \times 10^6 \text{ light-years}
Friedmann Equation

• Einstein’s equations can be solved for a homogeneous and isotropic Universe to yield the temporal development of the Universe, known as the Friedmann equation

\[ H^2 = \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N \rho}{3} - \frac{K c^2}{R^2} + \frac{\Lambda}{3} \]

• Note, to close universe (K=0) for a null cosmological constant (\(\Lambda=0\)), we require:

\[ H^2 = \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N \rho}{3} \]
Friedmann Equation

critical density ($\rho_c$)

- What matter density will just close the Universe (for $\Lambda=0$)?

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N \rho}{3} \quad \text{then } K=0$$

$$\rho_c = \frac{3}{8\pi G_N} H_0^2 = 1.88 \times 10^{-26} h_0^2 \text{ kg m}^{-3}$$

- The actual density is related to the critical density

$$\Omega \equiv \frac{\rho}{\rho_c} = 1 + \frac{K c^2}{H^2 R^2}$$

- so we have they relations

\[
\begin{align*}
K = 0 & \quad \Omega = 1 \\
K = 1 & \quad \Omega > 1 \\
K = -1 & \quad \Omega < 1
\end{align*}
\]
Friedmann Equation

**actual density**

- **Visible matter**
  \[ \rho_{\text{lum}} \simeq 2 \times 10^{-29} \text{ kg m}^{-3} \quad \Omega_{\text{lum}} \simeq 0.003 h_0^{-2} \]

- **Density of baryons (most invisible) inferred form baryogenisis**
  \[ \rho_{\text{baryon}} = (3 \pm 1.5) \times 10^{-28} \text{ kg m}^{-3} \quad \Omega_{\text{baryon}} \simeq (0.01-0.03) h_0^{-2} \]

- **Total matter density (from galactic rotation curves)**
  \[ \rho_m \geq 5 \times 10^{-27} \text{ kg m}^{-3} \quad \Omega_m \geq 0.3 \]

- **Conclusions**
  - most baryonic matter is non-luminous and most matter is non-baryonic
Cosmic Microwave Radiation: the Hot Big Bang

- Even though matter now dominates the Universe, earlier radiation would have dominated
  \[ \rho_m \propto R^{-3} \]
  \[ \rho_r \propto R^{-4} \text{, due to the redshift of the radiation} \]

- Early Universe, radiation dominant period
  \[
  \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N \rho}{3} - \frac{K c^2}{R^2} + \frac{\Lambda}{3}
  \]
  the energy density term will dominate over the \( K \) and \( \Lambda \) terms
  \[
  \dot{R}^2 = \frac{8\pi G_N}{3} \rho_r R^2
  \]
  \[
  \frac{\dot{\rho}_r}{\rho_r} = -4 \frac{\dot{R}}{R} = -4 \left( \frac{8\pi G_N \rho_r}{3} \right)^{1/2}
  \]
Cosmic Microwave Radiation: the Hot Big Bang

\[ \frac{\dot{\rho}_r}{\rho_r} = -4 \frac{\dot{R}}{R} = -4 \left( \frac{8\pi G_N \rho_r}{3} \right)^{1/2} \]

integrating:

\[ \rho_r = \left( \frac{3c^2}{32\pi G_N} \right) \frac{1}{t^2} \]

a photon gas in equilibrium:

\[ \rho_r = aT^4 = \frac{4\sigma}{c} T^4 = \left( \frac{\pi^4}{15} \right) \frac{(kT)^4}{\pi^2 h^3 c^3} \]

where the Stefan-Boltzmann constant is \( \sigma = \pi^2 k^4/60 \hbar^3 c^2 \)

leading to

\[ kT = \left( \frac{45\hbar^3 c^5}{32\pi^3 G_N} \right)^{1/4} \frac{1}{t^{1/2}} \approx \frac{1 \text{ MeV}}{t^{1/2}} \quad \text{or} \quad T \approx \frac{10^{10} \text{ K}}{t^{1/2}} \]

(\( t \) in seconds)

for \( t_0 \sim 10^{10} \) years \( \sim 3 \times 10^{17} \) seconds \( \quad T \approx 10^{10} \text{ K} / 5 \times 10^8 \sim 20 \text{ K} \)
Cosmic Microwave Radiation: the Hot Big Bang

- The 20 K estimate is expected to be an overestimate since the radiation will cool more quickly during the matter dominated era.

