Physics 662

Particle Physics Phenomenology

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Weak Interactions

• Classification
• Lepton Universality
• Nuclear $\beta$-decay: Fermi theory
• Inverse $\beta$-decay: neutrino interactions
• Parity nonconservation in $\beta$-decay
• Helicity of the neutrino
• The V-A interaction
• Conservation of weak currents
• The weak boson and Fermi couplings
• Pion and Muon decay
• Neutral weak currents
• Observation of $W^\pm$ and $Z^0$ bosons in pp collisions
• $Z^0$ production at $e^+e^-$ colliders
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Weak Interactions

- (continued)
- Weak decays of quarks. The GIM model and the CKM matrix
- Neutral K mesons
- CP violation in the neutral kaon system
- Cosmological CP violation
- $D^0$-$\bar{D}^0$ and $B^0$-$\bar{B}^0$ mixing
Classification

- Weak interactions are mediated by the "intermediate bosons" $W^\pm$ and $Z^0$
- Just as the EM force between two current carrying wires depends on the EM current, the weak interaction is between two weak currents, describing the flow of conserved weak charge, $g$
  \[ j \propto \psi^* \psi \]
- Two types of interactions:
  - CC (charged current)
  - NC (neutral current)
Classification

- Weak interactions occur between all types of leptons and quarks, but are often hidden by the stronger EM and strong interactions.

- Semi-leptonic
  
- Leptonic

- Non-leptonic
Lepton universality

- Unit of weak charge
  - all the leptons carry the same weak charge and therefore couple to the $W^\pm$ with the same strength
  - The quarks DO NOT carry the same unit of weak charge

- Muon decay

\[
\Gamma (\mu \rightarrow e\nu_e\bar{\nu}_\mu) = \frac{1}{\tau} \propto G^2 m_\mu^5
\]

\[
= \frac{G^2 m_\mu^5}{192\pi^3}
\]

- experimental: \( \tau_\mu = 2.197 \times 10^{-6} \text{ sec} \)
Lepton universality

- Tau decay

\[
\Gamma ( \tau \rightarrow e \nu_e \bar{\nu}_\tau ) = B(\tau \rightarrow e \nu \nu) \frac{1}{\tau} \propto G^2 m^5 \tau
\]

\[
= \frac{G^2 m^5}{192\pi^3}
\]

- \( B(\tau \rightarrow e \nu \nu) = 17.80 \pm 0.06\% \)

- Test universality:

\[
\left( \frac{g_\tau}{g_\mu} \right)^4 = B(\tau \rightarrow e \nu_e \bar{\nu}_\tau) \left( \frac{m_\mu}{m_\tau} \right)^5 \left( \frac{\tau_\mu}{\tau_\tau} \right)
\]

Physics 662, lecture 3
Lepton universality

- Test universality:

\[
\left( \frac{g_\tau}{g_\mu} \right)^4 = B(\tau \rightarrow e\nu_e\bar{\nu}_\tau) \left( \frac{m_\mu}{m_\tau} \right)^5 \left( \frac{\tau_\mu}{\tau_\tau} \right)
\]

With \( \tau_\mu = 2.197 \times 10^{-6} \text{ s}, \tau_\tau = (291.0 \pm 1.5) \times 10^{-15} \text{ s}, m_\mu = 105.658 \text{ MeV} \)
\( m_\tau = 1777.0 \text{ MeV} \) we have \( B(\tau \rightarrow e\nu\nu) = 17.80 \pm 0.06\% \)

\[
\frac{g_\tau}{g_\mu} = 0.999 \pm 0.003
\]

\[
\frac{g_\mu}{g_e} = 1.001 \pm 0.004
\]
Lepton universality

- Lepton universality also holds for the $Z$ couplings:

\[ Z^0 \rightarrow e^+e^- : \mu^+\mu^- : \tau^+\tau^- = 1 : 1.000 \pm 0.004 : 0.999 \pm 0.005 \]

- From the muon lifetime we can compute the Fermi constant, $G$:

\[ G/(\hbar c)^3 = 1.1664 \times 10^{-5} \text{ GeV}^{-2} \]
Nuclear $\beta$-decay: Fermi theory

- The decay of the neutron provides the prototype weak interaction

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

- The quark level interaction is

\[ d \rightarrow u + e^- + \bar{\nu}_e \]
Nuclear $\beta$-decay: Fermi theory

- Since $q^2 \ll M_W$, interaction is effectively pointlike

$$ W = \frac{2\pi}{\hbar} G^2 |M|^2 \frac{dN}{dE_0} $$

(Fermi's Golden Rule)

- $|M|^2 \approx 1$ if $J(\text{leptons}) = 0$ (Fermi transitions)
- $|M|^2 \approx 3$ if $J(\text{leptons}) = 1$ (Gamow-Teller transitions)

- $dN/dE$ is the density of states
Nuclear $\beta$-decay: Fermi theory

- **Density of states**
  - number of ways of sharing the available energy in the range $E_0 \rightarrow E_0 + dE_0$

