Physics 662
Problem Set 2

Due: February 6, 2014

1. Using the expressions for the Z^0-fermion couplings, estimate the total width of the Z^0 from the following expression
$$\Gamma_i(Z^0 \to f_i\overline{f}_i) = \frac{GM_Z^3\rho}{6\pi\sqrt{2}}(c_A^2 + c_V^2)F,$$
assuming $\rho = 1$, $M_Z = 91.2$ GeV and $\sin^2\theta_W = 0.23$. The quantity F is 1 for $Z^0 \to \nu\overline{\nu}$, $(1 + 3\alpha/4\pi)$ for $Z^0 \to \ell\overline{\ell}$, and $3(1 + \alpha_S/\pi)$ for $Z^0 \to Q\overline{Q}$.

2. The neutral-current cross-section for neutrino scattering by nucleons via the quark constituents is readily obtained from
$$\frac{d\sigma}{dy}(\nu_e e \to \nu_e e)|_{NC} = \frac{G^2 s}{\pi}[g_L^2 + g_R^2(1-y)^2]$$
$$\frac{d\sigma}{dy}(\overline{\nu}_e e \to \overline{\nu}_e e)|_{NC} = \frac{G^2 s}{\pi}[g_R^2 + g_L^2(1-y)^2]$$
(where $y = (E_e/E_\nu)$, the fractional energy acquired by the electron) if we replaces s = $2m_e$E by s = 2xME where M is the nucleon mass and x is the momentum fraction carried by the quark, and we use the results of Z^0-fermion couplings for the quarks. Neglecting sea quarks and using the analogous expressions for the charged-current cross-sections for LL $\to$ LL and RL $\to$ RL:
$$\frac{d\sigma}{dy}(\nu_e e \to \nu_e e)|_{CC} = \frac{G^2 s}{\pi}$$
$$\frac{d\sigma}{dy}(\overline{\nu}_e e \to \overline{\nu}_e e)|_{CC} = \frac{G^2 s}{\pi}(1-y)^2$$
show that the ratios of neutral to charged-current cross-sections on nucleons are
$$\frac{\sigma^{\nu N}(NC)}{\sigma^{\nu N}(CC)} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$$
and
$$\frac{\sigma^{\overline{\nu} N}(NC)}{\sigma^{\overline{\nu} N}(CC)} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{9}\sin^4\theta_W.$$
Assume the nucleons are isoscalar (equal numbers of neutrons and protons).

3. Show that the asymmetry in the scattering of polarized electrons by deuterons measured as a function of y $= (E_0 - E)/E_0$, the fractional energy loss of the electron in the collision, is

$$\frac{A}{q^2} = \frac{9G}{20\sqrt{2}\pi\alpha}\left\{a_1 + a_2\frac{1-(1-y)^2}{1+(1-y)^2}\right\}$$

where $a_1 = 1 - \frac{20}{9}\sin^2\theta_W$ and $a_2 = 1 - 4\sin^2\theta_W$.

The cross-section results from the sum of the photon exchange and Z^0 exchange amplitudes. In summing these, only the pure photon exchange term and the Z^0 photon interference term will contribute, assuming $q^2 \ll M_Z^2$. Assume also equal numbers of u and d quarks in the deuteron, and neglect antiquarks.

4. The Higgs mechanism predictions for the gauge boson masses are:

$$M_W = \frac{gv}{2} \qquad M_Z = \frac{v}{2}\sqrt{g'^2 + g^2},$$

where v is the vacuum expectation value defining the Electroweak scale.

   (a) From the measured values of M_W and $\sin^2\theta_W$, determine the value of v.

   (b) Repeat (a) using M_Z in place for M_W.

   (c) Comment on the comparison of the results for (a) and (b).

   (d) Electroweak theory says $v = (\sqrt{2}G_F)^{-1/2}$. G_F has been well measured for a long time. Using the well known value of G_F calculate v and compare it to the values found in (a) and (b).

5. (a.) For a Higgs boson of mass $M_h = 125$ GeV, identify the dominant decay mode.

   (b.) For this Higgs boson mass, apply the Higgs theory to determine the ratios of branching fractions of the Higgs to leptons over the branching fractions to quarks for each pair of lepton and quark flavors.

6. (related to Problem 9.6 of Martin and Shaw) In our discussion of the reaction $e^+ + e^- \to \mu^+ + \mu^-$, we completely neglected the Higgs exchange diagram of Figure 9.25 compared with the dominant diagrams of Figures 9.2(a) and (b). Justify this approximation for the case $M_H = 125$ GeV using the results of the previous question to estimate the total cross-section at $E_{CM} = M_Z$, which would arise from Figure 9.25 alone, and by comparing with estimates of the cross-sections, which would arise from Figure 9.2(a) alone and Figure 9.2(b) alone.

7. Consider the asymmetry at $\sqrt{s} = M_{Z^0}$ in the interaction

$$e^+e^- \to hadrons$$

between right- and left-handed electrons: $A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$

   (a.) Show $A_{LR} = \frac{2(1-4\sin^2\theta_W)}{1+(1-4\sin^2\theta_W)^2}$.

   (b.) What value would you expect to measure for this parameter?