- Since $\rho_m \propto R^{-3}$ (while $\rho_r \propto R^{-4}$), and the universe expands as $t^{2/3}$ during the matter dominated era, $kT$ of the radiation goes like $t^{-2/3}$ during that era.
Vacuum Dominated Era

- We now have discovered we reside in a vacuum dominated universe (dark energy)

\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N \rho}{3} - \frac{K c^2}{R^2} + \frac{\Lambda}{3}
\]

- During this era:

\[R(t) \propto e^{\sqrt{\Lambda/3} t}\]

- Exponential Expansion!!
Cosmic Microwave Radiation: the Hot Big Bang

- Energy density of the photons

\[ N(E)dp = \frac{p^2dp}{\pi^2 \hbar^3 \left[ \exp(E/kT) - 1 \right]} \left( \frac{8\gamma}{2} \right) \]

\[ N_\gamma = \frac{2.404}{\pi^2} \left( \frac{kT}{\hbar c} \right)^3 = 410.9 \left( \frac{T}{2.726} \right)^3 = 411 \text{ cm}^{-3} \]

\[ \rho_r = 0.261 \text{ MeV m}^{-3} \]

or

\[ \rho_r/c^2 = 4.65 \times 10^{-31} \text{ kg m}^{-3} \]

which is 4 orders of magnitude below the matter density
Cosmic Microwave Radiation: the Hot Big Bang

- Energy density of the fermions
  - we must consider the contribution of fermions to the energy density
    \[ N(E)dp = \frac{p^2 dp}{\pi^2 \hbar^3 \left[ \exp(E/kT) + 1 \right]} \left( \frac{g_f}{2} \right) \]

- In the relativistic limit, \( kT \gg mc^2 \)
  \[ \rho_f = \frac{7}{8} \left( \frac{\pi^4}{15} \right) \frac{(kT)^4}{\pi^2 \hbar^3 c^3} \left( \frac{g_f}{2} \right) \]

- so \( \rho_r + \rho_f = \left( \frac{\pi^4}{15} \right) \frac{(kT)^4}{\pi^2 \hbar^3 c^3} \frac{1}{2} \left( g_\gamma + \frac{7}{8} \sum g_f \right) \)
Cosmic Microwave Radiation: the Hot Big Bang

- All the fermions and bosons of the Standard Model would have been created in the Big Bang, as long as the temperature were high enough.
- As expansion lowered the temperature, heavy particles were lost through decay.
- When the temperature fell below about 100 MeV, even the relatively light antinucleons annihilated away, leaving a residue of nucleons.
- Today, the relativistic fermions contributing to the energy density are the electron and the three neutrinos, and their antiparticles. Therefore $\Sigma g_f = 4 + 2 + 2 + 2 = 10$,

$$N = \frac{1}{2} \left( g_\gamma + \frac{7}{8} \sum g_f \right) = 43/8$$

- so $\rho_r + \rho_f = \frac{43}{8} \left( \frac{\pi^4}{15} \right) \frac{(kT)^4}{\pi^2 \hbar^3 c^3}$ and $T \approx \frac{6.6 \times 10^9 \text{K}}{t^{1/2}}$
We can quantify these fluctuations.
Cosmic Microwave Radiation: the Hot Big Bang

The Moments

\[
\frac{\Delta T(\theta, \phi)}{T} = \sum_{\ell_m} a_{\ell m} Y_{\ell m}(\theta, \phi). \\
C_l = \frac{1}{2l+1} \sum_m |a_{lm}|^2
\]

Cosmic microwave background radiation
Cosmic Microwave Background

Acoustic peaks

Compression

Rarefaction
WMAP fits

WMAP only (TT+TE), flat LCDM (Spergel et al. 2003)

CMB appears to be Gaussian. (Komatsu et al.)

• 15% of CMB was re-scattered in a reionized universe.

• The estimated reionization redshift ~20, or 200 million years after the Big-Bang.

Flat LCDM still fits:

6 parameters fit 1348 points

\[ \frac{\chi^2}{\nu} = \frac{1431}{1342} = 1.07 \]

DM density \((2.25 \pm 0.38) \times 10^{-27} \text{Kg/m}^3\)

atomic density \((2.7 \pm 0.1) \times 10^{-7} \text{ cm}^{-3}\)

\[ \eta = (6.5^{+0.4}_{-0.3}) \times 10^{-10} \]

Age at decoupling \(372 \pm 14 \text{ Kyr}\)

\(\sigma_8 = 0.9 \pm 0.1\)

\(z_{\text{reion}} \approx 17\)

Fits not only the CMB but also a host of other cosmological observables.
More on the Expansion of the Universe

Type Ia SN to measure the expansion rate.

Expansion rate appears to be increasing
Fluctuations measured by WMAP are consistent with $\Omega_\Lambda + \Omega_m = 1.0$. 