- Consider the neutron decay

\[
P + q + p = 0
\]

\[
T + E_\nu + E = E_0
\]
Nuclear $\beta$-decay: Fermi theory

- $m_v \approx 0$, so $E_v \approx qc$
- $E_0 \approx 1$ MeV (very approx.), so $Pc \approx 1$ MeV
- $T = P^2 c^2 / (2Mc^2) = P^2 / (2M) \approx 10^{-3}$ MeV, which is negligible
- Energy, $E_0$, is shared entirely between electron and neutrino:
  \[ qc = E_0 - E \]
- Number of states available to an electron, in volume $V$, in momentum range $(p, p+dp)$ inside solid angle $d\Omega$:
  \[ \frac{V d\Omega}{(2\pi)^3 \hbar^3} \]

- Electron phase space factor is \[ \frac{4\pi p^2 dp}{(2\pi)^3 \hbar^3} \]
- and neutrino \[ \frac{4\pi q^2 dq}{(2\pi)^3 \hbar^3} \]
Nuclear $\beta$-decay: Fermi theory

- The proton momentum $P$ is fixed, once the electron and neutrino is specified (there is no more freedom):

$$P = -(p + q)$$

- so the number of final states is:

$$dN = \frac{(4\pi)^2}{(2\pi)^6\hbar^6} p^2q^2dpdq$$

- for a given $p$ and $E$ of the electron, the neutrino momentum is fixed $q = (E_0 - E)/c$ in the range $dq = dE_0/c$

- Hence:

$$\frac{dN}{dE_0} = \frac{1}{4\pi^4\hbar^6c^3} p^2(E_0 - E)^2dp$$
Nuclear $\beta$-decay: Fermi theory

\[ \frac{dN}{dE_0} = \frac{1}{4\pi^4\hbar^6c^3}p^2(E_0 - E)^2dp \]

- This expression gives the electron spectrum, since the matrix element $|M|^2$ is a constant when integrated over angle

\[ N(p)dp \propto p^2(E_0 - E)^2dp \]

$^3\text{H} \rightarrow ^3\text{He} + e^- + \overline{\nu}_e$

Langer and Moffatt (1952)
Nuclear β-decay: Fermi theory

- For a non-zero neutrino mass, the expression has a simple change

\[ N(p)dp \propto p^2(E_0 - E)^2 dp \]

\[ N(p)dp \propto p^2(E_0 - E)^2 \sqrt{1 - \left( \frac{m_\nu c^2}{E_0 - E} \right)^2} dp \]

\[ ^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e \]

Langer and Moffatt (1952)

Limit on electron neutrino mass (< 3eV)
**Inverse $\beta$-decay: neutrino interactions**

- The physical existence of the neutrino required a demonstration of its interaction
  
  \[ \bar{\nu}_e + p \rightarrow n + e^+ \]

  \[ \sigma(\bar{\nu}_e + p \rightarrow n + e^+) = \frac{G^2}{\pi} |M|^2 \frac{p^2}{v_i v_f} \]

  \[ \sigma(\bar{\nu}_e + p \rightarrow e^+ + n) \simeq 10^{-43} E^2 \text{ cm}^2 \quad (E \text{ in MeV}) \]

- Mean free path of a 1 MeV antineutrino is 50 light years of water!
Inverse $\beta$-decay: neutrino interactions

- Neutrinos from 1000 MW reactor core
- Flux $\sim 10^{13}$ cm$^{-2}$s$^{-1}$
- Target:
  - $\text{CdCl}_2$ + water
- Few events observed per hour

Reines and Cowan (1956)

- Events identified by sequence of pulses in scintillator separated by $\mu$sconds
  - Prompt pulse ($10^{-9}$s)
    - positron annihilates to gamma rays
  - Delayed pulse ($10^{-6}$s)
    - neutrons moderate and are radiatively captured by cadmium
Parity nonconservation in $\beta$-decay

- 1956 - Lee and Yang conclude weak interactions violate parity conservation (invariance under spatial inversion)
  \[ K^+ \rightarrow 2\pi \quad \text{(final state has even parity)} \]
  \[ K^+ \rightarrow 3\pi \quad \text{(final state has odd parity)} \]

- 1957 - Wu et al test this
  - $^{60}\text{Co}$ at 0.01K inside solenoid
  - high proportion of spin 5 cobalt nuclei are aligned with field
  \[ ^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \bar{\nu}_e \]

\[
I(\theta) = 1 + \alpha \frac{\sigma \cdot p}{E} = 1 + \alpha \frac{v}{c} \cos \theta
\]

experiment showed $\alpha = -1$
Parity nonconservation in β-decay

\[ I(\theta) = 1 + \alpha \frac{\sigma \cdot p}{E} \]

- The fore-aft asymmetry implies parity is violated
  - spin (\(\sigma\)) does not change sign under reflection
  - momentum (\(p\)) changes sign under reflection
  - therefore \(\sigma \cdot p\) changes sign under reflection

- If parity were conserved, the experiment would find \(\alpha = 0\)

\[ \alpha = \begin{cases} +1 & \text{for } e^+, \text{ thus } P = +v/c \\ -1 & \text{for } e^-, \text{ thus } P = -v/c \end{cases} \]

where \(P = \alpha \frac{v}{c}\)
Parity nonconservation in $\beta$-decay

$P = \alpha \frac{v}{c}$
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