Fluctuations and a Flat Universe
Implications of expansion rate

Supernova Cosmology Project
Perlmutter et al. (1998)

- No Big Bang
- 99%
- 95%
- 68%
- 42 Supernovae

Target Uncertainty

- Flat Universe $\Lambda = 0$
- expands forever
- recollapses eventually

mass density

vacuum energy density (cosmological constant)
We (and all of chemistry) are a small minority in the Universe.
Baryonic Matter

• Dominated by hydrogen and helium
  - helium abundance (~24%) too large to have been produced in stars

• \( \eta_B = \frac{n_B}{n_\gamma} \approx 10^{-9} \)
  - reveals early universe matter/antimatter asymmetry

• abundances of light elements consistent with theory
  - constrains number of neutrino species
Constrained number of neutrino species
Radiation and Matter Eras

• Very early, when $kT \ll Mc^2$ for all masses, all these particles would have been in thermal equilibrium and present in comparable numbers

• Thermal equilibrium requires the collision time be short compared to the age of the Universe (many collisions could occur)

$$W = \langle N \nu \sigma \rangle \gg t^{-1}$$

• Particle numbers will fall below the equilibrium number when
  - Temperature falls below the threshold energy
    for example when $kT \ll M_p c^2 \gamma \gamma \rightarrow p \bar{p}$
  - production cross-section is too small to sustain reaction rate
    this is the case for $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$ (weak interaction) when $kT < 3$ MeV ($t > 10^{-2}$ s)
    (this means the neutrino fireball decouples from matter and expands independently)
Radiation and Matter Eras

- For 100,000 years, matter (protons, electrons, hydrogen) were in equilibrium with the photons:
  \[ e^- + p \rightleftharpoons \gamma + H + Q \quad Q = 13.6 \text{ eV} \]

- At temperature $T$, the mean photon energy is $2.7kT$
  for $kT = 5 \text{ eV}$, $\langle E_\gamma \rangle \sim Q = 13.6 \text{ eV}$

- Since the photons outnumber the hydrogen by a billion to one, the process above will continue for much lower temperatures, as the tail of the photon energy spectrum has enough energy for the reaction.

- At $kT \sim 0.3 \text{ eV}$, the process turns off, and the matter become transparent to the photons (decoupling at $t \sim 10^{13} \text{ s} = 3 \times 10^5 \text{ yr.}$)
Radiation and Matter Eras

- Early ($t < 10^{13}$ sec)
  - radiation dominates
  - $kT \sim t^{-1/2}$

- Later ($t > 10^{13}$ sec)
  - radiation decouples

- Even later ($t > 3 \times 10^{13}$ sec)
  - matter density equals radiation density
  - from then on, matter dominates, since
    - $\rho_m \propto R^{-3}$
    - while $\rho_r \propto R^{-4}$

- Except, we now have discovered that today the dark energy is dominating
Relative abundances

\( \eta_{10} = \eta \times 10^{10} \)
\( = n_B / n_\gamma \times 10^{10} \)
\( \approx 6.1 \) (WMAP)
(baryon to photon ratio)
Nucleosynthesis in the Big Bang

- After \( t \sim 1 \) second, heavy particles were gone, and remaining particles were:
  - photons
  - leptons
  - neutrons
  - protons
- Numbers of protons and neutrons were determined by the weak processes
  \[
  \nu_e + n \rightleftharpoons e^- + p \\
  \bar{\nu}_e + p \rightleftharpoons e^+ + n \\
  n \rightarrow p + e^- + \bar{\nu}_e
  \]
- For \( kT < M_p c^2 \), the equilibrium ratio of \( n/p \) is
  \[
  \frac{N_n}{N_p} = \exp \left( \frac{-Q}{kT} \right), \quad Q = (M_n - M_p)c^2 = 1.293 \text{ MeV}
  \]
Nucleosynthesis in the Big Bang

- As the Universe cools, the weak reaction rate becomes too slow to maintain equilibrium.
- This happens when $kT = 0.87$ MeV

$$\frac{N_n(0)}{N_p(0)} = \exp\left(-\frac{Q}{kT}\right) = 0.23$$

- Then neutrons disappear through decay

- and the ratio of neutrons to protons decreases

$$\frac{N_n(t)}{N_p(t)} = \frac{0.23e^{-t/\tau}}{1.23 - 0.23e^{-t/\tau}} \quad \text{where} \quad \tau = 896 \pm 10 \text{ sec}$$
Nucleosynthesis in the Big Bang

- Nucleosynthesis locks the neutrons into nuclei before they can decay away

- First the formation of deuterium comes into equilibrium

\[ n + p \rightleftharpoons ^2\text{H} + \gamma + Q \quad Q = 2.22 \text{ MeV} \]

- this lasts until \( kT \sim Q/40 = 0.05 \text{ MeV} \) since there are so many more photons than nucleons

- Next, helium production takes over

\[
\begin{align*}
^2\text{H} + n & \rightarrow ^3\text{H} + \gamma \\
^3\text{H} + p & \rightarrow ^4\text{He} + \gamma \\
^2\text{H} + p & \rightarrow ^3\text{He} + \gamma \\
^3\text{He} + n & \rightarrow ^4\text{He} + \gamma
\end{align*}
\]

Neutron decay ceases
For \( kT = 0.05 \text{ MeV} \), \( t \sim 400 \text{ s} \)

\[ r = \frac{N_n}{N_p} = 0.14 \]
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- Baryon-antibaryon asymmetry
- Dark matter
- Inflation

Bergstrom and Goobar, Cosmology and Particle Astrophysics